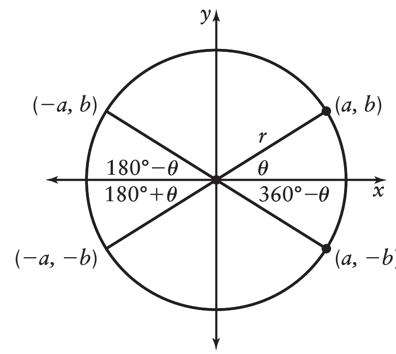
KEY CONCEPTS

- Exactly two angles between 0° and 360° have the same sine ratio.
  For example, $\sin \theta = \sin (180° - \theta) = \frac{b}{r}$
- Exactly two angles between 0° and 360° have the same cosine ratio.
  For example, $\cos \theta = \cos (360° - \theta) = \frac{a}{r}$
- Exactly two angles between 0° and 360° have the same tangent ratio.
  For example, $\tan \theta = \tan (180° + \theta) = \frac{b}{a}$



Example

Given $\cos \theta = \frac{4}{5}$, determine θ, where $0 \le \theta \le 360°$.
Then, determine $\sin \theta$ and $\tan \theta$.

Solution

Determine the measure of angle θ in quadrant I for which $\cos \theta = \frac{4}{5}$.

$$\begin{aligned}\cos \theta &= \frac{4}{5}\\ \angle\theta &= \cos^{-1}\left(\frac{4}{5}\right)\\ &= 36.8698\ldots°\\ &\doteq 36.9°\end{aligned}$$

The cosine ratio is positive in quadrants I and IV, so there is another angle for which $\cos \theta = \frac{4}{5}$ in quadrant IV.

$$\begin{aligned}\angle\theta &= 360° - 36.9°\\ &= 323.1°\end{aligned}$$

Given $\cos \theta = \frac{4}{5}$, the angle θ is approximately 37° or 323°.

If $\cos \theta = \frac{4}{5}$, $x = 4$ and $r = 5$. Determine y.

$$\begin{aligned}r^2 &= x^2 + y^2\\ 5^2 &= (4)^2 + y^2\\ 25 &= 16 + y^2\\ 9 &= y^2\\ \pm 3 &= y\end{aligned}$$

Write the sine and tangent ratios for $\angle\theta$.

$\sin \theta = \pm\frac{3}{5}$ $\qquad \tan \theta = \pm\frac{3}{4}$

A

Unless specified otherwise, all angles are between 0° and 360°.

1. Use a calculator to calculate each pair of ratios. Round decimal answers to four decimal places.

a) sin 58°, sin 122°

b) cos 117°, cos 243°

c) tan 238°, tan 58°

d) sin 310°, sin 230°

e) cos 82°, cos 278°

f) tan 266°, tan 86°

g) sin 65°, sin 115°

h) tan 109°, tan 289°

2. What do you notice about each pair of ratios in question 1? Explain.

3. Use a calculator to evaluate each ratio to four decimal places. Determine a second angle with the same ratio.

a) sin 89°

b) cos 335°

c) sin 132°

d) tan 140°

e) cos 155°

f) tan 305°

g) cos 307°

h) sin 13°

4. The coordinates of a point on the terminal arm of an angle θ are given. Determine the primary trigonometric ratios for θ. Round decimal answers to four decimal places.

a) A(5, 3) **b)** B(−4, 7)

c) C(−6, −2) **d)** D(2, −1)

e) E(10, 3) **f)** F(−5, −7)

g) G(−8, 6) **h)** H(−1, −2)

5. Use a calculator to determine the primary trigonometric ratios for each angle. Round decimal answers to four decimal places.

a) 80°

b) 110°

c) 200°

d) 324°

e) 47°

f) 192°

g) 217°

h) 345°

i) 13°

j) 270°

6. Find the values of θ, where $0° \leq \theta \leq 360°$.

a) $\sin \theta = \frac{\sqrt{3}}{2}$

b) $\cos \theta = \frac{1}{\sqrt{2}}$

c) $\tan \theta = \sqrt{3}$

d) $\sin \theta = 1$

e) $\cos \theta = \frac{\sqrt{3}}{2}$

f) $\tan \theta = 1$

B

☆**7.** Determine two angles between 0° and 360° that have a sine ratio of $\frac{\sqrt{3}}{2}$. Do not use a calculator.

