Chapter 5 Polynomial Functions

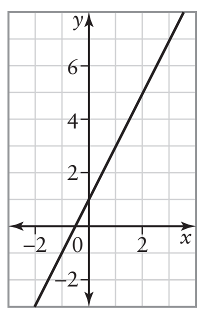
5.1 Identifying Polynomial Functions

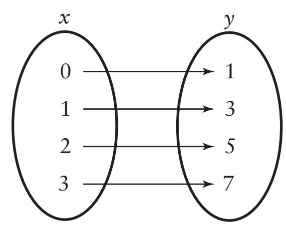
KEY CONCEPTS

- A polynomial expression is a series of terms in which each term is the product of a constant and a power of x that has a whole number as the exponent.
- A polynomial expression has the form $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$, where n is a whole number; x is the variable; the coefficients $a_0, a_1, \ldots, a_n$ are real numbers; the degree of the expression is n, the exponent of the greatest power of x; a_n, the coefficient of the greatest power of x, is the leading coefficient; and a_0, the term without a variable, is the constant term.
- A relation is a relationship between two variables that may be expressed as ordered pairs, an equation, a table of values, a graph, or a mapping diagram.

{(0, 1), (1, 3), (2, 5), (3, 7)} $y = 2x + 1$

x	y
0	1
1	3
2	5
3	7





- A function is a relation between two variables, such as x and y, in which every value of x is mapped onto exactly one value of y.
- A vertical line test can be used to determine if the graph of a relation is a function. If a vertical line intersects the graph of a relation at more than one point, then the relation is not a function.
- The domain is the set of first coordinates, typically the x-coordinate, of the ordered pairs in a relation.
- The range is the set of second coordinates, typically the y-coordinate, of the ordered pairs in a relation.
- A polynomial function is a function of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$.
- The degree of a polynomial function in one variable is the greatest exponent of the variable in any one term.
- A constant function is a polynomial function of degree 0.

Example

Identify whether each of the following functions is a polynomial function. If it is not a polynomial function, state the type of function that it is.

a) $f(x) = 3x^2 + 2x - 5$

b) $f(x) = 3^{x+2}$

c) $f(x) = 4$

d) $f(x) = \sin 2x$

e) $f(x) = 2x + 3$

f) $f(x) = \sqrt{4x^2 - 3x + 2}$

g) $f(x) = 4 \cos x$

h) $f(x) = \dfrac{1}{x^2 - 3}$

Solution

a) This is a polynomial function of degree 2.

b) This is not a polynomial function. It is an exponential function, since the base is a number and the exponent is a variable.

c) This is a polynomial function of degree 0, or a constant function.

d) This is not a polynomial function. It is a sinusoidal function.

e) This is a polynomial function of degree 1.

f) This is not a polynomial function. It is a square root function.

g) This is not a polynomial function. It is a sinusoidal function.

h) This is not a polynomial function. It is a rational function.

A

1. State the degree of each of the following polynomial expressions.

a) $5x^4$ **b)** $6x^2$

c) $-3x^5$ **d)** $\frac{1}{2}x^6$

e) 4 **f)** $-0.5x^3$

2. State the degree of each of the following polynomial expressions.

a) $4x^3 + 7x^2 - 3x$ **b)** $0.5x - 2$

c) $3x^2 - 8x + 2$ **d)** $-x^4 + 9x^3 - 5$

3. State the degree of each of the following polynomial functions.

a) $f(x) = 0.5x^2 + 5x$

b) $f(x) = -5x^4 + 2x^2 - 3$

c) $f(x) = 7$

d) $f(x) = 2x^3 - 4x^2 + 6x$

e) $f(x) = -\frac{1}{5}x + 9$

f) $f(x) = 7x^5 - 5x^3$

4. Determine whether each of the following relations is a function. Justify your answer.

a) $y = 5x$ **b)** $y = 4^x$

c) $y = -3x^2$ **d)** $x = y^2 + 3$

e) $y = 7$ **f)** $y = x^2 + 3x - 5$

g) $x = -3$ **h)** $x^2 + y^2 = 9$

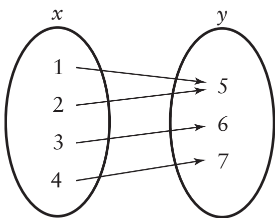
5. Describe how you can tell that an equation represents a non-function. Refer to question 4 in your answer.

B

6. For each relation, determine whether it is a function. Justify your answer.

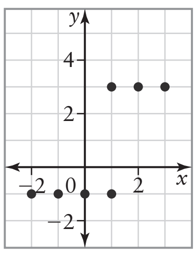
a) $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$

b)



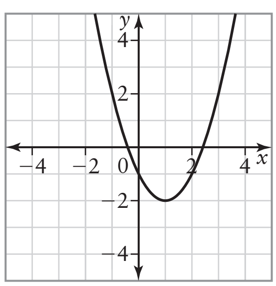
c) $\{(-1, 2), (2, 1), (2, 2), (3, 1)\}$

d)

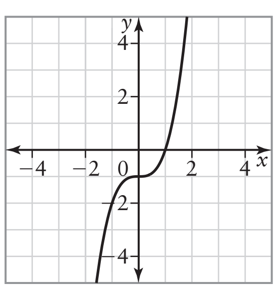


7. Determine which of the following graphs represent polynomial functions. If the relation is not a polynomial function, state the type of function that it is.

