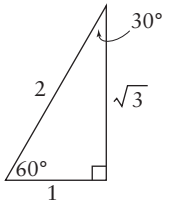
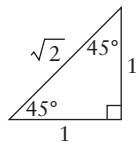
Chapter 1 Trigonometric Ratios

1.1 Sine, Cosine, and Tangent of Special Angles

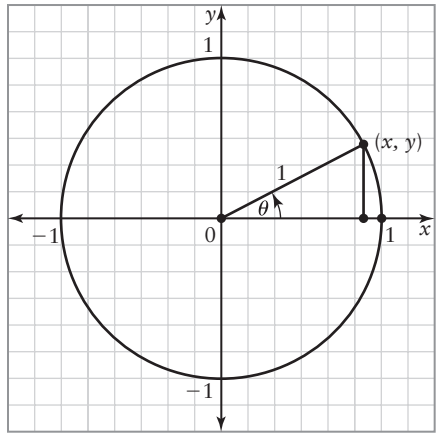
KEY CONCEPTS

- Exact trigonometric ratios for 30°, 45°, and 60° angles can be determined using special triangles.





- Any point (x, y) on a unit circle can be joined to the origin to form a radius 1 unit long.
- A rotation angle θ, in standard position, is formed by proceeding counterclockwise from the initial arm on the positive x-axis to the terminal arm through (x, y).
- For any rotation angle, the reference angle is the acute angle between the terminal arm and the x-axis.
- Given a point (x, y) on a unit circle, $\cos\theta = x$, $\sin\theta = y$, and $\tan\theta = \frac{y}{x}$.



Example

Determine the exact values of the primary trigonometric ratios for 135°.

Solution

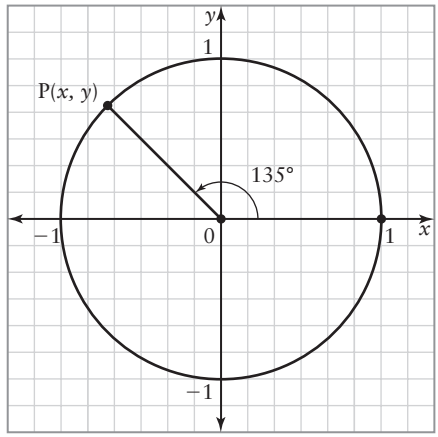
The measure of the reference angle is 180° − 135°, or 45°.

Use the special triangles to determine the sine and cosine ratios for the reference angle.

$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$

Since the terminal arm of a 135° angle in standard position is in quadrant II, the x-coordinate of P is negative.

$\sin 135° = \frac{1}{\sqrt{2}}$ $\cos 135° = -\frac{1}{\sqrt{2}}$ $\tan 135° = -1$



A

Unless otherwise specified, give all answers as exact values.

1. Determine the sine, cosine, and tangent ratios for each angle.

a) 30° **b)** 45° **c)** 60°

2. **a)** Copy and complete the table.

θ	sin θ	
	Exact	**Calculator**
0°		
30°		
45°		
60°		
90°		

b) Compare the exact values of the trigonometric ratios to the values from the calculator.

3. Repeat question 2 for cos θ and tan θ.

4. **a)** What reference angle should be used to find the primary trigonometric ratios for 150°?

b) Determine the primary trigonometric ratios for 150°.

5. **a)** Draw a 225° angle in standard position on a unit circle.

b) What reference angle should be used to find the primary trigonometric ratios for 225°?

c) Find the primary trigonometric ratios for 225°.

6. **a)** Draw a 240° angle in standard position on a unit circle.

b) State two other angles that have the same reference angle.

c) Find the primary trigonometric ratios of 240°.

d) State the primary trigonometric ratios for the angles in part b).

