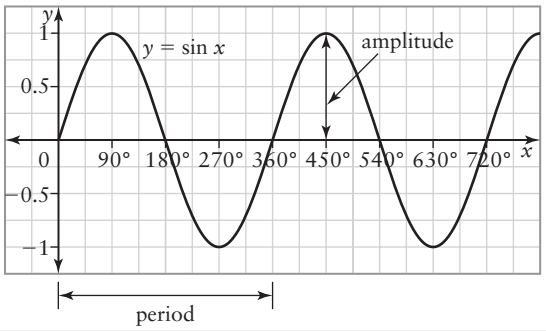
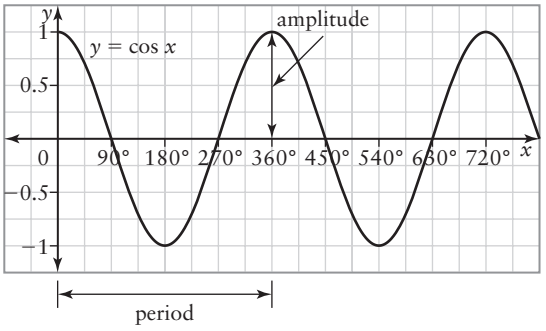
Chapter 2 Sinusoidal Functions

2.1 Graphs of Sinusoidal Functions

KEY CONCEPTS

- The sine function can be represented by the set of ordered pairs $(x, \sin x)$, where x is the angle in standard position measured in degrees. The equation is of the form $y = \sin x$ or $f(x) = \sin x$.
- The cosine function can be represented by the set of ordered pairs $(x, \cos x)$, where x is the angle in standard position measured in degrees. The equation is of the form $y = \cos x$ or $f(x) = \cos x$.
- The sine and cosine functions are periodic and their graphs have a wavelike appearance.
- The amplitude of the sine function and the cosine function is 1.
- The period of the sine function and the cosine function is 360°.





Example

Graph each of the following functions for $0° \leq x \leq 360°$. Then, describe the key features of each function. These include:

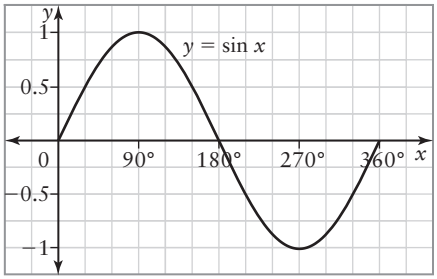
- the maximum and minimum values
- the amplitude
- the period
- the domain and range
- the x-intercepts and y-intercepts
- the intervals of increase and decrease

a) $y = \sin x$

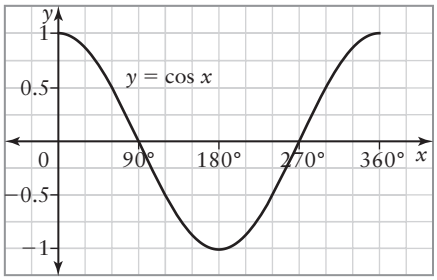
b) $y = \cos x$

Solution

a) The maximum is 1 and the minimum is –1.
The amplitude is 1.
The period is 360°.
The domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$.
The x-intercepts are 0°, 180°, and 360°.
The y-intercept is 0.
The intervals of increase are $0° < x < 90°$ and $270° < x < 360°$.
The interval of decrease is $90° < x < 270°$.

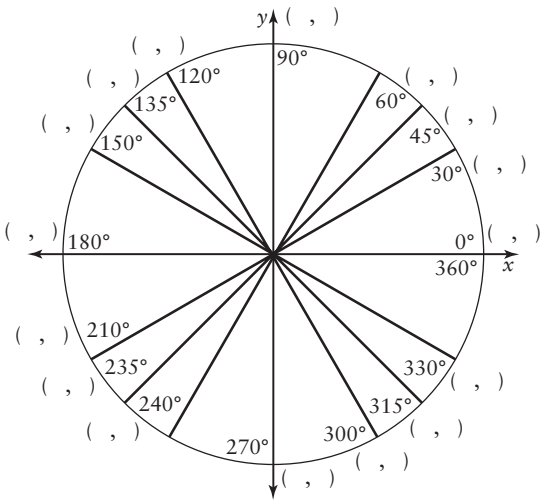


b) The maximum is 1 and the minimum is −1.
The amplitude is 1.
The period is 360°.
The domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$.
The x-intercepts are 90° and 270°. The y-intercept is 1.
The interval of increase is $180° < x < 360°$.
The interval of decrease is $0° < x < 180°$.



A

1. Copy the unit circle. Use the exact sine and cosine ratios for the special angles from 0° to 360° to complete the ordered pairs on the unit circle.



