7.5 Modelling Exponential Growth and Decay

Student Text Pages
395–405

Suggested Timing
80 to 160 min

Tools
- graphing calculators
- CBR™
- basketball or volleyball

Related Resources
BLM 7-10 Section 7.5 Modelling Exponential Growth and Decay
BLM 7-11 Section 7.5 Achievement Check Rubric
BLM T-6 The CBR™

Warm-Up

1. Identify each relation as linear, quadratic, or exponential.

a)

x	y
–1	1.5
0	1
1	1.5
2	3
3	5.5
4	9

b)

x	y
–1	0.25
0	1
1	4
2	16
3	64
4	256

c)

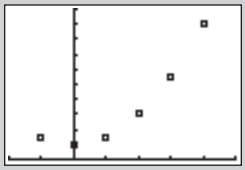
x	y
–1	5.5
0	4
1	2.5
2	1
3	–0.5
4	–2

2. Use a graphing calculator. Make a scatter plot of each relation in question 1.

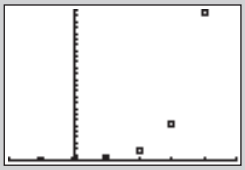
Warm-Up Answers

1. a) quadratic **b)** exponential **c)** linear

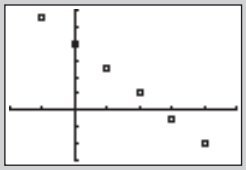
2. a)



b)



c)



Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Have students look at the photograph of the discarded computers and discuss how the mass of discarded computers is growing exponentially. Ask what this means and, after a short discussion, proceed with the Investigates to illustrate exponential growth and decay.

Investigate

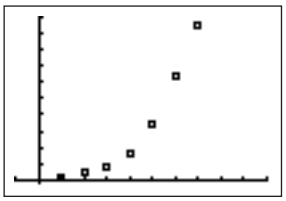
- The two Investigates serve different purposes. Investigate 1 uses secondary data to illustrate exponential population growth. Investigate 2 uses primary data to illustrate exponential decay.
- For Investigate 1, have students work through the regressions and compare their results. When entering the data, emphasize that students should only enter times from 1 to 7 in list L1.
- Discuss that, although r^2 indicates how well the equation fits the data, it does not indicate if the type of relation selected (linear, quadratic, exponential) is appropriate. That is left to other indicators, such as first and second differences, common ratios, and an analysis of the situation.

- For Investigate 2, it is important to use a large ball to get clean data from the CBR™. Have students practice a few times before collecting the data. Refer to **BLM T-6 The CBR™** for detailed instructions on how to use the CBR™.
- After students complete the Investigates, have a class discussion on exponential growth and decay. Ensure students understand that the rates of change for both depend on a growth or decay factor that is a common ratio between successive terms.

Investigate Answers (pages 395–398)

Investigate 1

1. b)



The graph increases from left to right at an increasing rate.

d) The line appears to only pass through two points on the scatter plot.

1. c), 2., 3. a)

Method	Expanded Form	Coefficient of Determination, r^2	Mass at Hour 8 (mg)	Mass at Hour 9 (mg)
Linear Regression	$y = 75.6x - 142.7$	0.859	462.43	538.07
Quadratic Regression	$y = 17.4x^2 - 63.8x + 66.4$	0.996	671.57	904.07
Exponential Regression	$y = 5.696(1.929)^x$	0.995	1090.9	2104.0

3. b) The initial mass is approximately 5.7 mg.

c) The exponential model is a better model than the linear or quadratic models. If the linear or quadratic models are used to estimate the initial mass, the value is negative.

4. The coefficient of determination for the quadratic model is 0.996 and the coefficient of determination for the exponential model is 0.995. This indicates that both models fit the data well. However, the quadratic model gives an estimate of initial mass that is negative, which does not make sense in this context. The exponential model fits the data best.

5. As time progresses, the linear model becomes less and less accurate.

Investigate 2

d) Horizontal axis: bounce number; vertical axis: height of bounce

e) As the bounce number increases, the maximum height of each bounce decreases. Each bounce is represented graphically by a parabola. Gravity is a factor in determining the shape.

f) Answers may vary.

g) Speed seems to be decreasing; height of next bounce decreases.

h) A curve starting from left sloping downward to right. If extended, the curve would reach zero where the ball stops bouncing.

i) The relationship between bounce number and height of the ball bounce is exponential. This is indicated by the shape of the curve and by the constant ratio of the differences in consecutive bounce heights.

