

1.4 Solving Problems Using Primary Trigonometric Ratios

KEY CONCEPTS

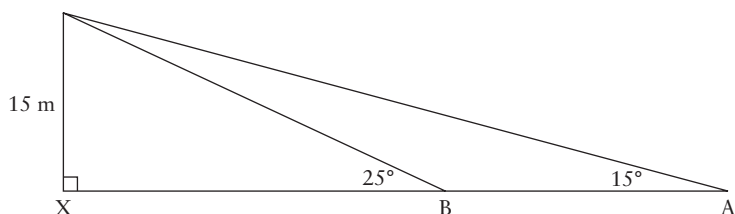
- The primary trigonometric ratios can be used to solve problems that involve right triangles.
- Problems involving right triangles in two dimensions and in three dimensions can be solved using the Pythagorean theorem and/or the primary trigonometric ratios.

Example

From an army tank, the angle of elevation to the top of a 15-m tower is 15° . After advancing toward the tower, the angle of elevation is 25° . How far did the tank advance?

Solution

Sketch and label a diagram.



Write an expression for the initial distance from the tank to the tower, AX.

$$\begin{aligned}\tan 15^\circ &= \frac{15}{AX} \\ AX &= \frac{15}{\tan 15^\circ}\end{aligned}$$

Write an expression for the new distance from the tank to the tower, BX.

$$\begin{aligned}\tan 25^\circ &= \frac{15}{BX} \\ BX &= \frac{15}{\tan 25^\circ}\end{aligned}$$

Determine the distance the tank advanced, $AX - BX$.

$$\begin{aligned}\text{Distance} &= AX - BX \\ &= \frac{15}{\tan 15^\circ} - \frac{15}{\tan 25^\circ} \\ &= 23.8131\dots \\ &\doteq 23.8\end{aligned}$$

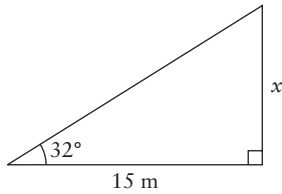
The tank advanced approximately 23.8 m.

A

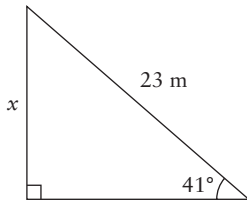
Unless specified otherwise, all angles are between 0° and 360° . Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

1. Determine the length of x .

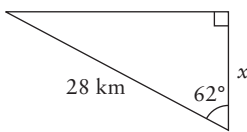
a)



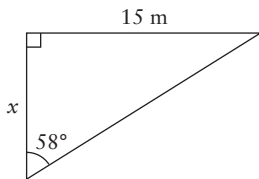
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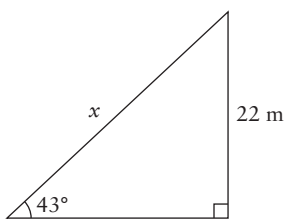
c)



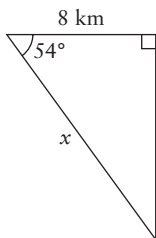
d)



e)

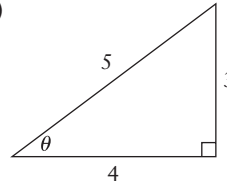


f)

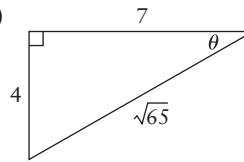


2. Determine $\sin \theta$, $\cos \theta$, and $\tan \theta$. Then, determine the measure of angle θ .

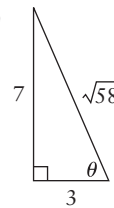
a)



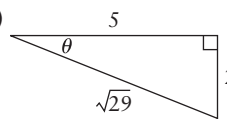
b)



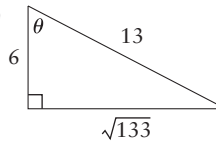
c)



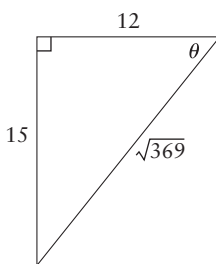
d)



e)



f)



3. Sketch each triangle and solve if possible.

a) In $\triangle ABC$, $\angle A = 90^\circ$, $\angle B = 36^\circ$, and $b = 15.3\text{ m}$

b) In $\triangle DEF$, $\angle D = 90^\circ$, $\angle E = 48^\circ$, and $e = 9.6\text{ cm}$

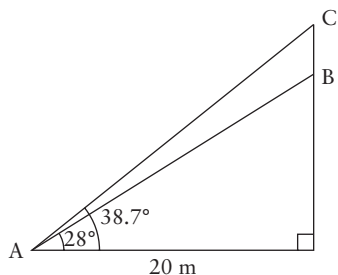
c) In $\triangle JKL$, $\angle J = 90^\circ$, $\angle K = 62^\circ$, and $j = 7.2\text{ km}$

d) In $\triangle PQR$, $\angle P = 53^\circ$, $\angle Q = 37^\circ$, and $p = 13.5\text{ m}$

4. From a point 5 m from the base of a tree, the angle of elevation to the top of the tree is 38° . **Draw a diagram, and** then determine the height of the tree.
5. The shadow of a building that is 28 m in height measures 20 m in length. **Draw a diagram, and** then determine the angle of elevation of the sun.
6. If you know a triangle contains a right angle, what other information is required to solve the triangle?

