7.3 Investigate Exponential Relationships

Student Text Pages
372–381

Suggested Timing
160 min

Tools
- paper
- grid paper
- temperature probes
- hot water
- large bowls

Related Resources
BLM 7-5 Section 7.3 Investigate Exponential Relationships
BLM 7-6 Section 7.3 Analogue Radio Dial
BLM 7-7 Section 7.3 Achievement Check Rubric
BLM G-1 Grid Paper

Link to Prerequisite Skills

You may wish to have students review Linear Relations and Quadratic Relations, questions 3 to 5, 7 and 8, before proceeding with this section. The material addressed in these questions is not directly required for this section, but students may benefit from reviewing the other types of relations they have studied before beginning to develop the concepts related to exponential relations.

Warm-Up

1. Calculate the first and second differences.

x	y	First Differences	Second Differences
0	16		
1	9		
2	4		
3	1		
4	0		
5	1		

Warm-Up Answers

1.

x	y	First Differences	Second Differences
0	16		
1	9	$9 - 16 = -7$	
2	4	$4 - 9 = -5$	$-5 - (-7) = 2$
3	1	$1 - 4 = -3$	$-3 - (-5) = 2$
4	0	$0 - 1 = -1$	$-1 - (-3) = 2$
5	1	$1 - 0 = 1$	$1 - (-1) = 2$

Teaching Suggestions

Warm-Up

- Write the Warm-Up question on the board or on an overhead. Have students complete the question independently. Then, discuss the solution as a class.

Section Opener

- Suggest to students that not all relationships are linear or quadratic. Many situations depend on the exponent varying, thus can be modelled using an exponential relation.

Investigate

- Students learn exponential relationships best by investigating them, rather than through teacher presentation.
- The Investigates and the Example could be set up as experiment stations where the students work through them, record their data, and proceed to the next station.
- Both Investigates should be done at the same time. Set up Investigate 2 initially, and have students record the results every 5 min for up to 60 min. It may be beneficial to stop after 45 min in order to finish both Investigates

in one class period. Alternatively, data collection could occur on one day with the analysis the following day. Meanwhile, students can focus on Investigate 1, looking at the three different ways to fold paper. The results allow students to compare linear, quadratic, and exponential relationships.

- Due to time and safety constraints, you may wish to conduct Investigate 2 as a demonstration. The results will be better if the data are collected using a temperature probe.
- Distribute copies of **BLM G-1 Grid Paper**.

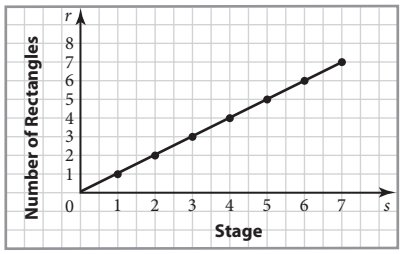
Investigate Answers (pages 372–374)

Investigate 1

1. a)–c)

Stage	Total Number of Rectangles
1	1
2	2
3	3
4	4
5	5
6	6
7	7

d)

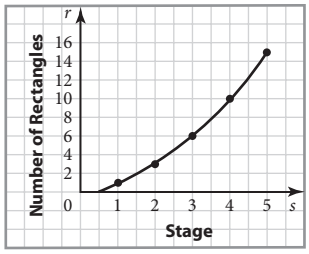


e) The points lie in a line. The graph appears to be linear.

2. a)–c)

Stage	Total Number of Rectangles
1	1
2	3
3	6
4	10
5	15

d)

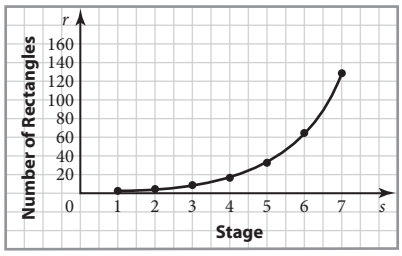


e) The points lie in a curve. The graph appears to be quadratic.

3. a)–c)

Stage	Total Number of Rectangles
1	2
2	4
3	8
4	16
5	32
6	64
7	128

d)



e) The points lie in a curve. The graph increases slowly, and then rapidly from left to right.

4. All the graphs increase from left to right, but the linear relation increases at a constant rate. The quadratic and the exponential relations increase at increasing rates. The rate at which the exponential relation increases is much greater than the rate at which the quadratic relation increases.

Investigate 2

1.–4. Answers may vary

5. a curve starting from upper left corner and sloping downward to the right and approaching room temperature

6. The quotients are approximately equal.

Examples

- Have students work through the Example as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Example independently or in small groups before reviewing them as a class.
- Discuss the Example. Stress the constant growth factor as an indicator of an exponential relationship. It is a good discussion point that, since these data are experimental, they are not perfect. This is an opportunity to discuss the idea of error in experimental data and mathematical modelling.

