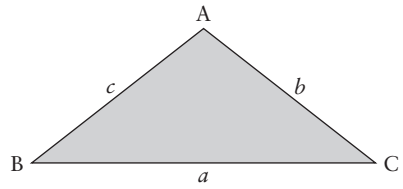
KEY CONCEPTS

- For any $\triangle ABC$, the sine law states that

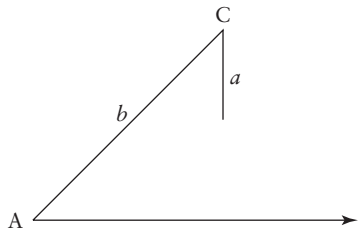
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

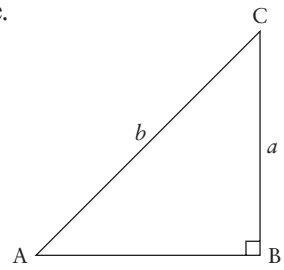


- A side length can be determined if the corresponding opposite angle and one other side-angle pair are known.
- An angle measure can be determined if the corresponding opposite side and one other side-angle pair are known.
- If the lengths of two sides and the measure of one angle are known, the ambiguous case is possible. Given $\triangle ABC$ with known side lengths a and b and known $\angle A$, if $a < b$, there are three possibilities:

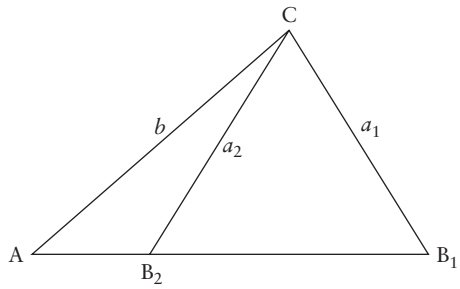
If $a < b \sin A$, then no triangle is possible.



If $a = b \sin A$, then only one right triangle is possible.

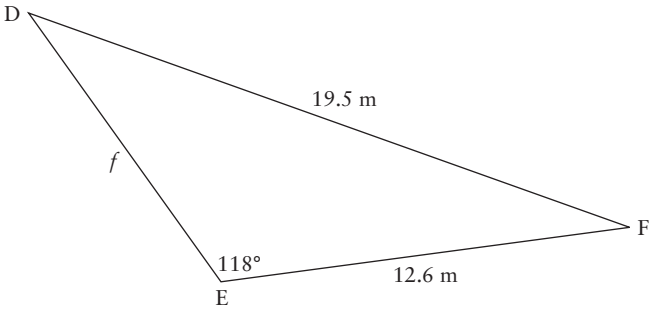


If $a > b \sin A$, then two triangles are possible. This is the ambiguous case.



Example

Determine the length of f to one decimal place.



Solution

Two side lengths and one angle measure are given. Check for the ambiguous case.

Side e is opposite the known angle.

Since $e > d$, only one triangle is possible.

Determine $\angle D$.

$$\frac{\sin D}{d} = \frac{\sin E}{e}$$

$$\frac{\sin D}{12.6} = \frac{\sin 118°}{19.5}$$

$$\sin D = \frac{12.6 \sin 118°}{19.5}$$

$$\angle D = \sin^{-1}\left(\frac{12.6 \sin 118°}{19.5}\right)$$

$$= 34.7864…°$$

$$\doteq 34.8°$$

Determine $\angle F$.

$$\angle F = 180° - (118° + 34.8°)$$

$$= 27.2°$$

Determine f.

$$\frac{f}{\sin F} = \frac{e}{\sin E}$$

$$\frac{f}{\sin 27.2°} = \frac{19.5}{\sin 118°}$$

$$f = \frac{19.5 \sin 27.2°}{\sin 118°}$$

$$= 10.0950…$$

$$\doteq 10.1$$

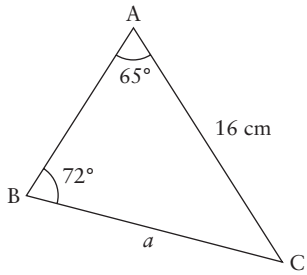
The length of side f is approximately 10.1 m.

A

Unless specified otherwise, all angles are between 0° and 360°. Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

1. Determine the length of each indicated side.

a)



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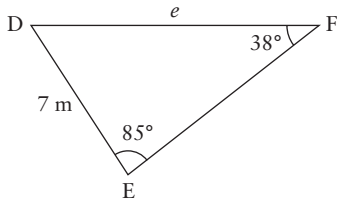

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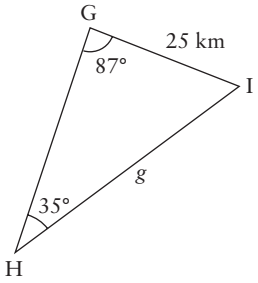
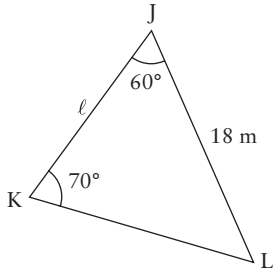
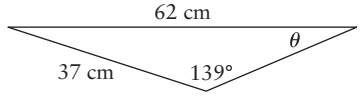


d)



2. Determine the measure of angle θ.

a)



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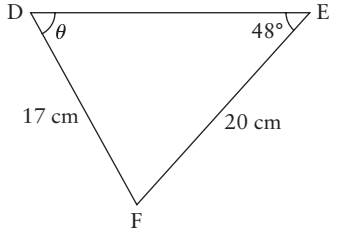


3. Determine the length of the indicated side.

a) In $\triangle ABC$, $\angle A = 52°$, $\angle B = 43°$, and $b = 9$ m. Determine a.

b) In $\triangle DEF$, $\angle D = 65°$, $\angle E = 49°$, and $e = 15.7$ cm. Determine f.

