

4.3 Solving Exponential Equations Numerically

KEY CONCEPTS

- Exponential equations in one variable can be solved by determining a common base.
- The solutions to exponential equations may be exact answers or approximate answers. When solutions to exponential equations cannot be easily determined by finding a common base, approximate solutions can be found using systematic trial on a scientific calculator.

Example

- Solve the exponential equation $2^{5x+2} = 8^x$ by determining a common base.
- Use substitution to verify your answer to part a).
- Use a graphing calculator and the **Intersect** operation to verify your answer to part a).
- Use a table of values for each of the corresponding functions to determine the solution to the exponential equation in part a).

Solution

- a) $2^{5x+2} = 8^x$ Rewrite 8 using a base of 2.
 $2^{5x+2} = (2^3)^x$ Apply the power of a power rule.
 $2^{5x+2} = 2^{3x}$

Since the bases are equal, the exponents must be equal.

$$\begin{aligned} 5x + 2 &= 3x && \text{Solve for } x. \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

The solution is $x = -1$.

- b) For $x = -1$,
- | | |
|-------------------|-----------------|
| L.S. = 2^{5x+2} | R.S. = 8^x |
| = $2^{5(-1)+2}$ | = $8^{(-1)}$ |
| = 2^{-5+2} | = 8^{-1} |
| = 2^{-3} | = $\frac{1}{8}$ |
| = $\frac{1}{2^3}$ | |
| = $\frac{1}{8}$ | |
| L.S. = R.S. | |

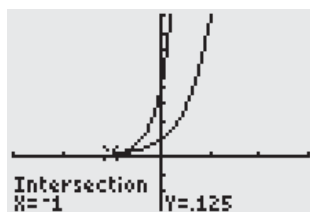
Therefore, $2^{5x+2} = 8^x$ for $x = -1$.

- c) Graph the functions $y = 2^{5x+2}$ and $y = 8^x$ using a graphing calculator. Use the window settings shown.

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WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-3
Ymax=8
Yscl=1
Xres=1
  
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Use the **Intersect** operation to determine the point of intersection of the two functions. The two graphs intersect at the point where $x = -1$ and $y = 0.125$.



- d) Using a table of values for the functions $y = 2^{5x+2}$ and $y = 8^x$ on a graphing calculator, the y -values are the same for the x -value of -1 .

X	Y ₁	Y ₂
-3	1.2E-4	.00195
-2	.00391	.01563
-1	.125	.125
0	4	1
1	128	8
2	4096	64
3	131072	512

X=-1

A

1. Solve each of the following equations by determining a common base.

- $5^x = 125$
- $2^x = 128$
- $3^x = 81$
- $7^x = 1$
- $4^x = \frac{1}{64}$
- $\left(\frac{2}{3}\right)^x = \frac{8}{27}$

2. Solve each of the following equations by determining a common base. Check your answers using graphing technology.

- $2^{4x} = 4^{x+3}$
- $25^{x-1} = 5^{3x}$
- $3^{w+1} = 9^{w-1}$
- $36^{3m+1} = 6^{m-8}$
- $4^{3x} = 8^{x-3}$
- $27^x = 9^{2x-3}$

3. **Use Technology** Use systematic trial to find an approximate solution to the following equations. Round your answer to one decimal place.

- a) $3^x = 4$
- b) $5^x = 7$
- c) $-4^x = -3$

B

4. Solve each of the following exponential equations by expressing both sides of the equation as a power with a common base. Express your answers as fractions in lowest terms.

- a) $\left(\frac{4}{25}\right)^x = \frac{125}{8}$
- b) $\left(3\frac{1}{2}\right)^{3x} = \frac{4}{49}$
- c) $\left(8\frac{1}{2}\right)^{x+2} = (64^2)^{x-1}$
- d) $(\sqrt{8})^{x+2} = (\sqrt[3]{32})^{x-3}$

- 5. a) Solve the exponential equation $3^{7x-3} = 81^x$ by determining a common base.
- b) Use substitution to verify your answer to part a).
- c) Describe two different ways of using a graphing calculator to verify the solution from part a).

★6. **Use Technology** Use systematic trial with a scientific calculator to determine the solution to the exponential equation $1.03^x = 1083$, to one decimal place.

7. **Use Technology** Use systematic trial with a scientific calculator to determine the solution to the equation $2.05^x = 78\,440$, to one decimal place.

8. A radioactive sample with an initial mass of 50 mg has a half-life of 4 days. Half-life is the time it takes for a quantity to decrease to half of its original amount.

a) Write an equation that models this exponential decay, where t is the time, in days, and A is the amount of the substance that remains.

b) What is the amount of radioactive material remaining in the sample after 10 days? Round your answer to the nearest tenth of a milligram.

c) How long will it take for the radioactive sample to decay to 6.25 mg?

9. Solve the exponential equation $9^{4x-2} = 3^{2(x+5)}$ by determining a common base. Express your answer as a fraction in lowest terms.

- ★10. a) Solve the exponential equation $25^{4x+3} = 125^{3x}$ by determining a common base.
- b) Use substitution to verify your answer to part a).

C

11. a) How are the equations $2^{2x} - 3(2^x) + 2 = 0$ and $a^2 - 3a + 2 = 0$ related?

b) Use your answer from part a) to solve $2^{2x} - 3(2^x) + 2 = 0$.

12. A chain hanging between two posts is in the shape of a catenary, which is a curve that approximates a parabola. The equation of the path of the catenary is $y = 0.1\left(2^{\frac{x}{2}} + 2^{-\frac{x}{2}}\right)$.

a) **Use Technology** Use a graphing calculator to graph this relation.

b) Is this relation a function? Justify your answer.

c) This relation is similar to a relation that you have studied before. State the name of this relation.

d) State the coordinates of the minimum point of the relation

$$y = 0.1\left(2^{\frac{x}{2}} + 2^{-\frac{x}{2}}\right).$$