KEY CONCEPTS

- When two or more forces act on an object, you can use vector addition or vector subtraction to find the magnitude of the resultant force.
- The direction of a resultant can be found using trigonometry. The direction may be expressed as an angle relative to one of the given vectors, or as a quadrant bearing.

Example

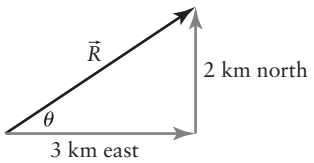
Mary is walking to the library, which is 3 km east and 2 km north of her current position.

a) Determine the magnitude of the resultant displacement to the nearest kilometre.

b) Determine the quadrant bearing of the resultant displacement to the nearest degree.

Solution

a) Draw a diagram to represent the situation.



Calculate the magnitude of the resultant displacement.

$$|\vec{R}|^2 = 3^2 + 2^2$$
$$= 9 + 4$$
$$= 13$$
$$|\vec{R}| \doteq 3.6$$

The magnitude of the resultant displacement is approximately 3.6 km.

b) Calculate the direction of the resultant displacement.

$$\tan\theta = \frac{2}{3}$$
$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$
$$\doteq 33.7°$$

Calculate the direction as a quadrant bearing.

$$\text{Bearing angle} = 90° - 33.7°$$
$$= 56.3°$$

The direction of the resultant displacement is N56.3°E.

A

Round all lengths to the nearest tenth of a unit and all angle measures to the nearest degree.

1. Determine the magnitude and direction of the resultant of each pair of vectors.

a) 12.3 m east and then 15.2 m north

b) 300 N north and then 250 N east

c) 42 km south and then 50 km west

d) 30 km/h west and then 12 km/h north

2. Draw a diagram to illustrate each situation. Then, determine the magnitude of the resultant force.

a) 32 N and 38 N acting at an angle of 43° to each other

b) 113 N and 247 N acting at an angle of 125° to each other

3. A powerboat is 5 km east and 7 km south of a lifeboat.

a) Determine the magnitude of the resultant displacement, to the nearest tenth.

b) Determine the bearing of the resultant displacement.

4. A car travels west at 100 km/h for 2 h, and then north at 80 km/h for 1 h.

a) Determine the magnitude of the resultant displacement.

b) Determine the quadrant bearing of the resultant displacement.

5. An airplane is flying with airspeed 435 km/h on a heading of 90°. There is a 40 km/h wind blowing from the direction 180°.

a) Draw a vector diagram to illustrate the situation.

b) Calculate the ground velocity of the airplane.

B

6. A car travels 16 km from Alton to Hart in 15 min, then 20 km from Hart to Lyon in 25 min, then 25 km back to Alton in 30 min. Find the average velocity for the entire trip.

☆7. An airplane is cruising at 180 km/h on a bearing of 240°. The wind is blowing at 30 km/h from a bearing of 330°.

a) Draw a diagram of this situation.

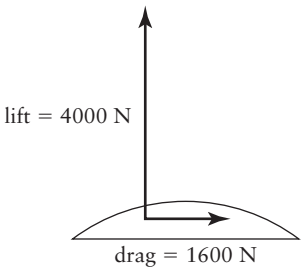
b) Find the ground velocity of the airplane.

☆8. An aircraft's takeoff velocity has a horizontal vector component of 218.7 km/h and a vertical vector component of 68.3 km/h.

a) Find the resultant velocity of the aircraft.

b) Determine the aircraft's displacement from the end of the runway after 6 min.

9. The force of the air moving past the wing of an airplane can be broken down into two forces: lift and drag. Use an appropriate scale drawing to approximate the resultant force acting on the airplane wing.



10. Wyatt and Jenna are pulling on two ropes attached to a trailer. The angle between the ropes is 90°. Wyatt pulls with a force of 310 N to the north and Jenna pulls with a force of 280 N to the east. Determine the magnitude and direction of the resultant force on the trailer.

11. Colton is on a dirt bike trail. He travels 500 m due east and then travels N50°E for 200 m. How far is Colton from his starting position, and at what bearing?

12. Yasmin and Marie are playing in a soccer game. Yasmin kicks the ball 30 m down the field parallel to the sideline, and then Marie kicks the ball 20 m farther at an angle 38° to the sideline. Determine the magnitude and direction of the resultant displacement of the soccer ball.

13. An airplane has a heading of N30°E and airspeed of 500 km/h, while the wind is 30 km/h in the direction S80°E.

a) Determine the angle between the heading of the airplane and the direction of the wind.

b) Determine the magnitude and direction of the airplane's velocity.

14. A boat travels 2 km N40°E, and then turns and travels 4 km N15°W. Find the resultant displacement of the boat.

C

15. Two tow trucks are pulling a transport truck due south. One tow truck exerts a force of 5000 N in a direction S20°W. The second tow truck exerts a force S30°E.

a) Find the magnitude of the force of the second tow truck if the transport truck must continue to move due south.

b) Find the magnitude of the resultant vector.

16. Two points, A (3, 5) and B (−2, 7), are joined by vector $\overrightarrow{AB}$. Determine the magnitude and direction of $\overrightarrow{AB}$.

Chapter 3: Checklist

By the end of this chapter, I will be able to:

- recognize a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors
- represent a vector as a directed line segment, with directions expressed in different ways, and recognize vectors with the same magnitude and direction but different positions as equal vectors
- resolve a vector represented as a directed line segment into its vertical and horizontal components
- represent a vector as a directed line segment, given its vertical and horizontal components
- determine, through investigation using a variety of tools and strategies, the sum (i.e., resultant) or difference of two vectors
- solve problems involving the addition and subtraction of vectors, including problems arising from real-world applications