2.3 Compare Experimental and Theoretical Probabilities

Student Text Pages
76–85

Suggested Timing
150 min

Tools
- coins
- grid paper
- graphing calculator

Optional
- spreadsheet software
- random number generator

Related Resources
BLM 2-5 Section 2.3 Compare Experimental and Theoretical Probabilities
BLM 2-6 Section 2.3 Achievement Check Rubric
BLM G-1 Grid Paper
BLM T-1 Microsoft® *Excel*

Link to Prerequisite Skills

Students should complete all the questions in the Prerequisite Skills before proceeding with this section.

Warm-Up

1. In a circle graph, how many degrees would represent
  a) a quarter of a circle?
  b) a third of a circle?
  c) 40% of a circle?
2. Consider the numbers: 0, 1, 1, 3, 2, 2, 1, 0.
  a) Find the mean of the numbers.
  b) What percent of the numbers are 0 or 1?

Warm-Up Answers

1. a) 90° **b)** 120° **c)** 144°
2. a) 1.25 **b)** 62.5%

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Pass out dice to the students and have them each roll their die six times. Ask how many rolled a 2 once. Discuss what went wrong (i.e. why they did not all roll 2 once). Collect the number of 2s from the entire class and find the total number of 2s divided by the total number of rolls. Is it exactly $\frac{1}{6}$? Is it close?

Investigate

- For Investigate 1, it may be necessary to fill in the first 2 or 3 rows together as a class to ensure students understand how to find the Average Number of Heads.
- For question 7, as the number of trials increases, students should see that the Average Number of Heads gets close to 5. Be aware that, since this is experimental probability, this trend toward an average of 5 heads might not always occur.
- As an extension, have students combine their data. Suggest working in pairs or groups of three to produce more trials. Or fill in a table similar to the one used, but using the average from 10 students (last entry in column 3) as the 10 entries in column 2.
- As an alternate activity, use technology to simulate tossing the coin.

- In Investigate 2, question 4, the probability of having 2 girls in a family of 3 children is $\frac{3}{8}$. See the tree diagram in Example 2 for an explanation. The simulation with 25 trials should produce a result close to this, roughly 9 times out of 25.
- To reinforce indirect thinking, ask students how the probability of having 1 boy and probability of having 2 girls are related.

Investigate Answers (pages 77-78)

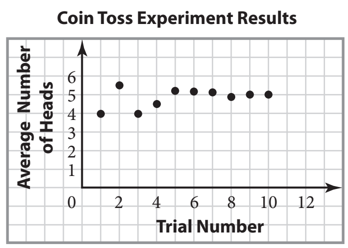
Investigate 1

1. 5 times; the probability of getting heads is $\frac{1}{2}$, so the most likely number of heads that will turn up when a coin is tossed 10 times is $10 \times \frac{1}{2} = 5$.

3. to 5. Answers may vary. Sample answer:

Trial Number	Number of Heads	Average Number of Heads
1	4	4
2	7	5.5
3	1	4
4	6	4.5
5	8	5.2
6	5	5.17
7	5	5.14
8	3	4.88
9	6	5
10	5	5

6.



7. The average number of heads gets closer and closer to 5 as the number of trials increases. This is usually the case. It will be the case if the trials are continued indefinitely.

Investigate 2

3. Answers may vary. Sample answer:

Trial Number	Number of Girls	Trial Number	Number of Girls
1	3	14	1
2	2	15	1
3	1	16	1
4	2	17	3
5	2	18	0
6	1	19	1
7	2	20	3
8	1	21	1
9	2	22	1
10	1	23	0
11	1	24	2
12	0	25	1
13	0		

4. Six times. No, I thought two girls would be more likely. There were 12 times where there was 1 girl (2 boys), which is twice as often as two girls.

5. The probability should be the same as the probability for having two girls out of three children. Both the outcomes of having a girl and having a boy are equally likely, so the outcomes of having two girls and having two boys are equally likely.

