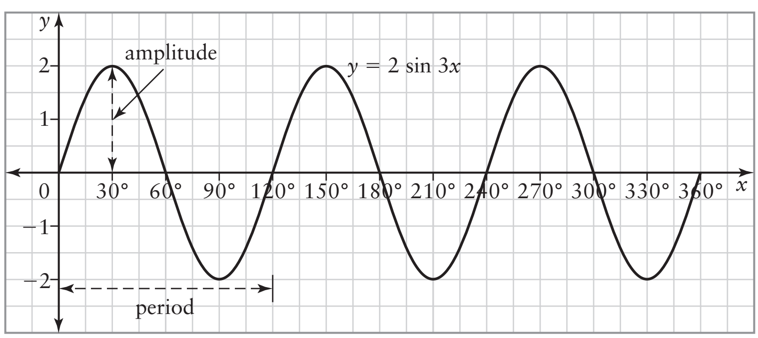
KEY CONCEPTS

- In an equation of the form $y = a \sin x$ or $y = a \cos x$, the coefficient a determines the vertical stretch or compression of the sine or cosine function.
  - If $0 < |a| < 1$, the function is compressed vertically by a factor of $|a|$.
  - If $|a| > 1$, the function is stretched vertically by a factor of $|a|$.
  - If $a < 0$, the function is reflected in the x-axis.
- In an equation of the form $y = \sin kx$ or $y = \cos kx$, the coefficient k determines the horizontal stretch or compression of the sine or cosine function.
  - If $0 < |k| < 1$, the function is stretched horizontally by a factor of $\frac{1}{|k|}$.
  - If $|k| > 1$, the function is compressed horizontally by a factor of $\frac{1}{|k|}$.
  - If $k < 0$, the function is reflected in the y-axis.
- For the functions $y = a \sin kx$ and $y = a \cos kx$, the amplitude, a, of the function is $|a|$ and the period, p, of the function is $p = \frac{360°}{|k|}$. For example, for the function $y = 2 \sin 3x$, the amplitude is 2 and the period is $\frac{360°}{3}$, or 120°.



Example

Make a table of values, and then graph $f(x) = \cos x$ and $g(x) = 3 \cos 2x$ on the same coordinate grid on the interval $0° \leq x \leq 360°$.

a) Determine the period and the amplitude of $f(x)$ and $g(x)$.

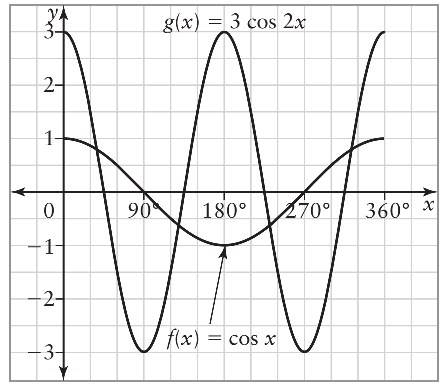
b) Determine the phase shift and the vertical shift of $g(x)$ relative to $f(x)$.

c) Do the transformations affect the period and amplitude? Explain.

d) Determine the domain and range of $f(x)$ and $g(x)$.

Solution

x	$f(x)$	$g(x)$
0°	1	3
45°	0.707	0
90°	0	–3
135°	–0.707	0
180°	–1	3
225°	–0.707	0
270°	0	–3
315°	0.707	0
360°	1	3



a) For $f(x)$, the amplitude is 1 and the period is 360°. For $g(x)$, the amplitude is 3 and the period is $\frac{360°}{2}$, or 180°.

b) There is no phase shift or vertical shift.

c) Yes, the transformations change the shape of the function: $g(x)$ is stretched vertically and compressed horizontally relative to $f(x)$. The position is not changed.

d) For $f(x)$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$.
For $g(x)$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, -3 \leq y \leq 3\}$.

