4.1 Modelling With Quadratic Relations

Student Text Pages
168–179

Suggested Timing
80 min

Tools
- calculators
- grid paper
- graphing calculators
- computers
- *The Geometer's Sketchpad®*

Related Resources
BLM 4-3 Section 4.1 Modelling With Quadratic Relations
BLM 4-4 Section 4.1 Achievement Check Rubric
BLM G-1 Grid Paper
BLM G-3 Four Quadrant Grids

Link to Prerequisite Skills

Students should complete questions 1 to 11 of the Prerequisite Skills before proceeding with this section.

Warm-Up

1. Graph each relation.

a)

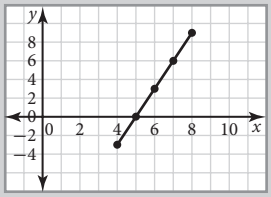
x	y
4	−3
5	0
6	3
7	6
8	9

b)

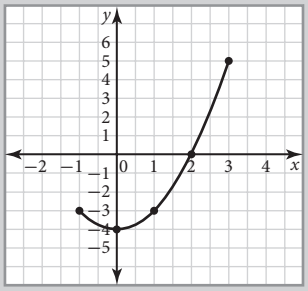
x	y
−1	−3
0	−4
1	−3
2	0
3	5

Warm-Up Answers

1. a)



b)



Teaching Suggestions

Warm-Up

- Write the Warm-Up question on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Quadratic relations were debated in the British Parliament after the president of a British teacher's union suggested ending mathematics as a compulsory subject, and cited quadratic equations as an example of an irrelevant topic. Several in parliament disagreed and argued that parliament should publicly rebut the statements. See the McGraw-Hill Ryerson Web-site at *www.mcgrawhill.ca/books/foundations11* for a link to the text of the debate.

Investigate

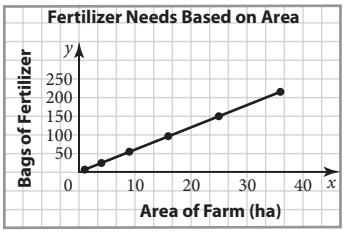
- Students can work individually or in pairs to complete the Investigate.
- You may wish to have copies of **BLM G-1 Grid Paper** and **BLM G-3 Four Quadrant Grids** available for students.

Investigate Answers (pages 168-169)

1.

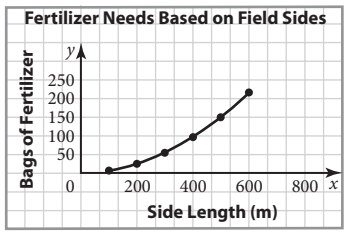
Side Length of Square Farm (m)	Area of Farm (ha)	Bags of Fertilizer Needed
100	1	6
200	4	24
300	9	54
400	16	96
500	25	150
600	36	216

2.



The graph is linear.

3. a)



b) The graph is non-linear.
864 bags

c) The number of bags of fertilizer needed is $\frac{6n^2}{10\,000}$, where n represents the side length of the field.

4. Use the table to find the first differences. If a relation is linear, the first differences are constant.

Examples

- The Examples show three different ways of representing quadratic relations: using a graph, a table of values, and an algebraic equation.
- Write the Examples on the board or on an overhead, and work through them as a class. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.

Key Concepts

- Where possible, students should be encouraged to represent quadratic relations numerically, graphically, and algebraically.

Discuss the Concepts

- Have students read the questions and record their solutions before starting a class discussion.

Discuss the Concepts Suggested Answers (page 173)

D1. To check if a relation is linear, determine whether its first differences are constant. To check if a relation is quadratic, determine whether its second differences are constant.

D2. Two examples of quadratic relations are $y = x^2$ and $y = 2x^2 + x$. These relations are quadratic since their second differences are constant.

Practise (A)

- Encourage students to refer to the Investigate and the Examples before asking for assistance.
- For **question 4, part c)**, ensure students understand that a relation can be quadratic even if only half a parabola is shown. Depending on the context, negative values may not make sense.

