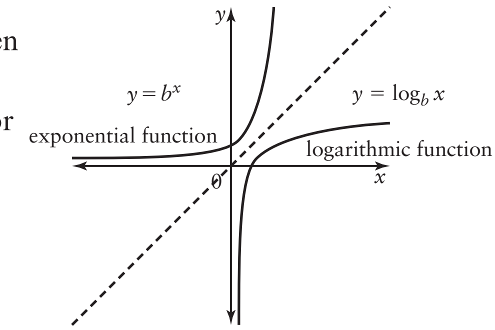
KEY CONCEPTS

- The inverse of $y = b^x$ is $x = b^y$, which can be written as $y = \log_b x$.
- The logarithmic function is defined as $y = \log_b x$, or the logarithm, base b, of x.
- The domain of the logarithmic function is $\{x \in \mathbb{R}, x > 0\}$.
- The range of the logarithmic function is $\{y \in \mathbb{R}\}$.



- The value of $\log_b x$ is equal to the exponent to which the base, b, must be raised to get the number, x.
- Logarithmic equations can be written in exponential form and vice versa.
  $y = \log_b x \longleftrightarrow x = b^y$
- Logarithmic and exponential functions are defined only for positive values of the base that are not equal to 1; i.e., $b > 0$, $b \neq 1$.
- Logarithms with a base of 10 are called common logarithms. For common logarithms, you do not have to write the base: $\log x$ is understood to mean $\log_{10} x$.
- Logarithmic equations can be solved algebraically and graphically.

Example

a) Rewrite $\log_3 81 = 4$ in exponential form.

b) Rewrite $5^3 = 125$ in logarithmic form.

c) Evaluate $\log_2 128$.

d) Evaluate $\log_{10} 1000$.

e) Use technology to determine the value of $\log_3 5$, to one decimal place.

Solution

a) If $\log_3 81 = 4$, then $3^4 = 81$.

b) If $5^3 = 125$, then $\log_5 125 = 3$.

c) You can evaluate $\log_2 128$ by determining to what exponent the base 2 must be raised to get the result 128. Since $2^7 = 128$, the exponent is 7. Therefore, $\log_2 128 = 7$.

Alternatively, let $\log_2 128 = y$.

$2^y = 128$ Express 128 as 2^7.

$2^y = 2^7$ Since the bases are equal, the exponents are equal.

$y = 7$

Therefore, $\log_2 128 = 7$.

d) You can evaluate $\log_{10} 1000$ by determining to what exponent the base 10 must be raised to get the result 1000. Since $10^3 = 1000$, the exponent is 3. Therefore, $\log_{10} 1000 = 3$.

Alternatively, let $\log_{10} 1000 = y$.

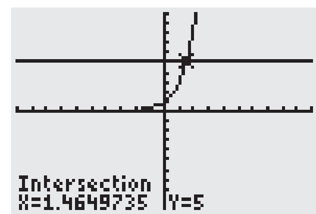
$10^y = 1000$	Express 1000 as 10^3.
$10^y = 10^3$	Since the bases are equal, the exponents are equal.
$y = 3$	

Therefore, $\log_{10} 1000 = 3$.

e) Let $\log_3 5 = x$.

Express $\log_3 5 = x$ in exponential form as $3^x = 5$.

Graph $y = 3^x$ and $y = 5$ using a graphing calculator.



The x-value of the point of intersection of the two graphs is approximately 1.5.

The value of $\log_3 5$ is 1.5, to one decimal place.

A

1. Write each equation in exponential form.

a) $\log_7 343 = 3$

b) $\log_2 16 = 4$

c) $\log_3\left(\frac{1}{81}\right) = -4$

d) $\log_2\left(\frac{1}{64}\right) = -6$

e) $0 = \log_5 1$

f) $\log_b b^2 = 2$

2. Explain why $\log_a \sqrt{a} = \frac{1}{2}$.

3. Rewrite each equation in logarithmic form.

a) $5^2 = 25$ **b)** $2^6 = 64$

c) $4^{-2} = \frac{1}{16}$ **d)** $8^0 = 1$

e) $216 = 6^3$ **f)** $\frac{1}{27} = 3^{-3}$

4. Evaluate each of the following logarithms.

a) $\log_2 128$ **b)** $\log_3 243$

c) $\log_5 625$ **d)** $\log_{10} 0.001$

e) $\log_6 216$ **f)** $\log_7\left(\frac{1}{49}\right)$

5. a) Evaluate $\log_2 1$.

b) Does the answer change if the base is changed? Explain why or why not.

6. Explain why the base on a logarithm cannot be negative, 0, or 1. Consider the exponential form as you answer the question.

7. a) Evaluate each logarithm.

i) $\log_5 5$

ii) $\log_7 7$

iii) $\log_{10} 10$

b) Why is the answer the same to all three?

8. Evaluate each common logarithm.

a) $\log 100$ **b)** $\log 0.1$

c) $\log\left(\frac{1}{1000}\right)$ **d)** $\log 10\,000$

e) $\log 0.01$ **f)** $\log\left(\frac{1}{10^{-6}}\right)$

9. a) Explain why $\log 0.01$ and $\log\left(\frac{1}{100}\right)$ are equivalent.

b) Which form do you find easier to evaluate? Give a reason for your answer.

B

10. Sketch a graph of the function $y = 3^x$ and its inverse on the same set of axes. Label each function.

11. Copy and complete the following table for the functions $y = 10^x$ and $y = \log x$.

	$y = 10^x$	$y = \log x$
Sketch of the graph		
Domain		
Range		
x-intercept(s)		
y-intercept(s)		
Vertical asymptote(s)		
Horizontal asymptote(s)		
Intervals of increase		
Intervals of decrease		

12. The point (2, 8) is on the graph of $y = a^x$. What would be the corresponding point on the graph of $y = \log_a x$? Explain your reasoning.

13. **a)** Explain how $y = 5^x$ can be used to evaluate $\log_5 13$.

    b) Use Technology Use a graphing calculator to determine an approximate value for $\log_5 13$. Round your answer to one decimal place.

14. **a)** State the x-intercept of the graph of $y = \log_7 x$.

    b) Explain how your answer from part a) can be used to explain why $\log_7 \left(\frac{1}{2}\right)$ must be less than 0.

    c) Use Technology Use a graphing calculator to determine an approximate value for $\log_7 \left(\frac{1}{2}\right)$. Round your answer to one decimal place.

C

15. **a)** Evaluate $\log_{10} 1000$.

    b) Perform the following calculations with a calculator.

    i) $\frac{\log 1000}{\log 10}$

    ii) $\frac{\ln 1000}{\ln 10}$

    c) What do you notice about your answers from part b)?

    d) Use graphing technology or systematic trial to evaluate $\log_3 20$.

    e) Perform the following calculations with a calculator.

    i) $\frac{\log 20}{\log 3}$

    ii) $\frac{\ln 20}{\ln 3}$

    f) What do you notice about your answers from part e)?

    g) Explain how you can use a calculator to evaluate $\log_2 50$.

16. An investment pays 4% interest, compounded semi-annually.

    a) Write a function to model the amount, A, of the investment in terms of time, t, in years.

    b) Determine how long it will take for this investment to be worth double its current value.