Examples

- Work through the Examples as a class. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- For Examples 1 and 2, explain that the growth factor is the base of the power. In Example 1, illustrate that the base 1.078 means a 7.8% increase each year, but the total population is 107.8% of the previous population. Similarly, for Example 2, the data are the remaining intensities, and 89.1% is the same as 100% – 10.9%, or 0.891 as the decay factor.
- If your class does not have access to graphing calculators, have students draw the exponential curve of best fit by hand.

Key Concepts

- Review the Key Concepts by referring to the Investigates and Examples. Remind students how the data were related by the growth or decay factors.

Discuss the Concepts

- Have students work in pairs to discuss these questions. Then, take them up as a class before moving on to the exercises. See the comments after Investigate 2, on page 398, to help students understand these concepts.

Discuss the Concepts Suggested Answers (page 401)

D1. Population growth, such as a fox population that grows by 2% per year, represents exponential growth since it has an initial value and increases by the same factor each time period.

D2. a) A 20% reduction in water clarity means that 80% (100% – 20%) clarity remains.

b) The graph will decrease from left to right and have *y*-intercept 100. To the left of the *y*-axis, the curve is above 100. To the right of the *y*-axis, the curve decreases quickly at first and then slowly approaches the x-axis.

Practise (A)

- Encourage students to refer to the Investigates and the Examples before asking for assistance.
- **Questions 1 to 3** should be assigned to consolidate students' understanding of the Examples. Take these up as a class before moving on to the rest of the exercise.

Apply (B)

- **Questions 6 to 10** require the analysis of data, as in the Investigates. Graphing calculators should be provided.
- **Question 7** can be used as an introduction to a discussion on environmental issues, such as global warming. Students may not know how to calculate percent increases. Explain that the total is divided by the original value then multiplied by 100.
- **Question 10** is an Achievement Check question. It can be used as a diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 7-11 Section 7.5 Achievement Check Rubric** to assist you in assessing your students.

Common Errors

- Some students may be intimidated by the lengthy nature of the problems.

R_x Have students concentrate on setting up the model step-by-step:
- Enter the data into lists on the graphing calculator.
- Make a scatter plot of the data.
- Look at the scatter plot and describe it.
- Look for evidence of exponential growth or decay.
- Answer the individual questions.

Accommodations

Spatial—have students work with a partner who can assist with sequencing steps for using the graphing calculator

Memory—provide simplified instructions for using the graphing calculator with a check box for each step completed

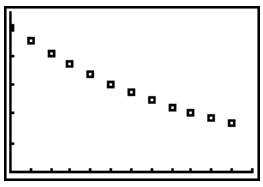
Language—have students read the instructions for using the graphing calculator aloud

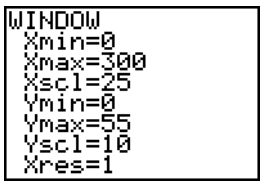
Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- **Question 11** requires additional graphing calculator skills. Pressing 2nd [CALC] **5:intersect** allows students to find the point of intersection of two graphs.
- **Question 12** extends the expectations to have students develop an equation. In part a), the equation would be $y = 4.5 \times 10^9(1.02)^x$, because the population is 102% of the previous population.

Achievement Check Answers (page 405)

10. a)





b) $P = 50(0.996)^t$

c) The satellite can operate for about 300 years before needing to be recharged.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	7, 11, 12
Reasoning and Proving	5–9
Reflecting	7
Selecting Tools and Computational Strategies	1–10
Connecting	3, 7–10, 12
Representing	1–4, 7–12
Communicating	7–9, 12

Ongoing Assessment

- Ask students what window settings are appropriate for each graph. Have them graph each set of data with different window settings to see which settings display the best graph.

Extra Practice

- You may wish to use **BLM 7-10 Section 7.5 Modelling Exponential Growth and Decay** for remediation or extra practice.