B

7. A rectangle is 16 cm long and 12 cm wide.
 - a) Determine the length of the diagonals of the rectangle.
 - b) Determine the measure of the angle formed by the length of the triangle and a diagonal.
8. Two guy wires from the top of a radio tower are fixed to the ground 20 ft apart, at points A and B. The tower is halfway between points A and B. From both points, the angle of elevation to the top of the tower is 72° .
 - a) Draw a diagram.
 - b) Determine the height of the tower.
 - c) Determine the total length of the two guy wires.



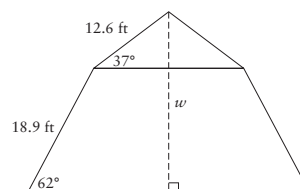
9. Kevin is standing at a point A, 20 m from the base of a building. From point A, the angle of elevation to point B is 28° , and the angle of elevation to point C is 38.7° . **Determine the vertical distance between points B and C.**

- ☆10. Zac is standing 10 m from the centre of a fountain. Water shoots straight up from the fountain's centre. Shortly after, the angle of inclination of the highest point of the water is 46° . **Fifteen seconds later, the angle of inclination is 60° .**

- a) Draw a diagram.
- b) Determine the height of the highest point of the water at each angle of inclination.
- c) Determine the average speed, in metres per second, at which the water shoots up.

11. Explain how you could find the height of a radio antenna on top of a tall observation tower using a measuring tape, a clinometer, and trigonometry.

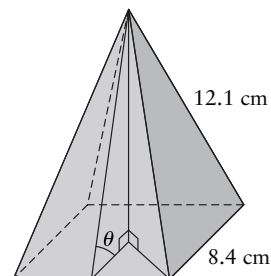
- ☆12. A patio has the shape shown. Determine the total width, w , of the patio.



13. Boat A is 5 km northeast of a lighthouse, L, and boat B is 7 km southeast of the lighthouse.

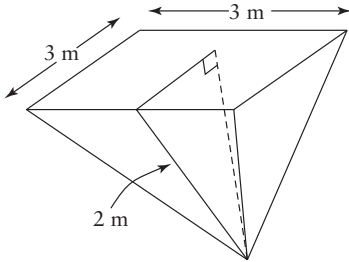
- a) Determine the distance between the boats.
- b) Determine the measure of $\angle ALB$.

14. The base of a square-based pyramid has length 8.4 cm. The slant height of the pyramid is 12.1 cm.



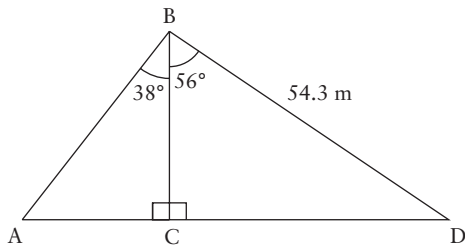
- a) Determine the height of the pyramid.
- b) What is the angle between one triangular face of the pyramid and the base?

15. Peter designed a garden pond, which is in the shape of an inverted square-based pyramid. The side lengths of the base are 3 m, and the slant height is 2 m.



- a) Determine the greatest depth of the garden pond.
 b) Determine the maximum volume of water the pond can hold.
16. The roof of a house is constructed in the shape of a square-based pyramid, and the lengths of the sides of the base are 26 ft. The angle of elevation formed by each side of the roof with the top of the house is 32° . Determine the height of the roof.

17. Determine the distance AC.



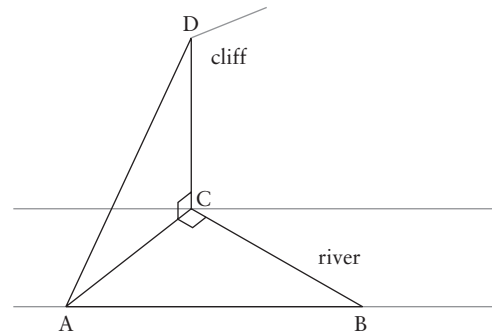
18. Ella is standing on a bridge. From her location, the angle of elevation to the top of a nearby building is 28° , and the angle of depression to the base of the building is 42° . The least distance between Ella and the building is 40 m. Determine the height of the building.

19. Adriana is standing at point A, directly across the river from point B, at the base of a cliff. From point A, the angle of elevation to point C at the top of the cliff is 60° . Dominic is standing at point D, on the same side of the river and 30 ft farther down from point A. $AB = DB$ and $\angle ABD = 90^\circ$.

- a) Is there enough information to determine, BC? Explain.
 b) If your answer to part a) is yes, determine BC. If your answer to part a) is no, explain what information is needed.

C

20. Louis is standing at point A on the shoreline of a river. From where he is standing, the angle of elevation to point D at the top of a cliff across the river is 25° . Louis walks to point B, which is 30 m east of point A, and determines that $\angle ACB$ is 90° and $\angle CAB$ is 38° . Determine the height of the cliff.



21. From point A on the level ground, the angle of elevation of the top of a building is 25° . From point B, 10 ft closer to the base of the building, the angle of elevation of the top of the building is 35° . Determine the height of the building.