7.6 Solve Problems Involving Exponential Growth and Decay

Student Text Pages
406–413

Suggested Timing
80 min

Tools
- dice, number cubes, or spinners
- calculators
- grid paper
- graphing calculators

Related Resources
BLM 7-12 Section 7.6 Solve Problems Involving Exponential Growth and Decay
BLM G-1 Grid Paper
BLM A-9 Communication General Scoring Rubric

Link to Prerequisite Skills

Students should have completed all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. Given the table of values for a relation, how can you tell whether the relation is exponential?
2. What is exponential growth?
3. What is exponential decay?
4. Classify each relation as exponential growth or exponential decay.
a) $A = 500(0.86)^n$
b) $A = 500(1.02)^n$

Warm-Up Answers

1. The ratios of successive y-values are equal.
2. When a quantity grows such that it has a constant growth factor of greater than 1, it is considered exponential growth.
3. When a quantity decays, or declines in quantity such that it has a constant decay factor of less than 1, it is considered exponential decay.
4. a) exponential decay **b)** exponential growth

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Discuss, in general, the exercise questions from section 7.5. Exponential relations can be developed to model situations and, if provided, can allow you to solve problems.

Investigate

- This is a fun activity to demonstrate exponential decay and half-life that gets the entire class involved. The activity requires at least 15 students to be effective. For a small class, you can have students flip a coin, so the half-life would be each individual flip.
- Have a discussion of the meaning of half-life and that the "half" means half of the previous quantity, not of the original quantity. Variations could use different shaped dice (for example, tetrahedral).
- If you do not have enough six-sided dice, you could use the random integer generator on a graphing calculator to simulate the roll. Press [MATH], use the arrow keys to highlight **PRB**, press [ENTER], then select **5:randInt(**. Type 1 [,] 6 [)]. Then press [ENTER] repeatedly. Each time [ENTER] is pressed, a different random number between 1 and 6 is displayed.

Investigate Answers (pages 406–407)

1.–4. Answers may vary.

5. a) Averages should be the same.

b) The decrease is $\frac{1}{2}$ of the original number of students. The ratio of the differences between first and second half-life is constant. Since this is experimental probability, the greater the number of trials, the closer the results will approach the theoretical probabilities.

6. Answers may vary.

7. No, a roll of 3 or 6 is equally likely.

Examples

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- Highlight the concepts of doubling time and half-life. Discuss the roles of the denominator of the exponent, and the coefficient of the power.
- If you would like to use an example of a common radioactive element in an example, smoke detectors use Americium Am-241, which has a half-life of 432 years.

Key Concepts

- Review the Key Concepts, particularly half-life and doubling time. Ask students to write the meanings of the terms in their own words.

Discuss the Concepts

- These questions can be answered quickly with students working individually. Discuss the answers as a class before assigning exercise questions.

Discuss the Concepts Suggested Answers (page 410)

D1. The half-life of the class is $M = M_0\left(\frac{1}{2}\right)^n$ where M_0 is the original number of students in the class and n is the number of trials. Use the base $\frac{1}{2}$ since the probability of tossing a head is $\frac{1}{2}$.

D2. The base for tripling would be 3.

Practise (A)

- Encourage students to refer to the Examples before asking for assistance.
- **Questions 1 to 3** are similar to the Examples. They consolidate understanding of the basic concepts.

Apply (B)

- **Question 5** refers to **question 4** for the half-life of carbon-14.
- You may wish to have copies of **BLM G-1 Grid Paper** available for **question 6**.
- **Questions 6 to 8, and 13**, are interesting examples of health-related situations involving exponential relations.
- **Questions 8 and 10 a)**, are Literacy Connects. You may wish to assign these questions as journal entries or discuss the question as a class. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics.

Common Errors

- Some students may be confused when solving longer problems.

R_x Have students read the problem and look for important points:
- Identify if the situation is exponential growth or decay.
- Identify the doubling time or half-life.
- Determine the indicated time to be substituted.
- Interpret the results.

Accommodations

Visual—allow students to use manipulatives to complete the Investigate

Language—increase the amount of scaffolding to ensure students are able to understand what is being asked

Spatial—provide a worksheet with the problems, including space for the answers

- **Question 9** links to the Chapter Problem. It highlights sound intensities relative to the threshold of human hearing. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up.
- **Questions 11 and 12** are good opportunities for students to research exponential relations as they apply to the social sciences.

Extend (C)

- Assign the Extend question to students who are not being challenged by the Apply questions.
- In **question 14**, ensure students recognise the doubling time is 15 years.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	1–7, 13, 14
Reasoning and Proving	8
Reflecting	11, 12
Selecting Tools and Computational Strategies	1–7, 9–14
Connecting	4–8, 10
Representing	6, 11, 12
Communicating	8

Ongoing Assessment

- Assess students' ability to communicate mathematically and to justify their thinking. You may wish to use **BLM A-9 Communication General Scoring Rubric** to assist you in assessing your students.

Extra Practice

- You may wish to use **BLM 7-12 Section 7.6 Solve Problems Involving Exponential Growth and Decay** for remediation or extra practice.