KEY CONCEPTS

- The amplitude, period, phase shift, and vertical shift can be determined for sinusoidal functions whose equations are given in the form $f(x) = a \sin [k(x - d)] + c$ or $f(x) = a \cos [k(x - d)] + c$.
- Graphs of $y = a \sin [k(x - d)] + c$ and $y = a \cos [k(x - d)] + c$ can be sketched by applying transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$.

Example

a) Describe the transformations that must be applied to the graph of $f(x) = \sin x$ to obtain the graph of $g(x) = -2 \sin [3(x - 30°)] - 1$.

b) State the amplitude and the period of $g(x)$.

c) State the domain and the range of $g(x)$.

d) Sketch a graph of $g(x)$.

Solution

a) Since $a = -2$, the graph of $f(x) = \sin x$ is reflected in the x-axis and is stretched vertically by a factor of 2.

Since $k = 3$, the graph of $f(x) = \sin x$ is horizontally compressed by a factor of $\frac{1}{3}$.

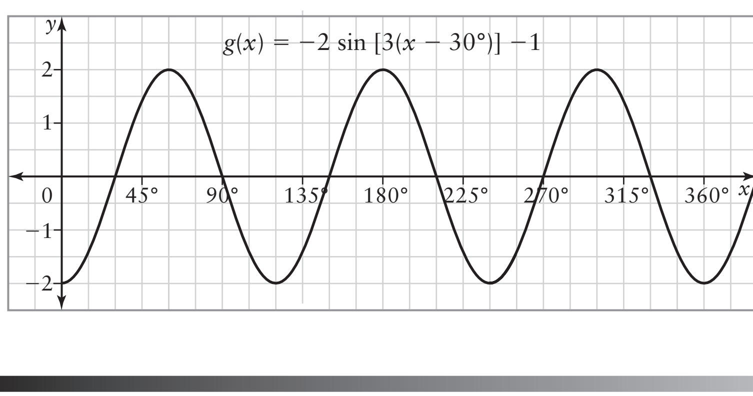
Since $d = 30°$, and $c = -1$, the graph of $f(x) = \sin x$ is shifted 30° right and down 1.

b) Because $g(x)$ has been stretched vertically by a factor of 2, the amplitude is 2.

Since it is horizontally compressed by a factor of $\frac{1}{3}$, the period is 3 times smaller, or $\frac{360°}{3}$, which is 120°.

c) The domain of $g(x)$ is $\{x \in \mathbb{R}\}$. Since the graph is shifted down 1 unit and the amplitude is 2, the range of the function is $\{y \in \mathbb{R}, -3 \le y \le 1\}$

d)



A

1. Determine the amplitude of each function.

   a) $y = 3 \cos x$

   b) $y = -4 \sin x$

   c) $y = \frac{1}{2} \sin x$

   d) $y = -\cos x$

2. Determine the period of each function.

   a) $y = \sin 2x$

   b) $y = \sin\left(\frac{1}{3}x\right)$

   c) $y = \cos\left(\frac{1}{4}x\right)$

   d) $y = \cos 3x$

3. Determine the amplitude and period of each function.

   a) $y = 5 \cos 4x$

   b) $y = -2 \cos\left(\frac{1}{2}x\right)$

   c) $y = \frac{1}{4} \sin 6x$

   d) $y = -\frac{2}{3} \sin\left(\frac{3}{4}x\right)$

4. Determine the amplitude, period, phase shift, and vertical shift of each function.

   a) $y = 3 \sin [2(x - 20°)] + 5$

   b) $y = -4 \sin [3(x + 50°)] - 2$

   c) $y = 2 \sin [9(x + 40°)] + 4$

   d) $y = \frac{1}{3} \sin\left[\frac{2}{5}(x - 30°)\right] - 1$

5. Determine the amplitude, period, phase shift, and vertical shift of each function.

   a) $y = 8 \cos [4(x + 45°)] - 3$

   b) $y = 5 \cos [6(x - 38°)] + 7$

   c) $y = -7 \cos [30(x + 75°)] - 5$

   d) $y = \frac{2}{7} \cos\left[\frac{3}{4}(x - 67°)\right] + 8$

6. a) Describe the transformations that must be applied to the graph of $f(x) = \sin x$ to obtain the graph of $g(x) = 4 \sin 5x + 1$. Sketch the graph of $g(x)$.

   b) State the domain and range of $f(x)$ and $g(x)$.

   c) Modify the equation for $g(x)$ to include a phase shift of 30° to the left. Call this function $h(x)$. Graph $h(x)$ on the same set of axes as $g(x)$.

   d) Use graphing technology to verify your graphs.

