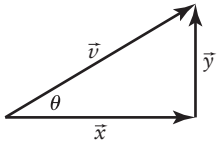
KEY CONCEPTS

- Any vector can be resolved into its perpendicular, or rectangular, vector components.
- A given vector, $\vec{v}$, at an angle of θ to the horizontal, is the resultant, or sum, of its perpendicular components $\vec{x}$ and $\vec{y}$. The trigonometric ratios and the Pythagorean theorem can be used to determine the magnitude and direction of the vectors.



- The magnitude of the horizontal and vertical components can be calculated using the primary trigonometric ratios $\frac{|\vec{x}|}{|\vec{v}|} = \cos\theta$ and $\frac{|\vec{y}|}{|\vec{v}|} = \sin\theta$ in the forms $|\vec{x}| = |\vec{v}|\cos\theta$ and $|\vec{y}| = |\vec{v}|\sin\theta$.
- A vector can be represented as a directed line segment with magnitude and direction if the vertical and horizontal components of the vector are given.

Example

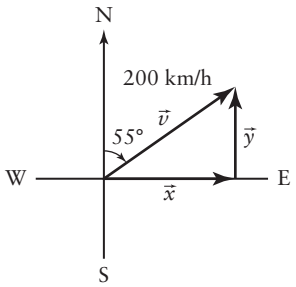
A train is travelling at 200 km/h in a direction N55°E.

a) Draw a diagram of the velocity vector and its perpendicular components.

b) Determine the magnitude of the horizontal and vertical components, to one decimal place.

Solution

a) The velocity can be resolved into horizontal and vertical components, $\vec{x}$ and $\vec{y}$.



b) Determine θ.

$\theta = 90° - 55°$

$= 35°$

$|\vec{x}| = |\vec{v}|\cos\theta.$

$= 200\cos 35°$

$\doteq 163.8$

$|\vec{y}| = |\vec{v}|\sin\theta$

$= 200\sin 35°$

$\doteq 114.7$

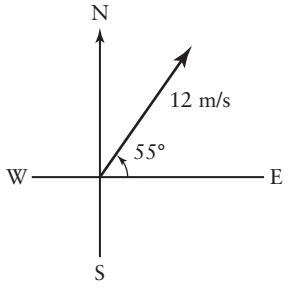
The magnitude of the horizontal component is approximately 163.8 km/h E, and the magnitude of the vertical component is approximately 114.7 km/h N.

A

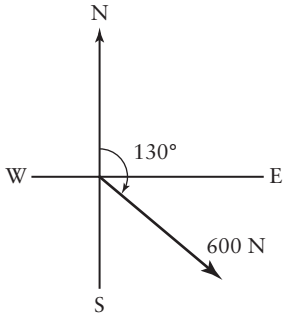
Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

1. Determine the horizontal and vertical components of each force.

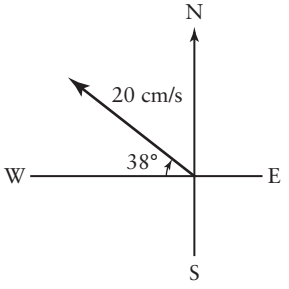
a)



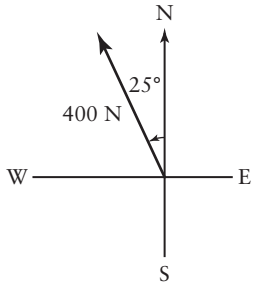
b)



c)



2. **a)** Determine the horizontal and vertical components of this force.



b) Explain why $\vec{x} \neq 400 \cos 25°$.

3. Determine the perpendicular components of each force.

a) 50 N on a bearing of 070°

b) 30 m/s on a quadrant bearing of S40°E

c) 800 m on a bearing of 065°

d) 220 N on a quadrant bearing of N20°W

4. **a)** Draw the vector that represents a 300-N force at an angle of 40° to the horizontal. Show the perpendicular components.

b) Draw the vector that represents a 300-N force at an angle of 50° to the vertical. Show the perpendicular components.

c) What do you notice about the answers for parts a) and b)?

d) Determine values for 300 cos 40° and for 300 sin 50°.

e) Determine values for 300 cos 50° and for 300 sin 40°.

f) Is there more than one way to resolve a force into horizontal and vertical components? Explain.

5. **a)** Draw a diagram resolving a 450-N force at 42° counterclockwise from the horizontal into its perpendicular components.

b) Determine the magnitude of the horizontal component.

c) Determine the magnitude of the vertical component.

6. A cable exerts a force of 475 N at an angle of 35.8° with the horizontal. Resolve this force into its perpendicular components.

7. A person is pushing on the handle of a lawn mower with a force of 300 N acting 28° below the horizontal.

a) Determine the magnitudes of the perpendicular components of the force pushing the lawn mower.

b) How would the person pushing the mower be affected if the angle were increased?

B

8. Lee drove 9 km east, and then 7 km north.

a) Draw the perpendicular component vectors and the resultant vector, $\vec{r}$.

b) Calculate the magnitude and direction, relative to the horizontal vector, of the resultant vector.

9. Kelsey is pulling a rope attached to her sled with a 60-N force at an angle of 32° to the horizontal.

a) Determine the rectangular vector components of the force.

b) What would happen to the angle if a longer rope were used? What would be the benefit?

☆**10.** After takeoff, an aircraft, travelling 160 miles per hour, climbed at an angle of 15° from the horizontal.

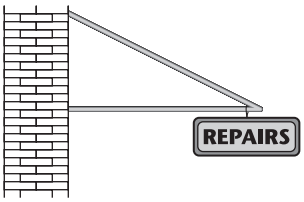
a) What is the speed of the aircraft's shadow on the ground, if the sun is directly overhead?

b) Calculate the height of the aircraft 2 min after takeoff.

☆**11.** A ship travels 4 km south and 3 km east from a lighthouse. Determine the magnitude and direction of the displacement vector of the ship.

☆**12.** A helicopter is 50 km from the airport on a quadrant bearing of N25.7°E. Determine the perpendicular components of the helicopter's displacement.

13. Hussain has a sign at the front of his store. The sign is suspended as shown.



The tension in the slanted rod supporting this sign is 130 N at an angle of 32° to the horizontal. What are the horizontal and vertical components of the tension vector?

14. A rocket is propelled at an initial velocity of 140 m/s at 78° from the horizontal.

a) Determine the horizontal and vertical vector components of the velocity.

b) Determine the height of the rocket 20 s after liftoff.

15. Two cars, one travelling south and one travelling east, collide at an intersection. The resulting momentum of the two cars together after the collision is 38 000 kg·m/s S30°E.

a) Find the momentum of each car before the collision.

b) Is it possible to determine the velocity of the cars before the collision? Explain your reasoning.

C

16. A box weighing 130 N is resting on a ramp that is inclined at an angle of 18°. Resolve the weight into rectangular vector components that keep the box at rest.

17. a) Resolve a 300-N force into two rectangular vector components such that the ratio of their magnitudes is 3:1.

b) Calculate the angle between the greater component and the 300-N force.

c) Would the angle change if the force was reduced to 200 N? Explain your reasoning.