Key Concepts

- Review the Key Concepts. Have students look at the behaviour of the graph—the curve approaches the *x*-axis at one end and grows or decays rapidly at the other end.

Discuss the Concepts

- Have students discuss these questions in small groups and take them up as a class before assigning the exercises.

Discuss the Concepts Suggested Answers (page 376)

D1. The relationship between speed and the collision rate is neither linear nor quadratic. The graphs do not increase at a constant rate and the rates of increase of the rate (second differences) are not constant. Estimating values from the graph, the ratios between consecutive values are approximately equal. This relationship could be exponential.

D2. On rural roads, the chance of a collision is the greatest. This may be due to the lack of enforcement of the speed limit.

Practise (A)

- Encourage students to refer to the Investigates and the Example before asking for assistance.
- **Questions 1 to 3** are good questions to consolidate understanding of exponential relationships.

Apply (B)

- **Question 4** is an excellent application of exponential relationships through analysis of data. This is an opportunity for kinaesthetic and visual learners to consolidate their understanding. You may wish to distribute copies of **BLM 7-6 Section 7.3 Analogue Radio Dial**.
  Some facts about frequencies and the spacing on a radio dial:
  – On a radio dial, stations are at positions measured in wavelengths by metres and in frequencies by kilohertzs (KHz). Each particular spot on the dial has a wavelength and a corresponding frequency.
  – To convert any frequency to a wavelength, divide the speed of light by the frequency. Use the approximate value of 3.0×108 for the speed of light. The formulas calculate or use frequencies measured in Hertzs (Hz).
  Wavelength $= 3.0 \times 10^8 \div$ frequency
  Frequency $= 3.0 \times 10^8 \div$ wavelength

  Why does it work this way?
  Imagine walking past a wall at 3 m/s (about 1.0×10^{-8} the speed of light) and drawing a wave on the wall with chalk by moving the chalk up and down. If you move the chalk up and down at a frequency of 3 times per second, this represents 3 Hz. The wave you have drawn on the wall has a wavelength (from peak to peak) of 1 m. Radio waves travelling at the speed of light and moving up and down at so-many millions of times a second

Common Errors

- Some students may confuse exponential and quadratic relationships.

R_x Have students compare the graphs from the Investigates and questions 2 and 3. Specifically have them look at the growth rates of the data and the two graphs. More comparisons of the types of relations will come in section 7.4 Exponential Relations.

Accommodations

Motor—have students work with a partner during the Investigate to assist with folding

Spatial—have students work in pairs to ensure that all steps of the Investigate are completed

Visual—allow use of a program, such as *Fathom*™, to record data and draw graphs

Gifted and Enrichment—challenge students to research fractals

have a corresponding wavelength drawn in space. At 3.0×10^8 m/s, a radio frequency of 300 mega Hertz (MHz) draws a line in space with peaks 1 m apart. This explains the spacing between frequencies on a radio dial.

- **Questions 6 to 8** illustrate the prevalence of exponential relationships in the field of science. Even those students who are not interested in science should try these questions.
- **Question 9** brings the concepts to common everyday experiences.
- **Question 10** is a Literacy Connect. It has applications to the social sciences. Literacy Connect questions offer the opportunity to explore literacy issues in the context of mathematics.
- **Question 11** is an Achievement Check question. It can be used as a diagnostic or formative assessment, or assigned as a small summative assessment piece.
- You may wish to use **BLM 7-7 Section 7.3 Achievement Check Rubric** to assist you in assessing your students.

Extend (C)

- Assign the Extend question to students who are not being challenged by the Apply questions.
- Question 12 will be of interest to students who like visual patterns. Fractals can also be found in many computer screen savers and mathematical models. It may help to discuss how the pattern develops, and to have the students extend the patterns.

Achievement Check Answers (page 381)

11. a) 128 s or about 2 min
b) 1 048 576 s or about 291 h or about 12 days
c) after approximately 12 school days
d) after approximately 16.4 school days

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9, 10, 12
Reasoning and Proving	1, 2, 5, 6, 10
Reflecting	3, 5
Selecting Tools and Computational Strategies	3, 7–12
Connecting	3, 4, 6, 10
Representing	1, 2, 4, 5, 7
Communicating	2, 3, 4, 6–10, 12

Ongoing Assessment

- Circulate as students work through the Investigates. Suggest students research Newton's Law of Cooling to confirm their results.
- You may wish to collect students' responses to the Discuss the Concepts questions as a formative assessment tool.

Extra Practice

- You may wish to use **BLM 7-5 Section 7.3 Investigate Exponential Relationships** for remediation or extra practice.