7. a) Describe the transformations that must be applied to the graph of $f(x) = \cos x$ to obtain the graph of $g(x) = 3 \cos 4x - 2$. Graph $f(x)$ and $g(x)$ on the same set of axes.

   b) State the domain and range of $f(x)$ and $g(x)$.

   c) Modify the equation for $g(x)$ to include a phase shift of 45° to the right. Call this function $h(x)$. Graph $h(x)$ on the same set of axes as $g(x)$.

   d) Use graphing technology to verify your graphs.

8. Describe the transformations that must be applied to the graph of $f(x) = \sin x$ to obtain the graph of $g(x) = 3 \sin 4x + 5$.

9. Describe the transformations that must be applied to the graph of $f(x) = \cos x$ to obtain the graph of $g(x) = 5 \cos 3x - 2$.

B

10. Consider the function $f(x) = 5 \cos [3(x - 60°)]$.

    a) Determine the amplitude, period, phase shift, and vertical shift of the function with respect to $f(x) = \cos x$.

    b) Determine the maximum and minimum values of the function.

    c) Determine the first two x-intercepts to the right of the origin.

    d) Determine the y-intercept of the function.

    e) **Use Technology** Use graphing technology to verify your answers.

11. Use Technology Consider the graph defined by $y = \sin(2x - 60°)$.

a) How is this equation written differently from the examples and previous questions?

b) Predict the phase shift.

c) Using technology, graph the function to verify your prediction.

d) Explain how to rewrite the equation in order to see the phase shift when there is also a horizontal stretch or compression.

12. Consider the function $f(x) = 4\sin[2x + 30°] - 2$.

a) Determine the amplitude, period, phase shift, and vertical shift of the function with respect to $f(x) = \sin x$.

b) Determine the maximum and minimum values of the function.

c) Determine the first two x-intercepts to the right of the origin.

d) Determine the y-intercept of the function.

e) Use Technology Use graphing technology to verify your answers.

13. a) Describe the transformations on $f(x) = \sin x$ that would result in $g(x) = 3\sin(x - 90°)$.

b) State the key features of $g(x)$.

c) Sketch the graph of $g(x)$.

d) Use graphing technology to verify your sketch of $g(x)$.

14. a) Describe the transformations on $f(x) = \cos x$ that would result in $g(x) = \cos 2x - 1$.

b) State the key features of $g(x)$.

c) Sketch the graph of $g(x)$.

d) Use Technology Use graphing technology to verify your sketch of $g(x)$.

☆**15. a)** Describe the transformations on $f(x) = \sin x$ that would result in $g(x) = 4\sin\left[\frac{1}{2}(x - 40°)\right] + 3$.

b) State the key features of $g(x)$.

c) Sketch the graph of $g(x)$.

d) Use Technology Use graphing technology to verify your sketch of $g(x)$.

☆**16. a)** Describe the transformations on $f(x) = \cos x$ that would result in $g(x) = \frac{1}{2}\cos[3(x + 50°)] - 2$.

b) State the key features of $g(x)$.

c) Sketch the graph of $g(x)$.

d) Use Technology Use graphing technology to verify your sketch of $g(x)$.

C

17. A roller coaster is designed so that the shape of one section can be modelled by the function $y = 15\cos(3.6x) + 2$. Describe how you would sketch a graph of this function. Include all the relevant information in your description.

18. Use Technology Use the Internet to research the amount of sway in a tall structure, such as the CN Tower. Describe how a sinusoidal function can be used to model the amount of sway. Relate the key features of the function to the situation.

19. Use Technology Use the Internet to research a situation—such as the motion of a weight on a spring, or tides—that can be modelled by a sinusoidal function. Describe how the key features of a sinusoidal function relate to the situation.