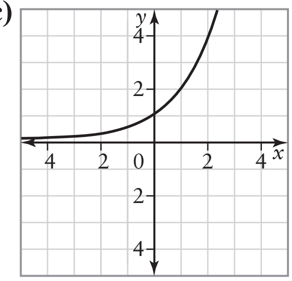
a)



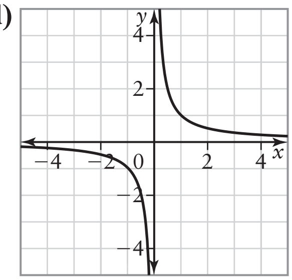
b)



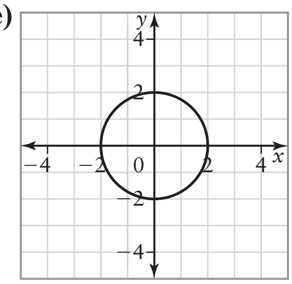
c)



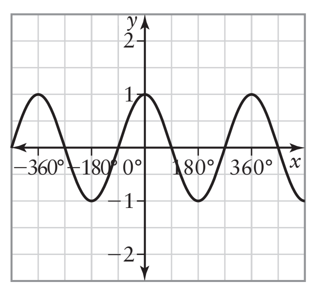
d)



e)



f)



8. Identify whether each of the following functions is a polynomial function. If it is not a polynomial function, state the type of function that it is.

a) $f(x) = 4^{x-3}$

b) $f(x) = 3x + 4$

c) $f(x) = 5 \sin x$

d) $f(x) = \sqrt{x^2 - 2x}$

e) $g(x) = -2x^2 - 3x + 5$

f) $s(x) = \cos 3x$

g) $h(x) = x^3 - 2$

9. The function $f(x) = \frac{1}{x^2}$ can be written as $f(x) = x^{-2}$. Does this mean it is a polynomial function? Explain why or why not.

10. Is the function defined by $y = (x - 1)(x + 2)$ a polynomial function? Explain your reasoning. Explain how this form, called factored form, might be useful when graphing the function.

☆ 11. Consider the functions $y = 2x - 5$, $y = 5x - 10$, and $y = 7x + 12$.

a) Explain why they are polynomial functions.

b) What do the graphs have in common with each other?

c) **Use Technology** Use a graphing calculator to graph the functions on the same set of axes. When x increases (that is, approaches positive infinity, written as $x \to \infty$), what happens to the y-values in each of the functions? What happens to the y-values as x decreases (that is, approaches negative infinity, written as $x \to -\infty$)?

☆ 12. Consider the functions $y = x^2 + 7$, $y = x^2 + 3x - 4$, and $y = 4x^4 - 9$.

a) Explain why they are polynomial functions.

b) What do the graphs have in common with each other?

c) **Use Technology** Use a graphing calculator to graph the functions on the same set of axes. When x increases (that is, approaches positive infinity, written as $x \to \infty$), what happens to the y-values in each of the functions? What happens to the y-values as x decreases (that is, approaches negative infinity, written as $x \to -\infty$)?

☆ 13. **Use Technology** Consider the functions $y = 2x - 1$, $y = x^3 + x^2 - 3$, and $y = x^5 - x^2 + 7$.

a) Use a graphing calculator to graph the functions on the same set of axes.

b) What do the graphs have in common with each other?

c) How do they all differ from the graphs in question 12?

14. What does $y = 2^x$ have in common with the functions in questions 11, 12, and 13? How is this function different? How is the function $y = \sin x$ different from all of the functions in questions 11, 12, and 13?

C

15. An oil slick on the ocean is increasing in a circular pattern. The circumference of the oil slick, C, in metres, can be modelled by the function $C(r) = 2\pi r$, where r is the radius of the oil slick, in metres, and $0 \le r \le 10$.

a) Graph the function for $0 \le r \le 10$.

b) Determine the domain and the range of the function.

c) Compare the graph of $C(r) = 2\pi r$ to the graph of $y = 2x$. How are the graphs alike? How are the graphs different?

16. The area, A, in square metres, of the oil slick in question 15 can be modelled by the function $A(r) = \pi r^2$, where r is the radius of the oil slick, in metres, and $0 \le r \le 10$.

a) Graph the function for $0 \le r \le 10$.

b) Determine the domain and the range of the function.

c) Compare the graph of $A(r)$ to the graph of $y = x^2$. How are the graphs alike? How are the graphs different?