A

1. Determine the amplitude.

a) $y = 3 \sin x$ **b)** $y = 4 \cos x$
c) $y = \frac{1}{2} \sin x$ **d)** $y = \frac{2}{3} \cos x$
e) $y = -3 \sin x$ **f)** $y = -2 \cos x$

2. Determine the value of k and the period.

a) $y = \sin 2x$ **b)** $y = \cos 5x$
c) $y = \sin (-4x)$ **d)** $y = \cos (-3x)$
e) $y = \sin \frac{1}{2} x$ **f)** $y = \cos \frac{3}{5} x$

3. Determine the vertical stretch or compression, horizontal stretch or compression, amplitude, and period.

a) $y = 3 \cos 4x$ **b)** $y = 2 \sin 3x$
c) $y = \sin \frac{1}{2} x$ **d)** $y = \frac{1}{3} \cos 6x$
e) $y = \frac{1}{4} \sin 4x$ **f)** $y = \frac{2}{5} \sin \frac{1}{3} x$
g) $y = 4 \cos \frac{1}{5} x$ **h)** $y = \frac{1}{2} \cos \frac{1}{4} x$

☆**4. Use Technology** Use graphing technology to investigate the graphs of $f(x) = a \sin x$ and $f(x) = a \cos x$ for different values of a. Describe the effects of changing a in terms of a transformation.

☆**5. Use Technology** Use graphing technology to investigate the graphs of $f(x) = \sin kx$ and $f(x) = \cos kx$ for different values of k. Describe the effects of changing k in terms of a transformation.

B

6. Determine the amplitude. Then, graph each function over two cycles.

a) $y = 3 \sin x$ **b)** $y = \frac{1}{2} \cos x$

7. Determine the period. Then, graph each function over two cycles.

a) $y = \sin 3x$ **b)** $y = \cos \frac{1}{2} x$

8. Determine the amplitude and period. Then, graph each function over two cycles.

   a) $y = 3 \sin 4x$

   b) $y = 1.5 \sin \left(\frac{1}{2}x\right)$

   c) $y = 2 \cos \left(\frac{1}{6}x\right)$

   d) $y = \frac{1}{2} \cos (3x)$

9. **Use Technology** Use graphing technology to graph each function in question 8.

10. a) How does a vertical stretch or compression affect the key features of the graph of $y = \sin x$ or $y = \cos x$?

    b) How does a horizontal stretch or compression affect the key features of the graph of $y = \sin x$ or $y = \cos x$?

11. The function $y = \sin x$ is transformed so the resulting function has amplitude 6 and period 720°. Write an equation of the transformed function. Is there more than one possible equation? Explain.

12. Write an equation of each transformed function.

    a) The function $y = \cos x$ is transformed so the resulting function has amplitude $\frac{1}{4}$ and period 180°.

    b) The graph of $f(x) = \sin x$ is stretched vertically by a factor of 4, compressed horizontally by a factor of $\frac{1}{2}$, and reflected in the x-axis.

    c) The graph of $f(x) = \cos x$ is compressed vertically by a factor of $\frac{1}{3}$, stretched horizontally by a factor of 5, and reflected in the y-axis.

13. Refer to your answers to question 12. Determine the key features of each transformed function.

C

14. The function $g(x)$, a sine function, has a period twice that of $f(x) = \sin x$.

    a) Predict the number of points of intersection of $g(x)$ and $f(x)$ on the interval $0° \leq x \leq 720°$. Explain.

    b) Write a possible equation for $g(x)$.

    c) **Use Technology** Use graphing technology to graph $f(x)$ and $g(x)$ on the same set of axes to check your prediction from part a).

15. The function $h(x)$, a cosine function, has a period half that of $k(x) = \cos x$. Predict the number of points of intersection of $h(x)$ and $k(x)$ on the interval $0° \leq x \leq 720°$. Justify your prediction.

16. According to the theory of biorythms, each of us has three periodic cycles—physical, emotional, and intellectual—that begin at birth and stay with us throughout our lives. The physical cycle has a period of 23 days, the emotional cycle has a period of 28 days, and the intellectual cycle has a period of 33 days. Whenever a rhythm crosses the time axis, a critical day occurs.

    a) Assuming an amplitude of 1, model each biorhythm—physical, emotional, and intellectual—with a sine function that begins at birth.

    b) Plot all three functions on the same set of axes for the first 90 days of life.

    c) Identify good days (when two or more biorhythms reach a maximum or close to a maximum).

    d) Identify bad days (when two or more biorhythms reach a minimum or close to a minimum).