KEY CONCEPTS

- A logarithm can be expressed in terms of base 10 using the change of base formula:
  $\log_b m = \dfrac{\log m}{\log b}, b > 0, b \neq 1, m > 0$
  This change of base formula can be used to evaluate logarithms or to graph logarithmic functions with any base using technology.
- Many problems based on real-world applications that can be modelled with exponential equations can be solved algebraically by rewriting them in logarithmic form. Logarithms are used to determine the exponent in an exponential equation.
- There are many applications of logarithms in business, such as compound interest, and in the sciences, such as the pH scale, decibel scale, and Richter scale.

Example

a) Evaluate $\log_{15} 20$. Round your answer to three decimal places.

b) Solve $2 = 1.04^t$ for t, to two decimal places.

c) Graph the function $f(x) = \log_4 x$ using a graphing calculator.

Solution

a) $\log_{15} 20 = \dfrac{\log 20}{\log 15}$

$\doteq 1.106$

b) $2 = 1.04^t$

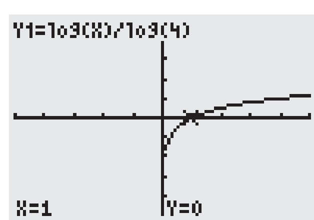
$t = \log_{1.04} 2$

$t = \dfrac{\log 2}{\log 1.04}$

$t \doteq 17.67$

c) $f(x) = \log_4 x$

$= \dfrac{\log x}{\log 4}$



A

1. Write each of the following as a single logarithm.

   a) $\dfrac{\log 14}{\log 6}$

   b) $\dfrac{\log 7}{\log 9}$

   c) $\dfrac{\log\left(\frac{1}{4}\right)}{\log\left(\frac{3}{5}\right)}$

   d) $\dfrac{\log 15}{\log\left(\frac{1}{3}\right)}$

2. Evaluate each of the following using logarithms. Round your answers to three decimal places.

   a) $\log_8 17$

   b) $\log_{12} 30$

   c) $\log_7 3$

   d) $-\log_{13} 5$

   e) $\log_{\frac{1}{4}} 10$

   f) $\log_{\frac{2}{3}} 12$

3. Solve each of the following for t, to two decimal places.

   a) $3^t = 8$

   b) $10.7^t = 4$

   c) $450 = 25^t$

   d) $400 = 50(1.04)^t$

4. **Use Technology**

   a) Graph $y = \log x$ using a graphing calculator.

   b) State the x-intercept.

   c) Use the **Trace** feature to find the value of x when $y = 1$ and when $y = 2$.

5. **Use Technology**

   a) Graph $y = \log_2 x$ using a graphing calculator.

   b) Use the **Trace** feature to find the value of x when $y = -1$.

   c) State two other x-values that result in y being integers. Explain how you arrived at these x-values. Use the **Trace** feature to verify your answer.

B

6. The number of people watching a certain online news site is doubling every day. The time, t, in days, for a number of people, P, to view the online news site is given by the equation $t = \log_2 P$. How long will it take until the number of people watching the online news site reaches 10 000 people? Round your answer to the nearest tenth of a day.

7. An investment of \$1000 earns 5% interest, compounded annually.

   a) Write a function to model the amount, A, that the investment is worth in terms of the time, t, in years.

   b) How much will the investment be worth after 3 years?

   c) How long will it take for the investment to double in value?

8. An investment of \$2000 earns 4.5% interest, compounded semi-annually.

   a) Write a function to model the amount, A, that the investment is worth in terms of the time, t, in years.

   b) How much will the investment be worth after 2 years?

   c) How long will it take for the investment to reach \$5000?

☆**9.** The pH scale is an application of logarithms used to determine how acidic or alkaline a substance is. The pH scale is defined as pH = –log $[H^+]$, where H^+ is the hydronium ion concentration in a substance, measured in moles per litre.

a) Lemon juice has a hydronium ion concentration of approximately 0.01 mol/L. Determine the pH of lemon juice.

b) Milk has a pH of approximately 6. Determine the concentration of hydronium ions in milk.

☆**10.** The difference in sound levels, in decibels, can be found using the equation $\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$, where $\beta_2 - \beta_1$ is the difference in sound levels, in decibels, and $\frac{I_2}{I_1}$ is the ratio of their sound intensities, where I is measured in watts per square metre (W/m^2). The sound level of a rock concert speaker is 150 dB and the sound level of a symphony, at its peak, is 120 dB. How many times as intense as the sound level of a symphony, at its peak, is the sound level of a rock concert speaker?

☆**11.** The sound level of normal conversation is approximately 60 decibels. The sound level of a shout is about 100 times as intense. What is the sound level of a shout, in decibels?

12. The magnitude, M, of an earthquake is measured using the Richter scale, which is defined as $M = \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the earthquake being measured and I_0 is the intensity of a standard, low-level earthquake.

a) How many times as intense as a standard earthquake is an earthquake measuring 3.5 on the Richter scale?

b) What is the magnitude of an earthquake that is 10 000 times as intense as a standard earthquake?

C

13. An investment earns 4% interest, compounded annually.

a) By what percent will the initial investment increase over 10 years?

b) How long will it take to double the investment?

c) Why does it not matter that the amount of the initial investment was not given?

14. Describe a situation involving half-life that can be modelled by the expression $3 = \log_{\frac{1}{2}}\left(\frac{1}{8}\right)$.

15. Solve the logarithmic equation $\log(x^2 - 3x) = 1$. Check your solution using a graphing calculator.

16. Use the Internet to research logarithms. Prepare a report for the class that includes the following information.

a) Who discovered logarithms? In what year were logarithms discovered?

b) List as many real-life applications of logarithms as you can discover in your research.

c) Until approximately what year were common logarithms and log tables used for calculations?

Chapter 4: Checklist

By the end of this chapter, I will be able to:

- simplify algebraic expressions containing integer and rational exponents using the laws of exponents
- determine, through investigation with technology, and describe the impact of changing the base and changing the sign of the exponent on the graph of an exponential function
- solve simple exponential equations numerically and graphically, with technology, and recognize that the solutions may not be exact
- pose problems based on real-world applications (e.g., compound interest, population growth) that can be modelled with exponential equations, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation
- solve exponential equations in one variable by determining a common base
- determine, through investigation using graphing technology, the point of intersection of the graphs of two exponential functions, recognize the x-coordinate of this point to be the solution to the corresponding exponential equation, and solve exponential equations graphically
- recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions
- make connections between related logarithmic and exponential equations, and solve simple exponential equations by rewriting them in logarithmic form
- determine, with technology, the approximate logarithm of a number to any base, including base 10
- pose problems based on real-world applications that can be modelled with given exponential equations, and solve these and other such problems algebraically by rewriting them in logarithmic form