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KEY CONCEPTS

- The point of intersection of the graphs of two exponential functions can be determined using graphing technology.
- An exponential equation can be solved graphically by determining the x-coordinate of the point of intersection of the two corresponding exponential functions.
- Any equation with one variable can be solved by forming and graphing corresponding functions. For example, to solve $3^x = 12$, determine the point of intersection of the graphs of $y = 3^x$ and $y = 12$. The solution to the equation will be the x-coordinate of the point of intersection.

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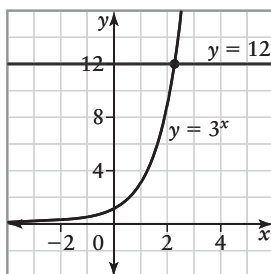
Example

a) Solve $2^{-x} = 4^{x+3}$ graphically, using technology, by determining the point of intersection of the graphs of the exponential functions $y = 2^{-x}$ and $y = 4^{x+3}$.

b) Solve the exponential equation $2^{-x} = 4^{x+3}$ algebraically.

c) What do you notice about the x-coordinate of the point of intersection found in part a) and the x-value found in part b)?
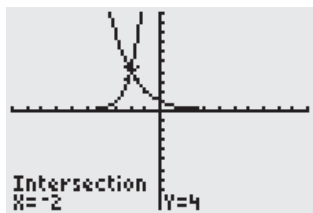
Solution

a) Graph the functions $y = 2^{-x}$ and $y = 4^{x+3}$ on a graphing calculator. Use the window settings as shown.

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Use the **Intersect** operation to determine the point of intersection of the two functions.

The point of intersection is $(-2, 4)$.

Therefore, the solution to the original equation is $x = 2$.


```
Intersection
X=-2          Y=4
```

b) Express each side of the exponential equation with a common base. Since the bases are equal, then the exponents are equal and the resulting linear equation in one variable can be solved to determine the solution to the exponential equation.

$$2^{-x} = 4^{x + 3}$$
$$2^{-x} = (2^2)^{x + 3}$$
$$2^{-x} = 2^{2(x + 3)}$$
$$2^{-x} = 2^{2x + 6}$$
$$-x = 2x + 6$$
$$-3x = 6$$
$$x = -2$$

Therefore, the solution is $x = -2$.

c) The x-coordinate of the point of intersection of the graphs of the two corresponding functions is the same as the x-value determined algebraically for the solution of the exponential equation.

A

1. Determine the point of intersection, if it exists, for each pair of functions.

a) $y = 5^{x + 1}$ and $y = 25^{x + 3}$

b) $y = 7^{5x - 2}$ and $y = 49^{x + 2}$

c) $y = 6^{4x + 2}$ and $y = 36^{x + 2}$

d) $y = 8^{x - 1}$ and $y = 4^{x + 2}$

2. Use an algebraic method to determine the point of intersection of each pair of exponential functions.

a) $y = 16^{x + 1}$ and $y = 64^x$

b) $y = 25^{4x}$ and $y = 125^{3x - 1}$

3. Can the exponential equation $4^{2x + 3} = 2^{4x - 7}$ be solved? Explain algebraically and graphically.

4. a) Graph the exponential function $y = 2^x$ and the linear function $y = 4$ on the same set of axes.

b) Determine the point of intersection of the graphs of the two functions.

c) State the x-coordinate for the point of intersection that you found in part b).

d) How could the solution be determined without graphing?

e) Could your method in part d) be used if the equation was $2^x = 25$? Explain.

B

5. a) **Use Technology** Use graphing technology to graph the exponential functions $y = 9^{x+1}$ and $y = 27^x$ on the same set of axes.

   b) Determine the point of intersection of the graphs of the two functions.

   c) State the x-coordinate for the point of intersection that you found in part b).

   d) Solve the exponential equation $9^{x+1} = 27^x$ using an algebraic solution.

   e) What do you notice about the results for parts c) and d)?

6. a) **Use Technology** Determine the point of intersection of the graphs of the exponential functions $y = 3^{x+2}$ and $y = 3^x + 24$ using technology.

   b) Solve $3^{x+2} = 3^x + 24$ algebraically.

7. a) **Use Technology** Determine the point of intersection of the graphs of the exponential functions $y = 9^{-x}$ and $y = 27^{x+5}$ using technology.

   b) Solve $9^{-x} = 27^{x+5}$ algebraically.

8. **Use Technology** Use a graphing calculator to show that

   a) $2^x = -x$ has exactly one solution

   b) $10^x = 10x$ has two solutions

9. Solve $0.4^x = 5^{x+2}$ by determining the x-coordinate of the point of intersection of the graphs of the exponential functions $y = 0.4^x$ and $y = 5^{x+2}$. Round your answers to one decimal place.

☆ 10. a) **Use Technology** Solve $2^{x+3} = 2^x + 14$ graphically using technology.

   b) Solve $2^{x+3} = 2^x + 14$ algebraically.

   c) What do you notice about the x-coordinate of the point of intersection found in part a) and the x-value found in part b)?

☆ 11. The table gives Taylor's weekly earnings over a 5-year period.

Year	Earnings ($)
2004	550.00
2005	566.50
2006	583.50
2007	601.00
2008	619.03

   a) Use the table to construct an exponential function that models the data.

   b) Predict Taylor's weekly earnings in the year 2012.

   c) Predict when Taylor might expect to earn more than $850 per week.

☆ 12. The table gives the value of Carly's car n years after she purchased it.

Year	Value of the Car ($)
1	12 000
2	9 600
3	7 680
4	6 140
5	4 920

   a) Use the table to construct an exponential function that models the data.

   b) Determine the most likely purchase price of the car.

   c) Determine the approximate value of the car in 2010, if year 1 represents 2003.

C

13. A tire with a slow puncture loses pressure at the rate of 5%/min.

   a) If the tire's pressure is 204 kPa to begin with, what is its pressure after 5 min?

   b) **Use Technology** Use graphing technology to determine when the tire's pressure will be 100 kPa, to the nearest minute.