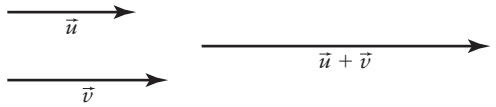
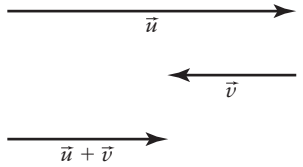
KEY CONCEPTS

- Two or more vectors can be added together to find a single vector, called the resultant.
- Vectors can be added by applying one vector after the other.
- Two vectors can be added using the head-to-tail (triangle) method or the parallelogram method.
- Given two parallel vectors, $\vec{u}$ and $\vec{v}$, in the same direction, $|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$ and $\vec{u} + \vec{v}$ is in the same direction as $\vec{u}$ and $\vec{v}$.



- Given two parallel vectors, $\vec{u}$ and $\vec{v}$, with opposite directions and $|\vec{u}| > |\vec{v}|$, $|\vec{u} + \vec{v}| = |\vec{u}| - |\vec{v}|$ and $\vec{u} + \vec{v}$ is in the same direction as $\vec{u}$.



- The zero vector, $\vec{0}$, has zero magnitude and no specific direction. Adding two opposite vectors results in the zero vector.
- For any vectors $\vec{u}, \vec{v}$, and $\vec{w}$,

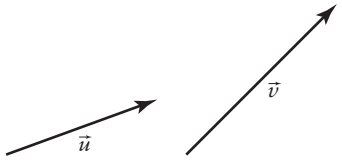
  $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (commutative property)

  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (associative property)

  $\vec{v} + \vec{0} = \vec{v} = \vec{0} + \vec{v}$ (identity property)

Example

Consider two vectors, $\vec{u}$ and $\vec{v}$.

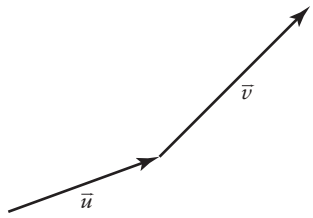


a) Find $\vec{u} + \vec{v}$ using the head-to-tail (triangle) method.

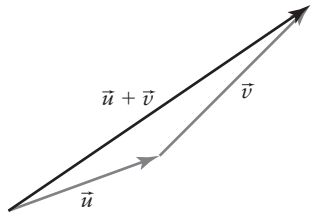
b) Find $\vec{u} + \vec{v}$ using the parallelogram method.

Solution

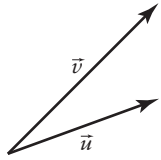
a) Translate $\vec{v}$ so that the tail of $\vec{v}$ touches the head of $\vec{u}$.



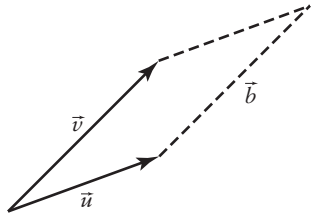
Find the sum by drawing and measuring the distance from the tail of $\vec{u}$ to the head of $\vec{v}$. This new vector is the resultant $\vec{u} + \vec{v}$, which is the sum of $\vec{u}$ and $\vec{v}$.



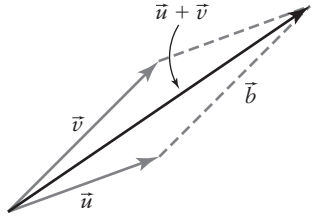
b) Translate $\vec{v}$ so that the tail of $\vec{v}$ touches the tail of $\vec{u}$.



Complete the parallelogram that has $\vec{u}$ and $\vec{v}$ as two of its sides.



Because of the properties of a parallelogram, $\vec{b}$ and $\vec{v}$ are equivalent vectors, so $\vec{u} + \vec{v} = \vec{u} + \vec{b}$.



A

Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

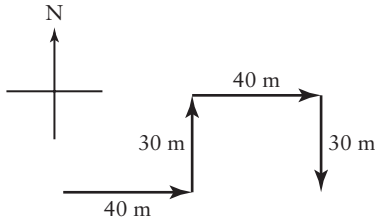
1. Draw the resultant vector.

a) 3 cm + 5 cm

b) 10 cm/s + 7 cm/s

c) 3 N + 6 N

2. Natasha and Mina went for a walk. The diagram shows their path.

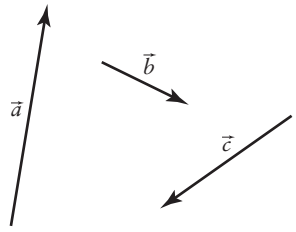


a) Determine the distance travelled and the displacement.

b) Are the distance and the displacement the same or different for each pair of vectors? Explain.

c) What must be true for the distance travelled and the magnitude of the displacement to be equal? Explain.

3. Consider vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.



a) Draw a diagram for each sum.

