KEY CONCEPTS

- For the functions $y = \sin(x - d) + c$ and $y = \cos(x - d) + c$, d represents the phase shift, or horizontal translation, and c represents the vertical shift, or vertical translation.
- For $y = \sin(x - d) + c$ and $y = \cos(x - d) + c$, the amplitude is 1, the period is 360°, the horizontal shift is d, the vertical shift is c, the domain is $\{x \in \mathbb{R}\}$, and the range is $\{y \in \mathbb{R}, (-1 + c) \leq y \leq (1 + c)\}$.
- When graphing transformations of trigonometric functions, it is helpful to transform key points, such as the x-intercepts and y-intercepts and the maximum/minimum points.

Example

Make a table of values, and then graph $f(x) = \sin x$ and $g(x) = \sin(x - 45°) + 2$ on the same coordinate grid on the interval $0° \leq x \leq 360°$.

a) Determine the period and the amplitude of $f(x)$ and $g(x)$.

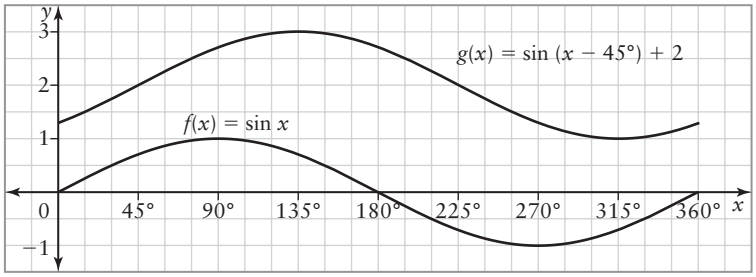
b) Determine the phase shift and the vertical shift of $g(x)$ relative to $f(x)$.

c) Do the translations affect the period and amplitude? Explain.

d) Determine the domain and range of $f(x)$ and $g(x)$.

Solution

x	$f(x)$	$g(x)$
0°	0	1.034
45°	0.707	2
90°	1	2.707
135°	0.707	3
180°	0	2.707
225°	–0.707	2
270°	–1	1.293
315°	–0.707	1
360°	0	2



a) Both $f(x)$ and $g(x)$ have amplitude 1 and period 360°.

b) The phase shift is 45° to the right and the vertical shift is 2 up.

c) No, the translations do not change the shape of the function, only the position.

d) For $f(x)$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$.
For $g(x)$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, 1 \leq y \leq 3\}$.

A

1. Use Technology

a) Graph $f(x) = \sin x$, $g(x) = \sin (x - 30°)$, and $h(x) = \sin (x + 60°)$ on the same screen. Use these window settings.

```
WINDOW
 Xmin=0
 Xmax=360
 Xscl=15
 Ymin=-1.25
 Ymax=1.25
 Yscl=.5
 Xres=1
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b) How do the graphs of $g(x)$ and $h(x)$ compare to the graph of $f(x)$? Refer to the key features of the functions in your answer.

c) Predict the key features of the graphs of $p(x) = \sin (x - 15°)$ and $q(x) = \sin (x + 45°)$.

d) Graph $p(x)$ and $q(x)$ to confirm your predictions.

2. Use Technology

a) Graph $b(x) = \sin x$, $c(x) = \sin x + 1$, and $d(x) = \sin x - 2$ on the same screen. Use these window settings.

```
WINDOW
 Xmin=0
 Xmax=360
 Xscl=15
 Ymin=-4
 Ymax=2
 Yscl=.5
 Xres=1
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b) How do the graphs of $c(x)$ and $d(x)$ compare to the graph of $b(x)$? Refer to the key features of the functions in your answer.

c) Predict the key features of the graphs of $f(x) = \sin x - 3$ and $g(x) = \sin x + 4$.

d) Graph $f(x)$ and $g(x)$ to confirm your predictions.

