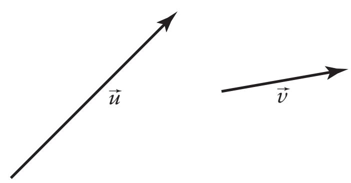
KEY CONCEPTS

- A vector can be subtracted from another vector to find the resultant vector.
- To subtract $\vec{u} - \vec{v}$, add the opposite of $\vec{v}$ to $\vec{u}$. Thus, $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$.

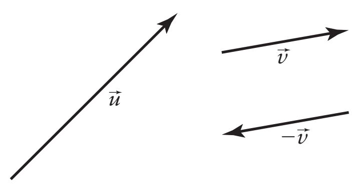
Example

Given vectors $\vec{u}$ and $\vec{v}$, determine $\vec{u} - \vec{v}$.

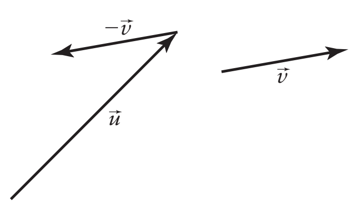


Solution

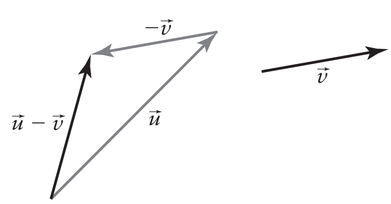
Draw the opposite of $\vec{v}$, $-\vec{v}$.



Translate $-\vec{v}$ so that the tail of $-\vec{v}$ touches the head of $\vec{u}$.



Draw the resultant $\vec{u} - \vec{v}$, from the tail of $\vec{u}$ to the head of $\vec{v}$. The sum is $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$.

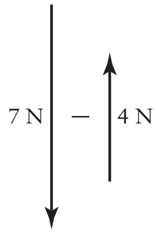


A

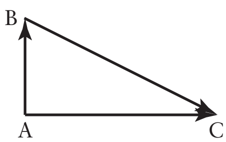
1. Draw a diagram to illustrate the resultant vector.

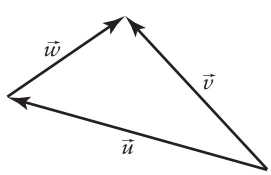
a) 

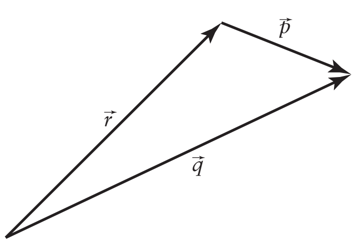
b) 

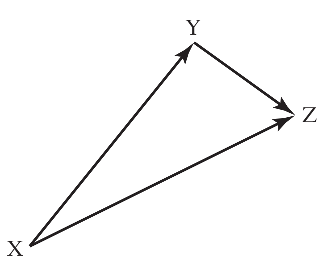
c) 

2. In each case, express one vector as a difference of the other two.

a) 

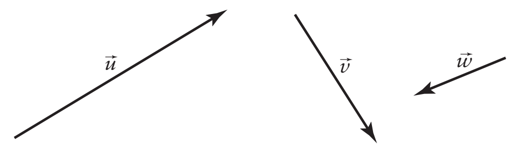
b) 

c) 

d) 

3. Consider a question in which you are given the resultant and one vector in the subtraction of two vectors. Explain how you would find the other vector.

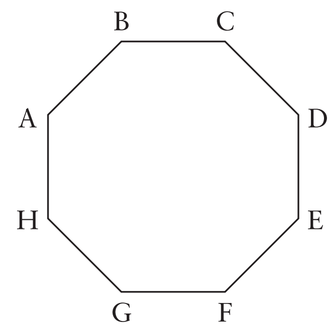
4. Use the vectors shown. Draw a diagram to represent each expression.



a) $\vec{u} - \vec{v} - \vec{w}$

b) $\vec{w} - \vec{v} - \vec{u}$

5. ABCDEFGH is a regular octagon. Name a single vector equivalent to each expression.



a) $\overrightarrow{AC} - \overrightarrow{AB}$

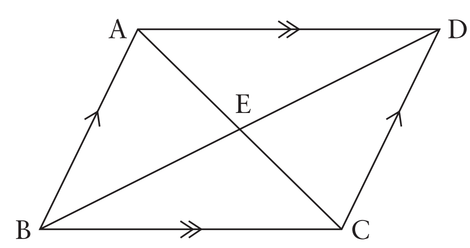
b) $\overrightarrow{HF} - \overrightarrow{GF}$

c) $\overrightarrow{GD} - \overrightarrow{ED}$

d) $\overrightarrow{DF} - \overrightarrow{DE}$

B

6. ABCD is a parallelogram, and E is the intersection point of diagonals AC and BD. Name a single vector equivalent to each expression.