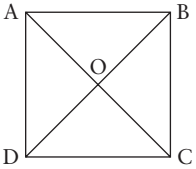
i) $\vec{a} + \vec{b} + \vec{c}$

ii) $\vec{b} + \vec{c} + \vec{a}$

iii) $\vec{c} + \vec{a} + \vec{b}$

b) What do you notice about the resultant vectors for parts i), ii), and iii)? Explain.

4. O is the centre of square ABCD. Name a vector equivalent to each sum.



a) $\overrightarrow{AO} + \overrightarrow{OC}$

b) $\overrightarrow{AO} + \overrightarrow{OB}$

c) $\overrightarrow{CD} + \overrightarrow{DA}$

d) $\overrightarrow{BD} + \overrightarrow{DC}$

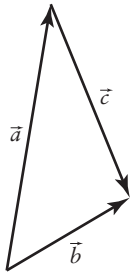
5. Refer to your answers to question 4.

a) In each case, what is true about the endpoint of the first vector and start point of the second vector?

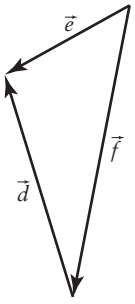
b) Explain how to find the endpoints of the resultant vector from the endpoints of the two given vectors.

6. In each case, express one vector as the sum of the other two vectors.

a)



b)



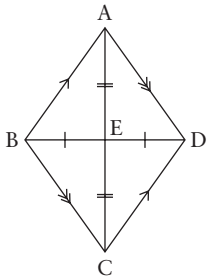
B

7. In rhombus ABCD, E is the intersection of diagonals AC and BD. Name a vector equivalent to each expression.

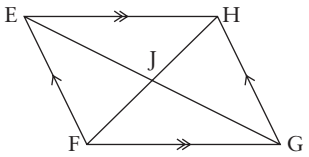
a) $\overrightarrow{CD} + \overrightarrow{AD}$

b) $\overrightarrow{BC} + \overrightarrow{BA}$

c) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$



8. In parallelogram EFGH, J is the intersection of diagonals EG and FH. Name a vector equivalent to each expression.



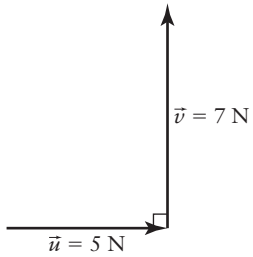
a) $\overrightarrow{HF} + \overrightarrow{FG}$

b) $\overrightarrow{EH} + \overrightarrow{HG} + \overrightarrow{GF}$

c) $\overrightarrow{EF} + \overrightarrow{FH} + \overrightarrow{HG}$

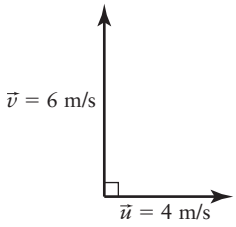
d) $\overrightarrow{FH} + \overrightarrow{HF}$

☆**9.** **a)** Determine the magnitude of the resultant vector, $\vec{u} + \vec{v}$.



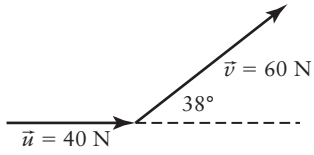
b) Determine the direction of $\vec{u} + \vec{v}$ relative to $\vec{u}$.

☆**10.** **a)** Determine the magnitude of the resultant vector, $\vec{u} + \vec{v}$.



b) Determine the direction of the resultant vector, $\vec{u} + \vec{v}$, relative to $\vec{u}$.

☆**11.** **a)** Determine the magnitude of the resultant vector, $\vec{u} + \vec{v}$.



b) Determine the direction of the resultant vector, $\vec{u} + \vec{v}$.

12. Vector $\vec{u}$ has magnitude 30 N and vector $\vec{v}$ has magnitude 40 N. When placed tail-to-tail, the angle between $\vec{u}$ and $\vec{v}$ is 50°.

a) Determine the magnitude of the resultant vector, $\vec{u} + \vec{v}$.

b) Determine the direction of the resultant vector, $\vec{u} + \vec{v}$.

13. A boat sails 20 km N20°E, and then turns and travels 12 km S30°E.

a) Determine the angle between the vectors that represent each leg of the journey.

b) Determine the magnitude of the resultant vector.

c) Determine the quadrant bearing of the resultant vector.

14. What force must be added to a 17-N force in the direction 039° to have a resultant force of 40 N in the direction 139°?

15. Let A, B, C, and O represent four different points.

a) Express each vector in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

i) $\overrightarrow{AB}$ **ii)** $\overrightarrow{BC}$

iii) $\overrightarrow{CA}$ **iv)** $\overrightarrow{AB} + \overrightarrow{BC}$

b) Show that $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

C

16. An airplane needs to travel 900 km due north at 400 km/h. There is a wind from the west at 20 km/h.

a) What heading must the plane take in order to head due north?

b) How long will it take the airplane to reach its destination?

17. Prove that the statement $|\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}|$ is true for all vectors.

18. ABCD is a parallelogram. P, Q, R, and S are the midpoints of AB, BC, DC, and DA, respectively. Use vector methods to prove that PQRS is a parallelogram.