7.4 Exponential Relations

Student Text Pages
382–394

Suggested Timing
80–160 min

Tools
- graphing calculators
- grid paper

Related Resources
BLM 7-8 Section 7.4 Exponential Relations
BLM G-1 Grid Paper

Link to Prerequisite Skills

Students should complete questions 6 to 8 before proceeding with this section.

Warm-Up

1. Sketch a graph of each relation.

a) $y = 2x + 1$

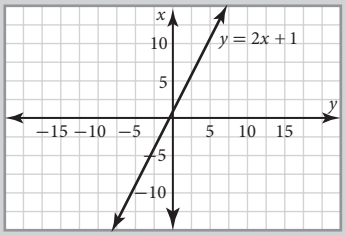
b) $y = -\frac{1}{2}x - 5$

c) $y = (x - 1)^2 + 3$

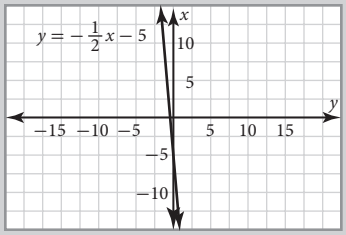
d) $y = -2(x + 3)^2 + 1$

Warm-Up Answers

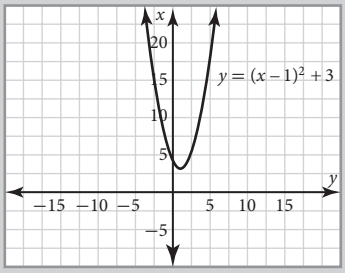
1. a)



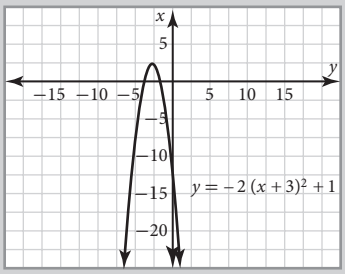
b)



c)



d)



Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Discuss the difference between a relationship and a relation. Remind students that a relationship is observed from data and can be modelled with a relation, which can be represented by an equation, a table of values, or a graph.

Investigate

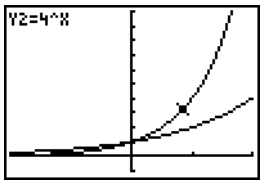
- It is important that students investigate different types of relations after having discussed exponential relationships.
- A graphing calculator, if available, eliminates the need to make a table of values and makes graphing less tedious. However, you may wish to have students draw at least one graph by hand to gain an appreciation of the growth of an exponential relation.

- If students are graphing by hand, have copies of **BLM G-1 Grid Paper** available.
- Allow students to use their own vocabulary when describing the graphs. You can introduce the mathematical language when working through the Examples.

Investigate Answers (pages 382–383)

Method 1: Use a Graphing Calculator

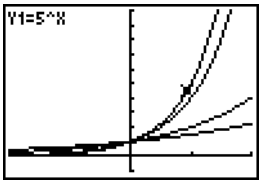
1.



2. The graphs have the same y-intercept at (0, 1).

3. To the left of the y-axis, both graphs are between 0 and 1. To the right of the y-axis, both graphs are above 1. The graph of $y = 4^x$ increases more rapidly than the graph of $y = 2^x$.

4. The y-intercept of both $y = 5^x$ and $y = 1.5^x$ should be 1. The graph of $y = 5^x$ should increase more rapidly than the graph of $y = 4^x$. The graph of $y = 1.5^x$ should increase less rapidly than the graph of $y = 2^x$.



Graphing the relations confirms my predictions.

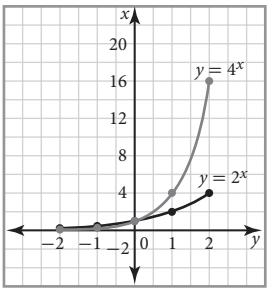
5. The graph of an exponential relation of the form $y = b^x$ increases from left to right and has y-intercept 1. To the left of the y-axis, the graph is between 0 and 1. To the right of the y-axis, the graph is above 1.

Method 2: Graph by Hand

1.

x	$y = 2^x$	$y = 4^x$
–2	0.25	0.0625
–1	0.5	0.25
0	1	1
1	2	4
2	4	16

2.

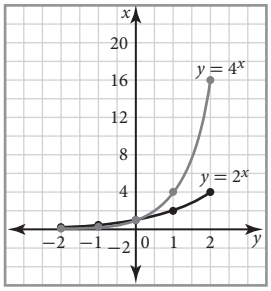


3. The graphs have the same y-intercept at (0, 1).

4. To the left of the y-axis, both graphs are between 0 and 1. To the right of the y-axis, both graphs are above 1. The graph of $y = 4^x$ increases more rapidly than the graph of $y = 2^x$.

