KEY CONCEPTS

- Problems based on applications involving a sinusoidal function may be posed and solved by using sinusoidal graphs, equations, or tables of values.
- Data collected from a primary source, such as a motion sensor, or from a secondary source, such as Statistics Canada, can sometimes be approximated by a sinusoidal function.

Example

Using data from Statistics Canada, determine if there is a period of time over which changes in the population of Canadians aged 20 to 24 can be modelled using a sinusoidal function.

Solution

Go to the Statistics Canada web site. Search for **Table 051-0001**. Under **Geography,** select **Canada**. Under **Sex**, select **Both** sexes. Under **Age group,** select **20 to 24** years. Set the reference period to **from 1976 to 2005**.

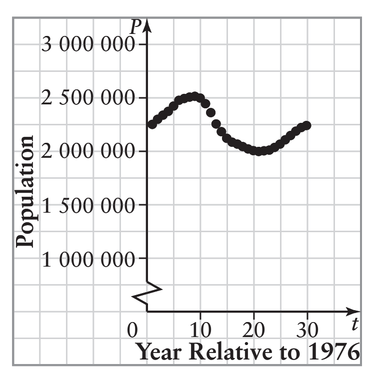
Population of Canada, Aged 20 to 24 Years, Both Sexes, by Year					
Year	**Population**	**Year**	**Population**	**Year**	**Population**
1976	2 253 367	1986	2 446 250	1996	2 002 036
1977	2 300 910	1987	2 363 227	1997	2 008 307
1978	2 339 362	1988	2 257 415	1998	2 014 301
1979	2 375 197	1989	2 185 706	1999	2 039 468
1980	2 424 484	1990	2 124 363	2000	2 069 868
1981	2 477 137	1991	2 088 165	2001	2 110 324
1982	2 494 358	1992	2 070 089	2002	2 150 370
1983	2 507 401	1993	2 047 334	2003	2 190 876
1984	2 514 313	1994	2 025 846	2004	2 224 652
1985	2 498 510	1995	2 009 474	2005	2 243 341

Source: Statistics Canada

Graph the data.

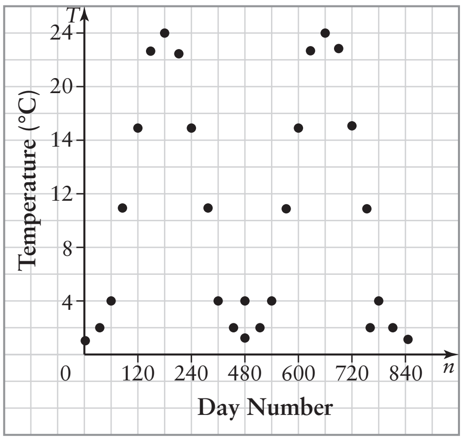
The population of people in Canada aged 20 to 24 from 1976 to 2005 appears to follow a sinusoidal model.

Population Aged 20 to 24 in Canada 1976 to 2005



A

1. The graph shows the daily high temperature as recorded every 30 days for the town of Sydenham for two years.



a) Does the graph appear to be periodic? Explain.

b) Determine the amplitude of the function.

c) Determine the period of the function. What does the period represent in this context?

d) What might be true about a town for which the graph has a significantly smaller amplitude?

2. **Use Technology**
The sinusoidal function $h(t) = 5 \sin [30(t - 4)] + 2$ models the height, h, in metres, of the tide in a particular location on a particular day at t hours after midnight.

a) Use technology to graph $h(t)$.

b) Determine the maximum and minimum heights of the tide.

c) At what times do high tide and low tide occur?

d) What is the depth of the water at 9:00 a.m.?

e) Explain how you can use the graph of $h(t)$ to model the height of the tide as a cosine function.

3. The table shows the mid-season high temperatures every three months over a three-year period for a town in Ontario.

Month	Temperature (° C)
February	−7
May	14
August	23
November	1
February	−12
May	15
August	25
November	1
February	–12
May	14
August	24
November	1

a) Graph the data.

b) Does the graph model a periodic function? Explain.

c) Construct a model for the mid-season high temperature by writing an equation using a sine function.

4. The number of students, s, visiting the Centre for the Sciences is modelled using the function $s = 200 \sin [30(t - 10)] + 500$, where t is the number of months since the first of January.

a) Find the maximum and minimum number of students visiting the Centre for the Sciences over the period of one year.

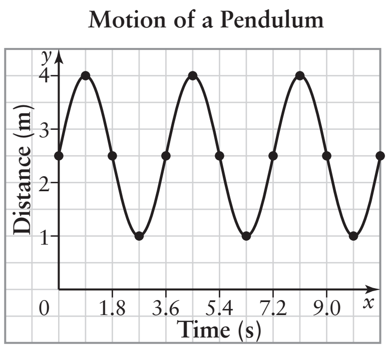
b) When is the number of students a maximum? When is it a minimum?

c) How many students visit the Centre for the Sciences on February 14?

d) In what month(s) is the number of students about 600?

e) Suggest a reason for the pattern of attendance represented by this function.

5. A student used a motion sensor to gather data on the motion of a pendulum. She exported the table of values to a computer and used graphing software to draw the graph.



a) Explain how the graph can be used to find the range, amplitude, and vertical shift.

b) Use the graph to determine the period, and then state the horizontal compression factor.

c) Use a sine function to write an equation that models the motion of the pendulum.

B

☆6. The height, h, in metres, above the ground of a rider on a Ferris wheel can be modelled by the function $h(x) = 30 \sin (x - 90°) + 35$, where x is the angle, in degrees, from the horizontal.

a) Determine the maximum and the minimum heights of the rider.

b) Use the cosine function to model the height of the rider on the Ferris wheel.

c) If the Ferris wheel makes one revolution per minute, find the speed of the rider in metres per second.