3. Determine the phase shift of each function.

a) $y = \sin (x - 60°)$

b) $y = \cos (x + 90°)$

c) $y = \sin (x + 30°)$

d) $y = \cos (x + 45°)$

e) $y = \sin (x - 90°)$

f) $y = \cos (x - 45°)$

4. Determine the vertical shift and state the range of each function.

a) $y = \sin x + 3$

b) $y = \cos x + 4$

c) $y = \sin x - 5$

d) $y = \cos x - 6$

e) $y = \sin x - 2$

f) $y = \cos x + 1$

5. Determine the phase shift and the vertical shift with respect to $y = \sin x$ for each function.

a) $y = \sin (x + 46°) + 2$

b) $y = \sin (x + 72°) - 3$

c) $y = \sin (x - 65°) + 4$

d) $y = \sin (x - 41°) - 5$

e) $y = \sin (x + 27°) - 6$

f) $y = \sin (x - 80°) + 1$

6. Determine the phase shift and the vertical shift with respect to $y = \cos x$ for each function.

a) $y = \cos (x - 32°) + 2$

b) $y = \cos (x + 55°) - 4$

c) $y = \cos (x + 73°) - 7$

d) $y = \cos (x - 42°) + 3$

e) $y = \cos (x - 18°) - 1$

f) $y = \cos (x + 64°) - 2$

B

7. Refer to your answers to questions 1 to 6. Explain how you can use the key features of sinusoidal functions to graph translated functions.

8. For each function, determine the horizontal or vertical shift, and then sketch two complete cycles.

a) $y = \sin x + 2$

b) $y = \sin (x - 45°)$

c) $y = \cos (x + 30°)$

d) $y = \cos x - 3$

9. For each function, determine the phase shift and the vertical shift, and then sketch two complete cycles.

a) $y = \sin (x - 25°) + 1$

b) $y = \sin (x + 35°) - 2$

☆**c)** $y = \cos (x + 50°) - 3$

d) $y = \cos (x - 20°) + 4$

e) $y = \sin (x + 60°) - 2$

f) $y = \cos (x + 45°) + 3$

10. How can you tell from the equation of a sinusoidal function if a translation represents a phase shift or a vertical shift?

11. Write the equation of each transformed function.

a) The function $y = \sin x$ is transformed so that it has a phase shift left 58° and a vertical shift down 4.

b) The function $y = \cos x$ is transformed so that it has a phase shift right 67° and a vertical shift up 5.

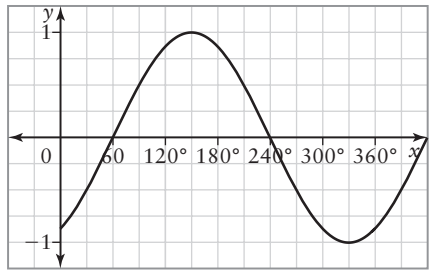
c) The function $y = \cos x$ is translated 41° left and 8 units down.

d) The function $y = \sin x$ is translated 15° right and 2 units up.

12. Graph $y = \sin x$ and $y = \cos (x - 90°)$ on the same set of axes. What do you notice? Explain.

13. For each graph, write two equations, one in the form $y = \sin (x - d) + c$ and one in the form $y = \cos (x - d) + c$.

a)



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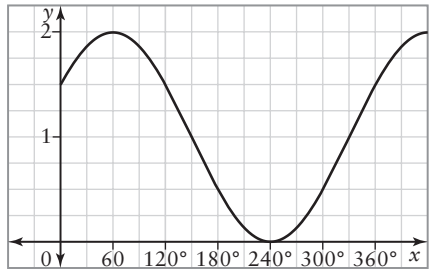

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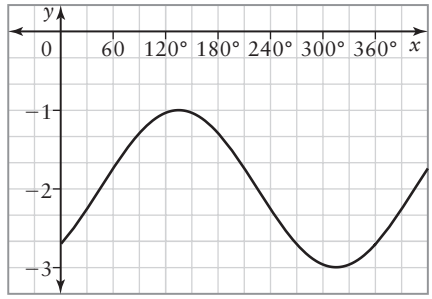

C

14. Use Technology

a) Graph $y = \sqrt{\sin x}$ over two cycles.

b) How do you think:

i) the graphs of $y = \sqrt{\sin x} + 2$ and $y = \sqrt{\sin x}$ will differ?

ii) the graphs of $y = \sqrt{\sin (x - 60°)}$ and $y = \sqrt{\sin x}$ will differ?

Check, using technology.

15. Use Technology

a) Graph $y = \sqrt{\cos x}$ over two cycles.

b) How do you think:

i) the graphs of $y = \sqrt{\cos x} - 3$ and $y = \sqrt{\cos x}$ will differ?

ii) the graphs of $y = \sqrt{\cos (x + 45°)}$ and $y = \sqrt{\cos x}$ will differ?

Check, using technology.