Examples

- For Example 1, suggest finding precipitation or temperature data for the region close to your school, or perhaps for a rainforest or other location students might find interesting.
- You can create a simple spinner by placing a pen or pencil at the centre of the circle, then using a paper clip as the spinner.
- To find the sector angle for a percent or fraction, multiply by 360°.
- Example 2 uses theoretical probability to explore the same scenario as in Investigate 2, which uses experimental probability. Method 2 in part c) uses the complement: the opposite of having at least 1 girl is having 0 girls. Point out that this method is often easier than calculating the more complex probability.

Key Concepts

- Have students experiment with using different types of technology to generate random numbers.
- Emphasize that experimental and theoretical probabilities are not always equal but that, as the number of trials increases, the experimental probability should approach the theoretical probability.

Discuss the Concepts

- Give students time to formulate their answers before discussing the questions as a class.

Discuss the Concepts Suggested Answers (page 81)

D1. Divide the circle into six equal sectors, each representing a different roll of the die. Each sector will have a sector angle of 60° (360° ÷ 6).

D2. As the number of trials increases, the reliability increases. With a larger number of trials, the experimental probability should approach the theoretical probability.

D3. It is possible to get 10 heads in 10 tosses, although it is highly unlikely. Because the probability of tossing heads is $\frac{1}{2}$, you would expect to get 5 heads out of 10. The results from performing the experiment with a real coin should not differ from the results when using a graphing calculator to simulate 10 tosses.

Practise (A)

- Encourage students to refer to the Investigates and the Examples before asking for assistance.

Apply (B)

- For **question 6**, remind students that experimental or theoretical probabilities are just estimates of what will occur. Students should understand that past results have no influence on future results.
- For **question 9, part b)**, remind students that one trial consists of performing the randInt command 10 times. Encourage students to work together and share results.
- Supply **BLM T-1 Microsoft® *Excel*** for spreadsheet basics for **question 10**.
- In **question 11**, remind students that human factors can affect probability. The dart thrower is aiming at the red circle, so the dart will not hit the target randomly.
- **Question 12** links to the Chapter Problem. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up.
- **Question 13** is an Achievement Check question. You may wish to use **BLM 2-6 Section 2.3 Achievement Check Rubric** to assist you in assessing your students.

Common Errors

- Some students may use theoretical probability when answering questions about experimental probability.

R_x Have students focus on the wording: experimental probability implies they should look at results of trials and not their knowledge of theoretical probability.

- Some students may have difficulty using technology.

R_x Demonstrate and discuss the available technology with the class. Encourage students to work in pairs or groups so students who are comfortable with technology can share their knowledge.

Accommodations

Spatial—provide a partner to help with the instructions for the Investigates

Perceptual—make appropriate manipulatives available to students who need them

Gifted and Enrichment—design several simulations to determine the number of boys and girls in a family

Language—demonstrate the experiments described in the exercises

Motor—encourage students to work with a partner who can record their explanations

Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- **For question 14**, suggest that students start by calculating the theoretical probability of rolling doubles in 6 rolls.
- For **question 15**, suggest drawing a tree diagram.
- For **question 16**, have students calculate the pairs of numbers that would result in each sum. They should see that rolling dice does not produce equally likely sums.

Achievement Check Answers (page 85)

13. a) 25%

b) 8%; this would be 1 roll.

c) The percents either increase or decrease toward a more equal distribution for each number. The equal distribution would be $\frac{1}{6}$ or 16.7% for each number.

d) The percents would be even closer to even distribution (all percents approximately 16.7%). The greater the number of experimental trials, the closer the results will be to the theoretical probability.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	6, 8–10, 12, 14–16
Reasoning and Proving	2, 4–6, 9–16
Reflecting	4, 6, 9–12, 15, 16
Selecting Tools and Computational Strategies	1–4, 5, 6, 8, 9, 11–16
Connecting	13
Representing	7–10, 15, 16
Communicating	2, 4, 6, 8, 10–12, 14, 16

Extra Practice

- You may wish to use **BLM 2-5 Section 2.3 Compare Experimental and Theoretical Probabilities** for remediation or extra practice.