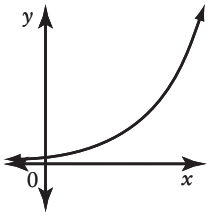
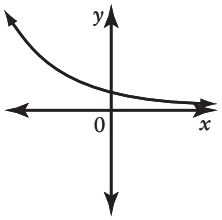
KEY CONCEPTS

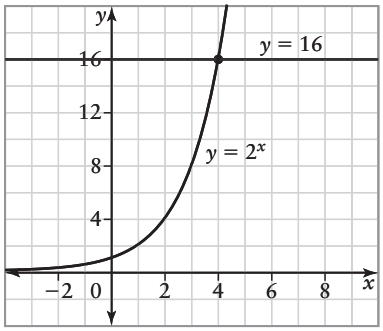
- A function of the form $y = ab^x$ is an exponential function for $b > 0$, $b \neq 1$.
- The graph of $y = ab^x$ represents exponential growth and is increasing if $a > 0$ and $b > 1$. The domain is $\{x \in \mathbb{R}\}$, the range is $\{y \in \mathbb{R}, y > 0\}$, the y-intercept is a, and the horizontal asymptote is $y = 0$.



- The graph of $y = ab^x$ represents exponential decay and is decreasing if $a > 0$ and $0 < b < 1$. The domain is $\{x \in \mathbb{R}\}$, the range is $\{y \in \mathbb{R}, y > 0\}$, the y-intercept is a, and the horizontal asymptote is $y = 0$.



- If you are given enough information about the graph or the properties of an exponential function, it is possible to write an equation or sketch a graph to model the function.
- Equations in one variable, such as $2^x = 16$, can be solved by finding the point of intersection of the corresponding graphs $y = 2^x$ and $y = 16$. The solution to the equation will be the x-coordinate of the point of intersection.



- Exponential equations in one variable can be solved graphically using technology.
- Many real-world applications can be modelled using an exponential function. Some examples are population growth, compound interest, and depreciation.

Example

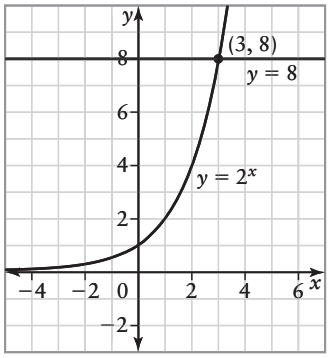
Use the graphs of $y = 2^x$ and $y = 8$ to solve the equation $2^x = 8$.

Solution

Method 1: Use Paper and Pencil

Graph the functions $y = 2^x$ and $y = 8$ on the same set of axes.

x	$y = 2^x$	$y = 8$
–3	$\frac{1}{8}$	8
–2	$\frac{1}{4}$	8
–1	$\frac{1}{2}$	8
0	1	8
1	2	8
2	4	8
3	8	8



The point of intersection appears to be (3, 8).

Check (3, 8) in the equation $2^x = 8$.

L.S. $= 2^x$ R.S. $= 8$

$= 23$

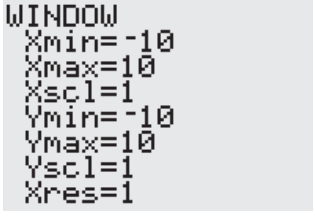
$= 8$

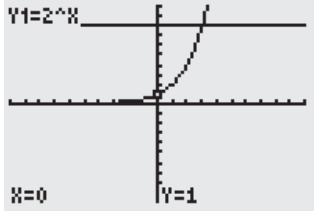
L.S. = R.S.

The point of intersection of the functions $y = 2^x$ and $y = 8$ is (3, 8), and the solution to the equation $2^x = 8$ is $x = 3$.

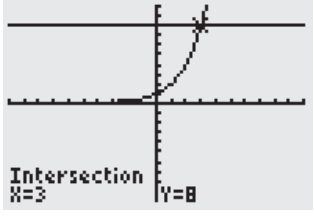
Method 2: Use Technology

Graph the functions $y = 2^x$ and $y = 8$ on a graphing calculator.





Use the **Intersect** operation to determine the point of intersection of the two functions.



The point of intersection of the functions $y = 2^x$ and $y = 8$ is (3, 8). Therefore, the solution to the equation $2^x = 8$ is $x = 3$.

A

1. **a)** Graph $y = 2^x$, $y = 5^x$, and $y = 10^x$ without technology.

b) Compare the three graphs. Discuss domain and range, asymptote, y-intercept, y-value when $x = 1$, and steepness or rate of increase.

2. **a)** Graph $y = \left(\frac{1}{3}\right)^x$, $y = \left(\frac{1}{2}\right)^x$, and $y = 0.8^x$ without technology.

b) Compare the three graphs. Discuss domain and range, asymptote, y-intercept, y-value when $x = 1$, and steepness or rate of decrease.