Apply (B)

- **Question 8** is a Literacy Connect question. This question offers students an opportunity to explore literacy issues in the context of mathematics.
- For **question 9**, students may recall that, for a rectangle with a fixed perimeter, a square gives the greatest area. However, students should be encouraged to follow the instructions to find a solution as this will develop their understanding of quadratic relations.
- **Question 10** links to the Chapter Problem. The general equation for projectile motion (neglecting air resistance) is $h = \frac{-4.9}{(V\cos\theta)^2}\ d^2 + (\tan\theta)d + R$, where V is the initial velocity of the projectile in metres per second, θ is the angle of the shot, h is the height above the ground, R is the release height, and d is the horizontal distance from the start, all in metres. You can use this equation to generate other relations that will follow projectile motion. Remind students to keep the solution to this question handy as it may help them complete the Chapter Problem Wrap-Up.
- **Question 11** is an Achievement Check question. Students can use toothpicks or straws to model the triangles. This question can be used as a diagnostic or formative assessment, or assigned as a small summative assessment. You may wish to use **BLM 4-4 Section 4.1 Achievement Check Rubric** to assist you in assessing your students.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- For **question 12,** encourage students to graph the relations on the same grid.
- Students may be surprised to discover that for **question 13**, the runner will win the race over short distances.
- **Question 14** exposes students to the idea that, although graphical representations are nice, they can be deceiving. These graphs could be mistaken as parabolic.
- In **question 15**, explain that although, a naturally hanging rope is a catenary, which can be approximated by a quadratic relation, once the suspension cables are attached to the deck, their shape becomes quadratic. This is true only of the cables and not the roadway. The roadway is still a catenary.

Common Errors

- Some students may be confused by the position of the squared term in equations such as $y = 3 + 2x - 15x^2$.

R_x Have students recall that the order of the terms in a polynomial is not important.

- Some students may find first and second differences without first checking that the differences between x-values are constant.

R_x Have students check that the differences between x-values are constant before they calculate first and second differences in the y-values.

Accommodations

Memory—begin development of a word wall and use colour for corresponding graphs and relations

Visual—allow the use of technology for graphing

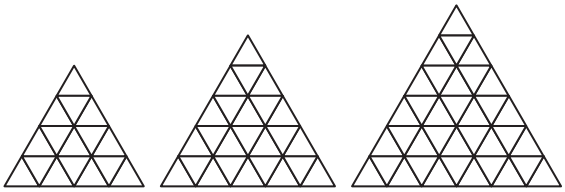
Spatial—provide the Achievement Check question on a handout and include space for responses

Perceptual—use of coloured pencils is recommended for all graphing

Gifted and Enrichment—encourage students to find other examples of quadratic relations on the Internet and prepare a poster for the class

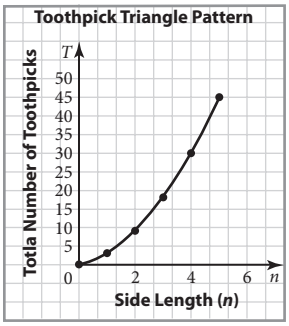
Achievement Check Answers (page 178)

a)



b)

Side Length, n	Total number of Toothpicks, T	First Differences	Second Differences
0	0		
1	3	3	
2	9	6	3
3	18	9	3
4	30	12	3
5	45	15	



c) The relationship between the side length of a triangle and the total number of toothpicks is quadratic, since the second differences are constant.

d) 630 toothpicks

e) 11 toothpicks

Literacy Connections

- Have students write a journal entry describing the different ways to represent a quadratic relation. Ensure they include at least one example of each representation.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9–13
Reasoning and Proving	1–3, 5–7, 11, 14
Reflecting	7, 11
Selecting Tools and Computational Strategies	1, 2, 4, 6, 7, 9–12
Connecting	8, 15
Representing	3, 5, 6, 9, 10, 13, 14
Communicating	3, 5, 8, 14, 15

Extra Practice

- You may wish to use **BLM 4-3 Section 4.1 Modelling With Quadratic Relations** for remediation or extra practice.