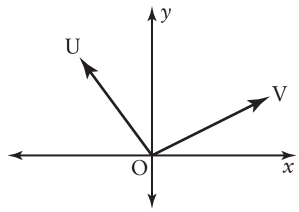
a) $\overrightarrow{AB} - \overrightarrow{AE}$

b) $\overrightarrow{CD} - \overrightarrow{BD}$

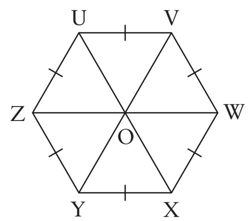
c) $\overrightarrow{CB} - \overrightarrow{CA}$

d) $\overrightarrow{AB} - \overrightarrow{DB}$

☆**7.** Two vectors, $\overrightarrow{OU}$ and $\overrightarrow{OV}$, are placed tail-to-tail as shown. Explain why $\overrightarrow{UV} = \overrightarrow{OV} - \overrightarrow{OU}$.



☆**8.** UVWXYZ is a regular hexagon, and O is the centre of the hexagon. Let $OU = \vec{u}$, $OV = \vec{v}$, and $OW = \vec{w}$.



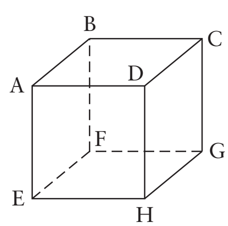
a) Write $\overrightarrow{UV}$ in terms of $\vec{u}$ and $\vec{v}$.

b) Write $\overrightarrow{OW}$ in terms of $\vec{u}$ and $\vec{v}$.

c) Write $\overrightarrow{WO}$ in terms of $\vec{u}$ and $\vec{v}$.

d) Write $\overrightarrow{XY}$ in terms of $\vec{u}$ and $\vec{v}$.

9. The diagram shows a cube.



a) Express $\overrightarrow{EG}$ in terms of $\overrightarrow{AG}$ and $\overrightarrow{AE}$.

b) Express $\overrightarrow{CG}$ in terms of $\overrightarrow{FG}$ and $\overrightarrow{FC}$.

c) Express $\overrightarrow{HD}$ in terms of $\overrightarrow{DB}$ and $\overrightarrow{BH}$.

d) Express $\overrightarrow{CA}$ in terms of $\overrightarrow{EC}$ and $\overrightarrow{AE}$.

10. ABCD is a parallelogram in which the diagonals AC and BD intersect at E. Write a single vector equivalent to each of the following.

a) $\overrightarrow{AB} - \overrightarrow{AC}$ **b)** $\overrightarrow{DC} - \overrightarrow{BC}$

c) $\overrightarrow{BC} - \overrightarrow{BE}$ **d)** $\overrightarrow{DB} - \overrightarrow{EB}$

11. The points A, B, C, D, and E are collinear. Express $\overrightarrow{BC}$ as a difference of vectors in two different ways.

12. Calculate the magnitude and direction of each force, $\vec{F}_1$, given $\vec{F}_2$ and the resultant force, $\vec{F}_1 + \vec{F}_2$.

a) $\vec{F}_2 = 120$ N on a bearing of 048°
$\vec{F}_1 + \vec{F}_2 = 110$ N on a bearing of 100°

b) $\vec{F}_2 = 60$ N on a quadrant bearing of N52°W
$\vec{F}_1 + \vec{F}_2 = 75$ N on a quadrant bearing of S85°W

c) $\vec{F}_2 = 110$ N on a quadrant bearing of S24°E
$\vec{F}_1 + \vec{F}_2 = 150$ N on a quadrant bearing of S15°W

d) $\vec{F}_2 = 20$ N on a bearing of 065°
$\vec{F}_1 + \vec{F}_2 = 40$ N on a bearing of 089°

e) $\vec{F}_2 = 130$ N on a bearing of 229°
$\vec{F}_1 + \vec{F}_2 = 140$ N on a bearing of 110°

C

13. Show that the expression $\overrightarrow{ST} - \overrightarrow{SQ} - \overrightarrow{QT}$ is equal to the zero vector.

14. When is the expression $|\vec{u} - \vec{v}| = |\vec{u} + \vec{v}|$ true? Use diagrams to explain.

15. a) Draw any vector $\vec{b}$ whose direction is *not* east. Then, draw any vector $\vec{c}$ such that $\vec{b} - \vec{c}$ is east.

b) Is more than one answer possible for part a)? Explain.

c) Suppose the magnitude of $\vec{c}$ must be a minimum. Draw $\vec{c}$ with minimum magnitude.

d) Suppose $\vec{b}$ has magnitude 8 m/s and direction S24°W, the direction of $\vec{b} - \vec{c}$ is east, and $|\vec{c}|$ is a minimum. Determine the magnitude and direction of $\vec{c}$, to one decimal place.