3. **Use Technology**

a) Use a graphing calculator to graph $y = 4^x$ and $y = \left(\frac{1}{4}\right)^x$ on the same set of axes. Describe how they are related.

b) Use a graphing calculator to verify that $y = \left(\frac{1}{4}\right)^x$ and $y = 4^{-x}$ represent the same function. Explain why this is the case.

4. **Use Technology** Use a slider in data software to investigate the graphs of $y = a^x$, where a is a slider. Describe the graph as a moves from 0 to 1, as a moves from being less than 1 to being greater than 1, and as a gets larger.

5. **a)** Sketch the graph of an exponential function that has the following properties:

- domain $\{x \in \mathbb{R}\}$
- range $\{y \in \mathbb{R}, y > 0\}$
- y-intercept 3
- horizontal asymptote $y = 0$
- decreasing function

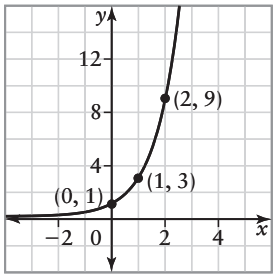
b) Is it possible to sketch the graph of another exponential function with the same properties? Explain.

6. **a)** Sketch the graph of an exponential function that has the following properties:

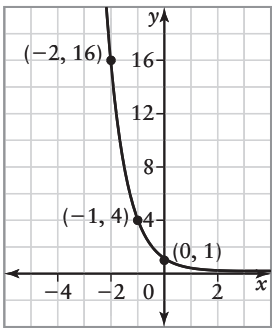
- domain $\{x \in \mathbb{R}\}$
- range $\{y \in \mathbb{R}, y > 0\}$
- y-intercept 4
- horizontal asymptote $y = 0$
- increasing function

b) Is it possible to sketch the graph of another exponential function with the same properties? Explain.

7. Write an exponential function to match the graph shown.



8. Write an exponential function to match the graph shown.



B

9. **a)** Use the graph of $y = 3^x$ to solve the equation $3^x = 81$.

b) Use the graph of $y = 4^x$ to solve the equation $4^x = \frac{1}{16}$.

10. a) Without a calculator, estimate the solution to the equation $2^x = 5$. Give reasons for your estimate.

b) Use Technology Use a graphing calculator to graph $y = 2^x$ and $y = 5$ to solve the equation $2^x = 5$. Round your answer to one decimal place.

11. Use Technology Solve each of the following equations using a graphing calculator.

a) $3^x = 9$ **b)** $4^x = 256$

c) $6^x = 216$ **d)** $\left(\frac{1}{2}\right)^x = 1$

e) $2^x = \frac{1}{32}$ **f)** $\left(\frac{3}{5}\right)^x = \frac{9}{25}$

☆**12.** The population, P, of Smalltown increases at a rate of 2% per year and is represented by $P = 2500 \times 1.02^n$, where n is the number of years after the year 2000.

a) Use Technology Use graphing technology to graph this function.

b) What does the coefficient 2500 represent on the exponential graph?

c) What does 2500 mean in the context of this question?

d) Use the graph to determine the year when the population will exceed 4000.

e) How would the equation of the function change if the population were to grow by 8% per year? What if it were to decrease by 5% per year?

13. Use Technology On January 1, one person discovered an interesting fact. The next day, that person told someone else. The next day, each of those people told someone.

a) Write an equation of a function that relates the number of people, N, who have learned the fact d days after January 1.

b) Use graphing technology to graph this function.

c) On what date will 256 people learn the fact?

d) After how many days will more than 2000 people know the fact?

☆**14. Use Technology** A car is purchased for $25 000 and its value decreases by 20% each year.

a) Write an equation of a function representing this situation.

b) Use graphing technology to graph the function.

c) After how many years will the car be worth 10% of its initial value?

d) By how much does the car depreciate during the first year? during the fifth year? Explain why these two amounts are significantly different.

C

15. For each of the following exponential functions,

i) sketch a graph of the function

ii) determine the domain and range of the function

iii) determine the intercept(s)

iv) determine the equation(s) of the asymptote(s)

v) determine the interval(s) of increase/decrease

a) $y = 4(2)^x$ **b)** $y = -\frac{1}{2}(3)^x$

16. Use Technology The wheel in a machine is rotating and speeding up. The number of revolutions per minute, R, after t minutes can be determined using the exponential function $R(t) = 200(2.5)^{2t}$.

a) Graph the function using technology.

b) Which value in the equation of the function indicates that the wheel is speeding up?

c) Determine the number of revolutions after

i) 1 min **ii)** 1.5 min