7. **Use Technology** Use geometry software to construct a circle with radius 1 unit. Label the origin O. Construct and label point B on the circle in quadrant I. Determine the coordinates of point B.

a) Construct segment OB. Determine the length of OB. Find the measure of $\angle AOB$.

b) Drag B until $\angle AOB = 30°$. Record the coordinates of point B, the length of OB, and the measure of $\angle AOB$.

c) Reflect B and OB in the x-axis. Record the coordinates of B′ and the coordinate distance OB′.

d) Construct segment BB′. Determine the coordinate distance BB′. What type of triangle is $\triangle OBB'$?

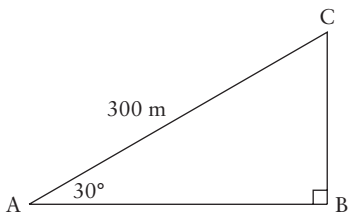
e) Construct the midpoint, D, of BB′. Then, construct segment OD. Determine the coordinate distances OD and DB.

f) How do the distances OD and DB compare to the coordinates of B? How do the distances OD and DB′ compare to the coordinates of B′?

g) Drag point B around the unit circle. What do you notice?

B

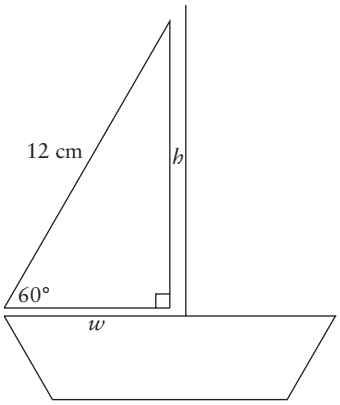
8. Devan started at point A and walked 300 m across a park to a store at point C. Sonal started at point A and walked east to point B and then walked north to the store at point C.



a) Determine the distance AB.

b) Determine the distance BC.

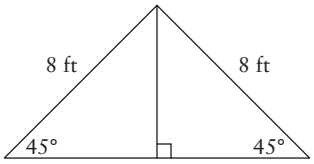
9. Doug designed a sail for a model sailboat. The sail is in the shape of a right triangle.



a) Determine the height, h, of the sail.

b) Determine the width, w, of the sail.

10. Alicia is on an overnight camping trip. At the front of her tent, the distance from the top of the tent to the ground on either side is 8 ft.



a) Determine the height of Alicia's tent.

b) Determine the width of the floor of her tent.

☆**11.** A 3-m long brace is placed against a wall so the bottom of the brace makes an angle of 60° with the ground.

a) Draw a diagram to represent this situation.

b) How far up the wall is the top of the brace?

12. A 4-m long ramp is placed against a wall. The ramp makes an angle of 30° with the ground. How far from the wall is the bottom of the ramp?

13. Boat A is 5 km north of a marina. Boat B is 5 km east of the marina.

a) Determine the distance between the two boats.

b) Describe an alternative method that can be used to solve this problem.

☆**14.** A patio in the shape of a regular hexagon has side lengths 4 m. Determine the area of the patio.

15. Use Technology Use geometry software to construct a circle with radius 1 unit. Construct point A on the circle in quadrant I. Construct a segment joining A to the origin, O, to form angle θ in standard position. Determine the coordinates of A.

a) Calculate the sine, cosine, and tangent ratios of $\angle\theta$, using x and y.

b) Drag point A around the circle. What do you notice?

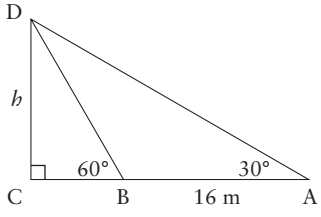
☆**16.** Determine the exact value of $\cos 30° \times \sin 240° + \sin 330°$.

C

17. Determine all the possible measures of θ, where $0° \leq \theta \leq 360°$.

a) $\cos\theta = \frac{\sqrt{3}}{2}$ **b)** $\sin\theta = \frac{1}{2}$

18. Determine h.



☆**19.** Given $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$, show that $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

20. Given $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $x^2 + y^2 = r^2$, show that $\sin^2\theta + \cos^2\theta = 1$.