5. The y-intercept of both $y = 5^x$ and $y = 1.5^x$ should be 1. The graph of $y = 5^x$ should increase more rapidly than the graph of $y = 4^x$. The graph of $y = 1.5^x$ should increase less rapidly than the graph of $y = 2^x$.

Graphing the relations confirms my predictions.



6. The graph of an exponential relation of the form $y = b^x$ increases from left to right and has y-intercept 1. To the left of the y-axis, the graph is between 0 and 1. To the right of the y-axis, the graph is above 1.

Examples

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- Students often think of exponential relations as half of a parabola. This is because they may not graph enough points at the extremes. Make sure students observe that the curve approaches a horizontal line at one end and approaches a vertical line at the other end. The term *asymptote* is not a required part of students' vocabulary, so use *horizontal/vertical line* as a descriptor.
- It is important to include contextual examples. Examples 2 and 4 examine population growth and the musical scale, which are such examples. Populations do not grow at a constant rate. This is because as the population of an area increases, the number of births increases proportionally and therefore the rate of growth of the population increases as well. Stress the importance of not rounding the base, as rounding can give inaccurate results.
- Example 3 provides is an important comparison of linear, quadratic, and exponential relations. Stress multiple representations of relations—words, tables, graphs, and numbers.

Key Concepts

- Review the Key Concepts by discussing each point as a class.

Discuss the Concepts

- These questions provide an important assessment of student understanding. Have students discuss the questions with a partner before discussing as a class.

Discuss the Concepts Suggested Answers (page 389)

D1. The graph of an exponential relation does not have an x-intercept. The curve approaches the x-axis to the left of the y-axis, but never touches it.

D2. Consider the graph of $y = x^2$. The y-values are always greater than or equal to 0. When x is positive, the value of y decreases as x decreases. When x is negative, the value of y increases as x decreases. Consider the graph of $y = 2^x$, the value of y increases as x increases. When x is negative, the value of y is always between 0 and 1. When x is positive, the value of y is always greater than 1.

Practise (A)

- Encourage students to refer to the Examples before asking for assistance.
- **Questions 1 to 7** are good practice of the essential skills. Students may wish to check their answers with a graphing calculator, especially answers with decimal bases.
- Students may have difficulty with **question 5**, as it requires simplifying the exponent before applying it. With a graphing calculator, it requires placing the exponents in brackets. This may also be true about the Apply questions.

Apply (B)

- **Questions 8 to 13** are good applications of exponential relation that cover the expectations. All of these questions are accessible to students working at levels 2, 3, or 4. You may have to discuss some of the vocabulary.
- For **question 8**, ask students to discuss the significance of the bases of the powers in each formula. Why do they start with the digit 1? Have students research population change in their own town or city to see if it is exponential.

Common Errors

- Some students may confuse exponential relations with quadratics.

R_x Have students graph a greater number of points if that is the cause of their errors. Otherwise, go over Example 3 carefully and compare the rates of change for both types of relations. Make sure you assign question 9 from the Apply exercises.

Accommodations

Visual—provide a graphic organizer for the Key Concepts

Perceptual—encourage students to use different colours when graphing more than one relation on a coordinate grid

Language—have students work with a reading buddy for Key Concepts and Apply problems

If not, ask how the numbers would need to change to make it exponential.

- **Question 10** refers to the Chapter Problem. It provides an interesting insight into the danger loud noises pose to hearing. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up.
- **Question 13** is a Literacy Connect. Discuss the patterns and help students decide on the next diagram. Literacy Connect questions offer the opportunity to explore literacy issues in the context of mathematics.
- In **question 13**, the rate of change can be expressed as the growth factor.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 14** is accessible to students working at levels 3 and 4. It requires graphs of altitude versus temperature and altitude versus atmospheric pressure. For **part c)**, you may wish to have students research the effects of increasing altitude on the atmosphere.
- Ensure students consider the exponent rules as they work through **question 15**.

Literacy Connections

- Distribute copies of **BLM 7-9 Section 7.4 Literacy Connect**. Have students work in pairs to complete this read and understand opinion piece. You may wish to use BLM A-18 Opinion Piece Checklist to assess students' responses.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	12
Reasoning and Proving	1–3, 8, 9, 11–15
Reflecting	9, 10, 11, 14
Selecting Tools and Computational Strategies	6, 7, 10, 13
Connecting	11, 13
Representing	3–7, 8–11, 13, 15
Communicating	1, 2, 9, 12

Onging Assessment

- Collect students' answers to one of questions 8 to 13 to assess their understanding.

Extra Practice

- You may wish to use **BLM 7-8 Section 7.4 Exponential Relations** for remediation or extra practice.