7. The average monthly depth, d, in metres, of water in a lake can be modelled by the function $d = 6 \sin [33.5(n + 55)] + 13$, where n is the number of months since January 1, 2005. Identify and explain the restrictions on the domain and range of this function.

☆8. The movement of a piston in an automobile engine can be modelled by the function $y = 60 \sin 10\,800t + 10$, where y is the distance, in millimetres, from the crankshaft and t is the time, in seconds.

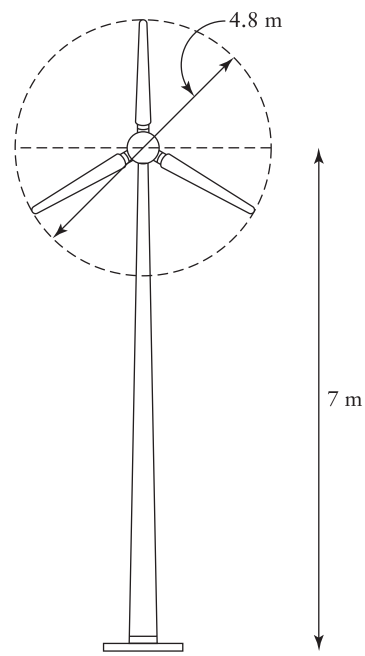
a) What is the period of the motion?

b) Explain how the period of the function relates to the revolutions per minute (RPM) of the engine.

c) Which number in the equation would change if the driver increased the speed? Would the number increase or decrease? Explain.

d) Determine the maximum, minimum, and amplitude.

9. The blades on a wind turbine rotate counterclockwise at 30 revolutions per minute. The rotor diameter is 4.8 m and the hub height is 7 m.



a) Write an equation to model the height of the tip of one of the blades above the ground over time. Assume the blade starts at the point when it is the least distance from the ground.

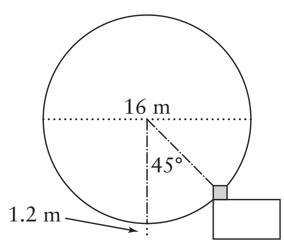
b) After how long does this blade reach the maximum height above the ground?

10. a) The owner of a bakery kept records of average daily sales for each month for the past year, as shown, beginning in January. Construct a sinusoidal function to model the average daily sales versus the month.

Month	Daily Sales ($)
1	35
2	105
3	185
4	280
5	350
6	370
7	345
8	275
9	195
10	95
11	32
12	8

b) Over what domain and range is your model valid? Explain.

11. A Ferris wheel with diameter 16 m rotates clockwise. At its lowest point, the wheel is 1.2 m above the ground. Riders enter the cars from a platform at a point that is 45° before the lowest point.



a) Write a cosine equation to model the height of a car above the ground relative to the angle from vertical.

b) Write a sine equation to model the height of a car above the ground relative to the angle from vertical.

c) Suppose the platform is moved so that riders enter the car at the lowest point on the Ferris wheel. How do the equations from parts a) and b) change?

12. a) Use a watch to determine how many breaths you take in 1 min. Use this measurement as the length of one breathing cycle.

b) Assuming an amplitude of 1 unit, use a sine function to construct a model of the volume of air in the lungs over time.

c) Use the library or the Internet to research and determine if a sinusoidal function is a good model for human breathing.

13. Visit the Statistics Canada web site. Select a data table and pose a question that can be answered by constructing a sinusoidal model of the data.

C

14. Use a sound probe to collect primary data on sound, or research secondary data.

a) Graph the data.

b) Determine the key features of the graph and write an equation to model the data.

c) Determine the impact of increasing the amplitude of a sound wave.

d) What is the effect of changing the frequency of sound? How does this relate to the period of the function?

15. Use Technology

a) Use the Internet to research suppliers of electricity in Ontario and the plans that they offer.

b) Prepare a report on the advantages and disadvantages of the different plans offered by suppliers of electricity in Ontario.

16. The diameter of Marni's bicycle tire is 66 cm. Marni rides her bicycle through a strip of wet paint. After 500 m, what is the height above the ground of the paint on Marni's tire?

Chapter 2: Checklist

By the end of this chapter, I will be able to:

- make connections among different representations of sinusoidal functions
- demonstrate an understanding that sinusoidal functions can be used to model some periodic phenomena, and solve related problems
- make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship with or without technology, defining the relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining why the relationship is a function
- sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures in degrees, and describe their key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)
- determine, through investigation using technology, the roles of the parameters d and c in functions of the form $y = \sin (x - d) + c$ and $y = \cos (x - d) + c$, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ with angles expressed in degrees (i.e., vertical and horizontal translations)
- determine, through investigation using technology, the roles of the parameters a and k in functions of the form $y = a \sin kx$ and $y = a \cos kx$, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ with angles expressed in degrees (i.e., reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)
- determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form $f(x) = a \sin [k(x - d)] + c$ or $f(x) = a \cos [k(x - d)] + c$, and sketch graphs of $y = a \sin [k(x - d)] + c$ and $y = a \cos [k(x - d)] + c$ by applying transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$
- represent a sinusoidal function with an equation, given its graph or its properties
- collect data that can be modelled as a sinusoidal function from primary sources, using a variety of tools (e.g., concrete materials, motion sensors), or from secondary sources (e.g., Statistics Canada, E-STAT), and graph the data
- identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions on the domain and range
- pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology, in degree mode, or from a table of values, or from its equation