8. Use a diagram to determine two angles between 0° and 360° that have a cosine ratio of $-\frac{1}{2}$. Do not use a calculator.

9. The tangent ratio of each of two angles between 0° and 360° is $-\frac{1}{\sqrt{3}}$. Without using a calculator, determine the angles.

10. Two angles between 0° and 360° have a tangent ratio of −1. Without using a calculator, determine the angles.

11. The point T(3, 4) is on the terminal arm of ∠B in standard position.

a) Draw and label a diagram.

b) Explain how you would determine the primary trigonometric ratios for ∠B.

c) Determine the three primary trigonometric ratios for ∠B.

d) Explain how you would determine the measure of ∠B.

e) Determine the measure of ∠B to the nearest degree.

f) How would the answer for parts a), c), and e) change if point T was reflected in the x-axis?

g) How would the answer for parts a), c), and e) change if point T was reflected in the y-axis?

12. Consider an angle, ∠C, that lies in quadrant III, such that tan C = 0.4663.

a) Draw a diagram to represent this situation.

b) Determine the measure of ∠C to the nearest degree. Explain how you determined the measure of ∠C.

13. Use a calculator to find the values of θ to the nearest degree, where $0° \leq \theta \leq 360°$.

a) $\sin \theta = 0.7312$ **b)** $\cos \theta = 0.4538$

c) $\tan \theta = -1.7321$ **d)** $\sin \theta = 0.9534$

e) $\cos \theta = 0.8862$ **f)** $\tan \theta = 1$

g) $\sin \theta = -0.7317$ **h)** $\cos \theta = -0.3640$

i) $\tan \theta = 2.4751$ **j)** $\sin \theta = -0.9511$

k) $\cos \theta = 0.1829$ **l)** $\tan \theta = 0.0543$

14. Determine another angle that has the same trigonometric ratio as each given angle. Draw a sketch with both angles labelled.

a) sin 75° **b)** cos 190°

c) tan 355° **d)** sin 252°

15. Draw a diagram, and then determine values for the other primary trigonometric ratios, to four decimal places.

a) sin A = 0.9138; ∠A lies in quadrant I

b) cos B = −0.2145; ∠B lies in quadrant II

c) tan C = −8.144; ∠C lies in quadrant IV

16. Determine the approximate measures of all angles from 0° to 360° in each case.

a) The sine ratio is 0.3195.

b) The tangent ratio is 1.4385.

c) The cosine ratio is −0.7431.

☆**17. a)** If $\cos \theta = \frac{1}{3}$, find two possible values for $\sin \theta$.

b) For each value of $\sin \theta$ from part a), find the value(s) of θ.

☆**18.** The point S(−5, −6) is on the terminal arm of ∠A.

a) Determine the primary trigonometric ratios for ∠A.

b) Determine the measure of ∠A.

c) Determine the primary trigonometric ratios for ∠B such that sin B = sin A.

d) Determine the measure of ∠B.

C

☆**19. a)** Solve $2x^2 - x - 1 = 0$.

b) Explain how the equation in part a) is related to $2 \sin^2 \theta - \sin \theta - 1 = 0$.

c) Solve $2 \sin^2 \theta - \sin \theta - 1 = 0$.

20. Determine all the possible measures of θ, where $0° \leq \theta \leq 360°$.

a) $\cos^2 \theta - 1 = 0$ **b)** $\tan^2 \theta = 3$

21. Given $\tan A = \frac{a + b}{a - b}$ and ∠A in quadrant I, determine expressions for sin A and cos A. State any restrictions on the values of a and b.