7.2 Zero and Negative Exponents

Student Text Pages
364–371

Suggested Timing
80 min

Tools
• calculators

Related Resources
BLM 7-4 Section 7.2 Zero and Negative Exponents

Link to Prerequisite Skills
Students should have completed questions 1 and 2 before proceeding with this section.

Warm-Up
1. Evaluate the powers in each set.
a) $5^2, 5^3, 5^4$ **b)** $3^2, 3^3, 3^4$ **c)** $10^2, 10^3, 10^4$ **d)** $\left(\frac{1}{2}\right)^1, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3$

2. Use the exponent rules to evaluate.
a) $5^2 \times 5$
b) $3^3 \times 3^2$
c) $2^2 \times 2^4$
d) $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

Warm-Up Answers

1. a) 25, 125, 625 **b)** 9, 27, 81 **c)** 100, 1000, 10 000 **d)** $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

2. a) 125 **b)** 243 **c)** 64 **d)** $\frac{1}{32}$

Teaching Suggestions
Warm-Up
- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener
- Remind to students that they are familiar with exponents being used for large numbers. Small numbers can also be expressed using exponents.

Investigate
- Have students work through the Investigate individually, then discuss their results with their classmates.
- Some students may find it helps to write the operation between terms as $\times \frac{1}{2}$ rather than as $\div 2$.
- You may wish to ask students to describe the patterns on the left and right sides of the equal sign.
- Some students may benefit from plotting a graph comparing the powers to their values to see that the value of a power with a negative exponent is not negative.
- As an alternative to the Investigate, use the exponent rules to develop the concepts. Have students expand both the numerator and denominator of fractions, such as $\frac{2^2}{2^3}$, and compare the result to the result when they subtract the exponents, 2^{2-3}. Have students complete additional examples to consolidate their understanding.

Investigate Answers (page 364)

1. a) $2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$

b) $3^5 = 243$
$3^4 = 81$
$3^3 = 27$
$3^2 = 9$
$3^1 = 3$
$3^0 = 1$

c) $10^5 = 100\,000$
$10^4 = 10\,000$
$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$

2. The value of a power with a zero exponent is always 1.

3. a) $2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

b) $33 = 27$
$32 = 9$
$31 = 3$
$30 = 1$
$3^{-1} = \frac{1}{3}$
$3^{-2} = \frac{1}{9}$

c) $10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = \frac{1}{10}$
$10^{-2} = \frac{1}{100}$

4. To evaluate a power with a negative exponent x^{-a}, first write the power with a positive exponent $\frac{1}{x^a}$, then evaluate the power.

Examples

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- Work through numerous examples of evaluating simple powers as well as evaluating expressions. For the bases, include only integers or fractions with 1 as the numerator.
- It might help students to visualize fractional answers by placing them on a number line and seeing how their values decrease without becoming negative.
- Remind students that $\frac{-1}{4} = \frac{1}{-4} = -\frac{1}{4}$.
- For Example 3, explain that plutonium is a radioactive, metallic chemical element. Its most stable isotope is plutonium-244, which decays so slowly that trace amounts still exist from the creation of the solar system.

Key Concepts

- Review the Key Concepts as a class. Have students make up their own examples for class discussion.

Discuss the Concepts

- Have students answer these questions with a partner then discuss them as a class before assigning the exercises.
- After students complete question D2, you may wish to ask them to create their own question that requires them to find the value of each power. This gives students an opportunity to look back and consolidate their knowledge to formulate an appropriate answer.
- For question D3, have students make up examples such as $3^{-1} \times 3^{-2}$, then evaluate by expanding and by combining the exponents.

Discuss the Concepts Suggested Answers (page 367)

D1. a) First, write the power with a positive exponent, $\frac{1}{2^3}$. Then evaluate the power, $\frac{1}{2^3} = \frac{1}{8}$.

b) First, write the power with a positive exponent, 3^4. Then evaluate the power, $3^4 = 81$.

D2. a) $(-2)^4 = 16$ **b)** $-2^4 = -16$ **c)** $2^{-4} = \frac{1}{16}$

In part a), the power has base –2 and exponent 4. In part b), the power has base 2 and exponent 4. The negative sign in front of the power applies to the whole power. In part c), the power has base 2 and exponent –4.

D3. Answers may vary.

Product	Expanded Form	Number of Factors	Single Power
$5^{-3} \times 5^{-2}$	$\frac{1}{5 \times 5 \times 5} \times \frac{1}{5 \times 5}$	5	5^{-5}
$3^{-4} \times 3^{-3}$	$\frac{1}{3 \times 3 \times 3 \times 3} \times \frac{1}{3 \times 3 \times 3}$	7	3^{-7}
$(-7)^{-1} \times (-7)^{-2}$	$\frac{1}{(-7)} \times \frac{1}{(-7) \times (-7)}$	3	$(-7)^{-3}$

Practise (A)

- **Questions 1 to 5**, **and 8** are good practice of the basic skills. In particular, **question 4** assesses students' understanding of the patterning approach to the development of the concepts.
- In **question 3**, students might have trouble with part e) on their calculator. The result of evaluating 500^{-1} may be shown in scientific notation. Explain that a display, such as [**2.0 –3**] means 2.0×10^{-3}, which is equal to 0.002.
- **Questions 6 and 7** involve an approach to the concepts using the exponent rules.

Apply (B)

- **Question 10** is a Literacy Connect. You may wish to assign this question as a journal entry or to discuss the question as a class. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics.
- **Questions 10 and 12** add an interesting context to the numbers, involving understanding very small numbers. The context of **question 12** is good considering many students like to listen to loud music.
- Many students can relate to **question 11** because of their attachment to electronic devices, such as iPods and video games. You may wish to display a table, on the board or overhead, showing the relationships among the various prefixes. For example, one kilobyte is equivalent to 2^{10} bytes, and one megabyte is equivalent to 2^{20} bytes.
- **Question 12** links to the Chapter Problem. It is an interesting comparison of the intensities of different sounds. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up. Alternatively, you may wish to use the Chapter Problem as a summative assessment at the end of the chapter.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 13** refers to **question 12** and adds complexity to the concept of sound intensity.

Common Errors

- Some students may evaluate powers with negative exponents by assuming the result is negative, or with zeros exponent by assuming the results equal zero.

R_x Have students use an alternate method of developing the rules. If patterning was used, refresh using the exponent rules, and vice versa. Alternatively, have students investigate results using a graphing calculator with fraction capabilities.

Accommodations

Spatial—provide a photocopy of a table for the Investigate

Perceptual—take extra care when writing on the board as the symbolic nature of this section can be difficult

Gifted and Enrichment—have students work in small groups on the Extend questions and present their solutions to the class

- **Question 14** is accessible to all students. The extension is to a rational base, but can be solved with ease using a calculator.
- Students may have difficulty with **question 17**, as it involves variables. Remind students to substitute the powers as given, use the exponent rules to simplify, then evaluate.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9
Reasoning and Proving	12
Reflecting	4, 5
Selecting Tools and Computational Strategies	1–3, 5–17
Connecting	4, 12
Representing	n/a
Communicating	13

Ongoing Assessment

- Circulate as students work through the Investigate to see how well they understand the concepts.
- You may wish to collect students' responses to the Discuss the Concepts questions to use as a formative assessment tool.

Extra Practice

- You may wish to use **BLM 7-4 Section 7.2 Zero and Negative Exponents** for remediation or extra practice.