4. Solve each triangle.

a) In $\triangle ABC$, $\angle A = 72°$, $\angle B = 47°$, and $a = 15$ km.

b) In $\triangle DEF$, $\angle D = 65°$, $\angle F = 51°$, and $d = 8$ m.

c) In $\triangle JKL$, $\angle K = 52°$, $k = 25$ cm, and $\ell = 28$ cm.

d) In $\triangle PQR$, $\angle R = 35°$, $p = 30$ m, and $r = 37$ m.

5. In $\triangle HJK$, $h = 7.2$ m, $j = 8.4$ m, and $\angle H = 68°$.

a) Calculate $j \sin H$.

b) How many solutions are possible for $\triangle HJK$?

6. In $\triangle ABC$, $a = 4.5$ cm, $b = 5.2$ cm, and $\angle A = 37°$.

a) Calculate $b \sin A$.

b) How many different $\triangle ABC$ are possible?

c) Solve $\triangle ABC$.

7. In $\triangle PQR$, $p = 5.3$ km, $q = 10.6$ km, and $\angle P = 30°$.

a) Determine the number of possible solutions for $\triangle PQR$.

b) Solve $\triangle PQR$.

B

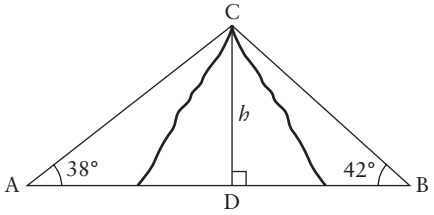
★8. Given $\triangle MNP$ with $m = 6.7$ cm, $n = 12.4$ cm, and $\angle M = 26°$, determine p.

9. Sketch each triangle. Then, calculate the length of the third side.

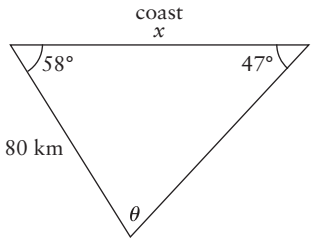
a) In $\triangle ABC$, $c = 10$ cm, $b = 12$ cm, and $\angle B = 44°$.

b) In $\triangle RST$, $r = 6.2$ m, $s = 8.1$ m, and $\angle R = 39°$.

★**10.** Sheila is standing at point A west of a mountain in the Kootenay region of British Columbia. From point A, the angle of elevation of the top of the mountain is 38°. From point B, which is 10 560 ft to the east of point A, the angle of elevation of the top of the same mountain is 42°. Determine the height of the mountain.



★**11.** A triangular delta has been formed at the mouth of a large river by sediment deposits. The distance from the coast to the starting point of the delta is 80 km, and the angles formed by the coastline and two sides of the triangular delta are 58° and 47°. Determine the length of the delta along the coastline.



12. A triangular deck is to be added to the back wall of a house. The length of the side of the deck that will be attached to the house is 20 ft, and the other two sides form angles of 32° and 65° with the side that is attached to the wall of the house.

a) Draw a diagram to represent this situation.

b) Determine the length of the shortest side of the deck.

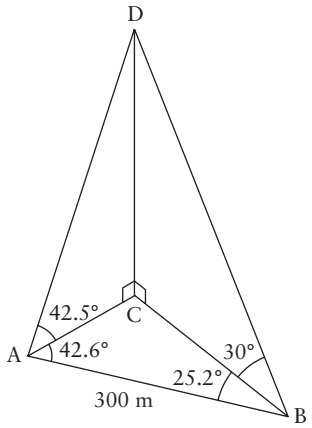
13. Michael is an architect who has designed a cottage with a roof that has two sides with different slopes. The shorter side of the roof is 12 ft in length and makes an angle of 68° with the top of the building. The longer side of the roof makes an angle of 28° with the top of the building.

a) Draw a diagram to represent this situation.

b) Determine the length of the other side of the roof.

C

14. A rocket, launched vertically from point C, is tracked by two tracking stations at A and B. Data from the launch were recorded according to the diagram below.



a) Determine the height, h, of the rocket as calculated from tracking station A.

b) Determine the height, h, of the rocket as calculated from tracking station B.

c) Are the approximate results for the calculated heights of the rocket the same or different for parts a) and b)? Explain.

15. Two boats, A and B, are in a harbour close to a marina at point C. Boat A is 38 ft from the marina. The angle between AC and BC is 51°, and the angle between AC and AB is 65°. From boat A, the angle of depression to the anchor directly below boat B is 50°. Determine the distance from boat A to the anchor directly below boat B.