2. a) Refer to your answer to question 1. Determine the exact values for sin x. Then, use a calculator to determine the approximate values for sin x, to three decimal places, for each angle. Record your answers in a table.

x	$y = \sin x$	
	Exact	**Approximate**
0°		
30°		
45°		
60°		
90°		

b) Plot the ordered pairs (x, sin x) on a graph for $0° \leq x \leq 360°$, using the approximate values for sin x. Draw a smooth curve through the points.

c) Predict the shape of the graph past 360°. Justify your prediction.

3. Use Technology

a) Use a graphing calculator. Graph the function $y = \sin x$ for $0° \leq x \leq 360°$.

b) Explain how you can use the graphing calculator to generate the coordinates of points on $y = \sin x$.

c) Determine the approximate values for $\sin x$ for the special angles and their related angles. Record the values in a table.

x	$y = \sin x$
0°	
30°	
45°	
60°	
90°	

d) Compare the approximate values for $\sin x$ that you determined using the unit circle to the approximate values for $\sin x$ that you calculated using a graphing calculator. What do you notice about the approximate values?

4. a) Refer to your answer to question 1. Determine the exact values for $\cos x$. Then, use a calculator to determine the approximate values for $\cos x$, to three decimal places, for each angle. Record your answers in a table.

	$y = \cos x$	
x	**Exact**	**Approximate**
0°		
30°		
45°		
60°		
90°		

b) Plot the ordered pairs $(x, \cos x)$ on a graph for $0° \leq x \leq 360°$, using the approximate values for $\cos x$. Then, draw a smooth curve through the points.

c) Predict the shape of the graph past 360°. Justify your prediction.

5. Use Technology

a) Use a graphing calculator. Graph the function $y = \cos x$.

b) Explain how you can use the graphing calculator to generate the coordinates of points on $y = \cos x$.

c) Determine the approximate values for $\cos x$ for the special angles and their related angles. Record the values in a table.

x	$y = \cos x$
0°	
30°	
45°	
60°	
90°	

d) Compare the approximate values for $\cos x$ that you determined using the unit circle to the approximate values for $\cos x$ that you calculated using a graphing calculator. What do you notice about the approximate values?

6. Refer to your answers for questions 2 to 5.

a) Compare the graphs of $y = \sin x$ and $y = \cos x$. How are the graphs similar? How are they different?

b) Is $y = \sin x$ a function? Justify your answer.

c) Is $y = \cos x$ a function? Justify your answer.

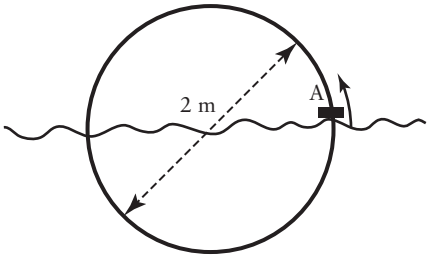
B

☆ **7.** **a)** Sketch the graphs of $y = \sin x$ and $y = \cos x$ on the same set of axes.

b) How many times do the graphs intersect on the interval $0° \leq x \leq 360°$?

c) Refer to the unit circle. Explain why graphs of the sine and cosine functions have the same values at the points where they intersect.

8. Karen has a water wheel with diameter 2 m in her garden pond. The water wheel rotates counterclockwise. Displacement is measured relative to the centre of the wheel.



a) Sketch a graph of the horizontal displacement of point A versus the angle of rotation for one rotation of the wheel. Which function models the horizontal displacement? Justify your choice.

b) Sketch a graph of the vertical displacement of point A versus the angle of rotation for one rotation of the wheel. Which function models the vertical displacement? Justify your choice.

☆ **9.** The hour hand of a clock on a tower has length 1 m. Displacement is measured relative to the centre of the clock.

a) Sketch a graph of the horizontal displacement of the tip of the hour hand versus the angle (from vertical) through which the hand rotates for 36 h, beginning at 12 p.m. Which function models the horizontal displacement? Justify your choice.

b) Sketch a graph of the vertical displacement of the tip of the hour hand versus the angle (from vertical) through which the hand rotates for 36 h, beginning at 12 p.m. Which function models the vertical displacement? Justify your choice.

c) How many cycles appear in each graph?

10. Compare the graphs for part a) of each of questions 8 and 9. Explain any differences in the graphs with reference to the situations.

C

11. Use Technology Use a graphing calculator to graph $y = \sin x + \cos x$, $0° \leq x \leq 360°$.

a) Describe the shape of the graph.

b) Determine the maximum and minimum values of the function as both exact and approximate values. At what x-values do these values occur?

c) Determine the amplitude of the function.

d) Determine the y-intercept of the function.

e) Determine the x-intercept(s) of the function.

f) How can the answers to part b) be used to answer part e)?

g) How can the unit circle be used to answer this question?

12. Use Technology

a) Graph $y = (\sin x)^2$ and $y = (\cos x)^2$, $0° \leq x \leq 360°$, using technology.

b) How are the graphs alike? How are they different?

c) Determine if the relations $y = (\sin x)^2$ and $y = (\cos x)^2$ are functions. Justify your answer.