KEY CONCEPTS

- The equation of a sinusoidal function can be determined given its graph or a description of its properties.
- Transformations can be used to adjust the basic sine and cosine functions to match given period, phase shift, intercepts, amplitude, vertical shift, and range.
- The graph of $y = \sin x$ is a horizontal translation of the graph of $y = \cos x$. Similarly, the graph of $y = \cos x$ is a horizontal translation of the graph of $y = \sin x$.

Example

Write an equation of the form $y = a \sin [k(x - d)] + c$ that represents the graph of $y = \sin x$ after it has been reflected in the x-axis, vertically stretched by a factor of 4, horizontally compressed by a factor of $\frac{1}{3}$, shifted 45° right, and translated 5 units up.

Solution

The graph is reflected in the x-axis and vertically stretched by a factor of 4, so $a = -4$.
The equation becomes $y = -4 \sin x$.

The graph is horizontally compressed by a factor of $\frac{1}{3}$, so $k = 3$.
The equation becomes $y = -4 \sin 3x$.

The graph is shifted 45° to the right, so $d = 45°$.
The equation becomes $y = -4 \sin [3(x - 45°)]$

The vertical shift is 5 units up, so $c = 5$.
The equation of the transformed graph is $y = -4 \sin [3(x - 45°)] + 5$.

A

1. **Use Technology** Graph the functions in each set on the same screen. Compare the functions.
   a) $f(x) = \sin x$, $f(x) = \cos (x - 90°)$, and $f(x) = \cos (x + 270°)$
   b) $f(x) = \cos x$, $f(x) = \sin (x + 90°)$, and $f(x) = \sin (x - 270°)$

2. **a)** Write an equation of a sine function with amplitude 3 and period 180°. Is more than one equation possible? Explain.
   b) State the key features of the function from part a).
   c) Sketch the function from part a), showing all the key features.

3. **a)** Write an equation of a sine function with amplitude $\frac{1}{4}$ and period 720° through $\left(180°, \frac{1}{4}\right)$. Is more than one equation possible? Explain.

b) State the key features of the function from part a).

c) Sketch the function from part a), showing all the key features.

4. **a)** Write an equation of a cosine function with amplitude 6 and period 120°. If more than one equation is possible, write another equation.

b) State the key features of the function(s) from part a).

5. **a)** Write an equation of a cosine function with amplitude $\frac{1}{3}$ and period 1080°. If more than one equation is possible, write another equation.

b) State the key features of the function(s) from part a).

6. A sinusoidal function has amplitude 4, period 180°, and a maximum at $\left(0°, \frac{5}{2}\right)$.

a) Represent the function as a transformed sine function.

b) Represent the function as a transformed cosine function.

c) Refer to your answers to parts a) and b). Which was easier? Explain.

d) Sketch one full cycle of the function.

7. A sinusoidal function has amplitude 5, period 120°, and a maximum at (30°, 3).

a) Explain how you can use the given information to determine the other key features of the function.

b) Sketch a graph of the function.

c) Represent the function as a transformed sine function.

d) Represent the function as a transformed cosine function.

B

8. **a)** **Use Technology** Graph $f(x) = 4 \sin [3(x - 30°)]$ and $g(x) = 4 \cos [3(x - 30°)]$, using graphing technology. How do the graphs compare?

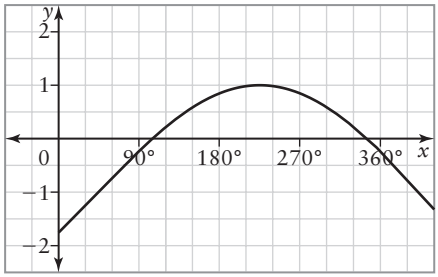
b) What phase shift on $g(x)$ would make it represent the same graph as $f(x)$?

c) Use your answer to part b) to rewrite $g(x)$ as a sine function.

☆**9.** Represent the graph of $f(x) = 4 \sin [3(x - 30°)]$ with an equation using a cosine function.

10. Represent the graph of $f(x) = 5 \sin [2(x + 45°)]$ with an equation using a cosine function.

11. Consider this function.



a) Determine the amplitude, maximum and minimum values, vertical shift, period, and phase shift relative to the graph of $y = \sin x$.

b) Write the equation of a transformed sine function that represents the graph.

12. A sinusoidal function has amplitude 4, period 180°, and a maximum at (45°, 6).

a) Explain how you can use the information to identify other key features of the graph.

b) Sketch a graph of a function with these key features.

c) Represent the function in two different ways, using first the sine function and then the cosine function.

13. Write an equation of the form $y = a \sin [k(x - d)] + c$ that represents the graph of $f(x) = \sin x$ after it has been vertically stretched by a factor of 3, horizontally compressed by a factor of $\frac{1}{2}$, shifted 55° to the left, and translated 5 units down.

14. Write an equation of the form $y = a \cos [k(x - d)] + c$ that represents the graph of $f(x) = \cos x$ after it has been vertically compressed by a factor of $\frac{1}{5}$, horizontally stretched by a factor of 3, shifted 38° to the right, and translated 2 units up.

15. Write an equation of the form $y = a \sin [k(x - d)] + c$ that represents the graph of $y = \sin x$ after it has been reflected in the x-axis, vertically stretched by a factor of 7, horizontally stretched by a factor of 2, reflected in the y-axis, shifted 20° to the left, and translated 3 units down.

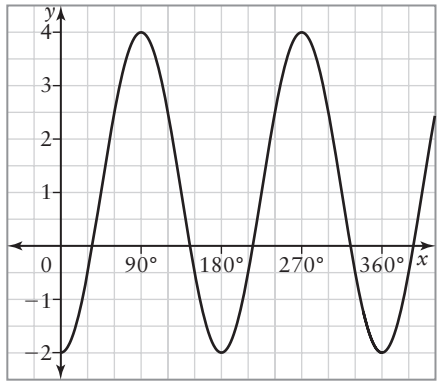
16. Write an equation of the form $y = a \cos [k(x - d)] + c$ that represents the graph of $y = \cos x$ after it has been reflected in the x-axis, vertically compressed by a factor of $\frac{2}{3}$, horizontally compressed by a factor of $\frac{1}{4}$, reflected in the y-axis, shifted 49° to the right, and translated 2 units up.

17. A sinusoidal function has amplitude 3, period 180°, a maximum at (45°, 5), and a minimum at (135°, –1).

a) Represent the function with an equation using a sine function.

b) Represent the function with an equation using a cosine function.

☆**18.** Consider the following graph.



a) Write an equation of a sine function that represents the graph.

b) Write an equation of a cosine function that represents the graph.

C

19. Jacob is recording data for the time and height of high tide and low tide. He records a high tide of 18 m at 8:00 a.m. and low tide of 8 m at 2:00 p.m.

a) Construct a sinusoidal model for the water depth using a cosine function, where time is measured in hours past high tide.

b) Construct a sinusoidal model for the water depth using a sine function, where time is measured in hours past low tide.

20. Consider the function $y = a \sin [k(x - d)] + c$.

a) Write expressions for the coordinates of the maxima.

b) Write expressions for the coordinates of the minima.

21. Repeat question 20 for $y = a \cos [k(x - d)] + c$.