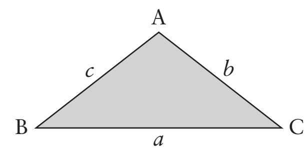
KEY CONCEPTS

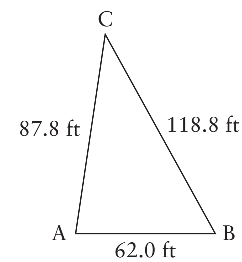
- The cosine law states that for any △ABC,
  $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$



- A side length can be determined if the lengths of the other two sides and the measure of the contained angle are known.
- An angle measure can be determined if all three side lengths are known.

Example

The lengths of the sides of a triangular property on Sydenham Lake are 62.0 ft, 87.8 ft, and 118.8 ft. Determine the angles formed between the sides of the triangular property to the nearest tenth of a degree.



Solution

For △ABC, the side lengths are $a = 118.8$, $b = 87.8$, and $c = 62.0$.
Use the cosine law to determine ∠A.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{87.8^2 + 62.0^2 - 118.8^2}{2(87.8)(62.0)}$$
$$= -0.2351\ldots$$
$$\angle A = 103.6030\ldots°$$

Check for the ambiguous case. Given ∠A and $a > b$, only one triangle is possible.
Use the sine law to determine ∠B.

$$\frac{\sin B}{87.8} = \frac{\sin 103.6030\ldots°}{118.8}$$
$$\sin B = \frac{87.8 \sin 103.6030\ldots°}{118.8}$$
$$= 0.7183\ldots$$
$$\angle B = 45.9164\ldots°$$

Determine ∠C.

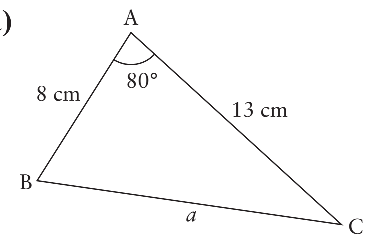
$$\angle C = 180° - (103.6030\ldots° + 45.9164\ldots°)$$
$$= 30.4805\ldots°$$
$$\doteq 30.5°$$

A

Unless specified otherwise, all angles are between 0° and 360°. Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

1. Determine the length of the indicated side.

a)



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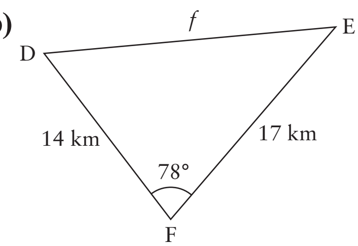

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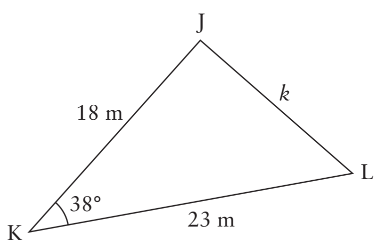

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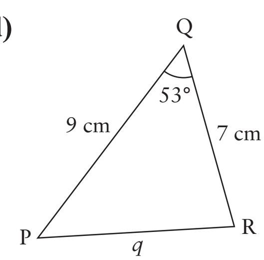

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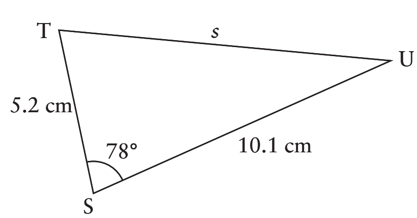
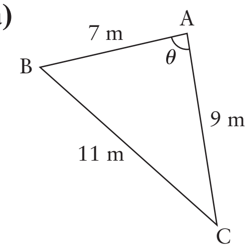


2. Determine the measure of angle θ.

a)



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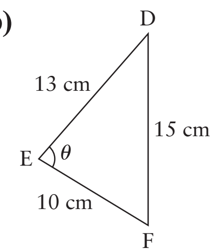
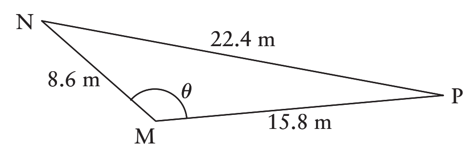
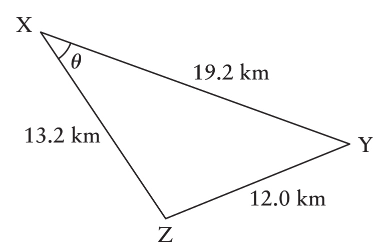


c)



d)



3. Determine the length of the indicated side.

a) In △ABC, ∠A = 32°, b = 22 cm, and c = 15 cm. Determine a.

b) In △DEF, ∠F = 71°, d = 9 m, and e = 5 m. Determine f.

4. Sketch, and then solve if possible.

a) In △ABC, ∠A = 48°, b = 18 cm, and c = 12 cm.

b) In △JKL, j = 23 cm, k = 25 cm, and l = 27 cm.

c) In △PQR, p = 15 m, q = 17 m, and r = 20 m.

d) In △DEF, ∠D = 62°, e = 12 m, and f = 10 m.

B

5. An air traffic controller is tracking two airplanes, flying at the same altitude, on the radar screen. Airplane A, en route to Saskatoon, is 23.8 km away in a direction 60° west of north. Airplane B, en route to Calgary, is 25.2 km away in a direction 70° west of north.

a) Draw a diagram to illustrate this situation.

b) How far apart are the two airplanes?

☆**6.** Building plans show a triangular cottage property with side lengths 42.7 m, 49.4 m, and 58.2 m.

a) Sketch a diagram of this property.

b) Determine the angles formed between the sides of the triangular property.

7. A funnel is in the shape of a right circular cone. The angle at the vertex of the cone is 36°. Find the diameter of the funnel at a point on the face 6 in. from the vertex.

☆**8.** Two ships, A and B, both started out at point C. Ship A is now 40 km from point C in a direction 30° west of north. Ship B is now 60 km from point C in a direction 40° west of south. Determine the distance between the two ships.

☆**9.** Two checkpoints, A and B, in an orienteering course are 5 km apart. Another checkpoint, C, is 3 km from checkpoint A. The angle between AB and AC is 25°. Determine the distance between checkpoints B and C.

10. The pendulum on a clock is 30 cm long. The pendulum moves a horizontal distance of 5 cm from one end of each swing to the other. Find the angle through which the pendulum swings.

11. Bill started at point A and walked 22 m to a point B, then walked 17 m to a point C, and then walked back to point A. The path of his walk formed a triangle with ∠BAC = 42°. Leslie calculated that the distance Bill walked from point C to point A was 24.9 m. Kelly calculated that the distance that Bill walked from point C to point A was 7.8 m.

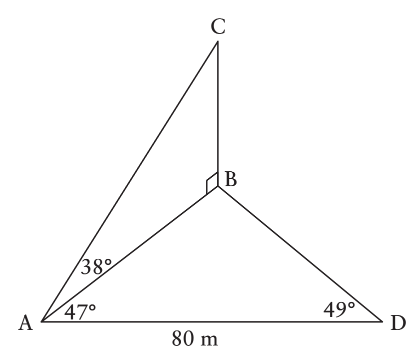
a) Do you think that Leslie's, Kelly's, or both girls' calculations are correct?

b) Draw a diagram to represent Leslie's calculations for the distance Bill walked from point C to point A.

c) Draw a diagram to represent Kelly's calculations for the distance Bill walked from point C to point A.

d) Explain the differences between Leslie's calculations and Kelly's calculations.

12. Faiza would like to calculate the height of a cliff. From point A where she is standing, the angle of elevation to point C at the top of the cliff is 38°. If Faiza walks 80 m east to point D, △ABD is formed, where B is the base of the cliff, such that ∠DAB = 47° and ∠ADB = 49°.



a) Determine the height, BC, of the cliff.

b) Determine the distance from point A, where Faiza is standing, to point C at the top of the cliff.

13. A drill bit is in the shape of a cone. The angle at the vertex of the cone is 15°. The length of the slanted side of the drill bit is 13 mm. Determine the diameter at the top of the drill bit.

14. In order to measure the height of a cliff, AB, a surveyor uses a baseline, CD, and records the following data: ∠BCD = 64.3°, ∠BDC = 55.2°, CD = 240 m, and ∠ACB = 28°. Draw a diagram to illustrate this situation, and then determine the height of the cliff.

C

15. Two roads intersect at an angle of 38°. Two cars, A and B, leave the intersection at the same time and both travel in an easterly direction. After 2 h, car A has travelled 140 km and car B has travelled 160 km. At this time, a hot-air balloon is directly above the line between the two cars. The balloonist notices that the angle of depression to the faster car, which is 1 km away from the hot-air balloon, is 25°. Determine the distance from the hot-air balloon to the slower car.

Chapter 1: Checklist

By the end of this chapter, I will be able to:

- determine the values of the trigonometric ratios for angles between 0° and 360°
- solve problems using the primary trigonometric ratios, the sine law, and the cosine law
- determine the exact values of the sine, cosine, and tangent ratios of the special angles 0°, 30°, 45°, 60°, 90°, and their multiples
- determine the values of the sine, cosine, and tangent ratios of angles from 0° to 360°, through investigation using a variety of tools and strategies
- determine the measures of two angles from 0° to 360° for which the value of a given trigonometric ratio is the same
- solve multi-step problems in two and three dimensions, including those that arise from real-world applications by determining the measures of the sides and angles of right triangles, using the primary trigonometric ratios
- solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (including the ambiguous case) and the cosine law