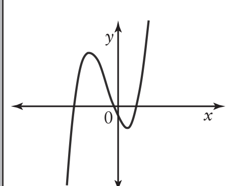
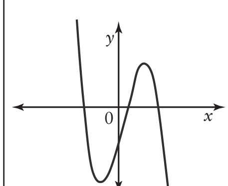
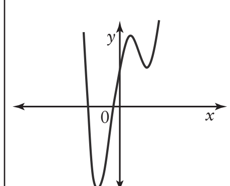
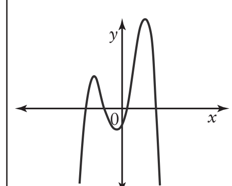
KEY CONCEPTS

- The degree of a polynomial function
  $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$
  determines the end behaviour as x approaches positive infinity ($x \to \infty$) and as x approaches negative infinity ($x \to -\infty$).
- The leading coefficient is the coefficient of the term that is used to determine the degree of a polynomial function. It may be a positive number or a negative number.
- A polynomial function may be an odd-degree polynomial or an even-degree polynomial, as shown in the chart.

	Odd-Degree Polynomial		Even-Degree Polynomial	
Leading Coefficient	positive	negative	positive	negative
End Behaviour	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$ (similar to the graph of $y = x$)	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to -\infty$ (similar to the graph of $y = -x$)	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to \infty$ (similar to the graph of $y = x^2$)	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to -\infty$ (similar to the graph of $y = -x^2$)
Sketch				
Domain	$\{x \in \mathbb{R}\}$		$\{x \in \mathbb{R}\}$	
Range	$\{y \in \mathbb{R}\}$		$\{y \in \mathbb{R}, y \geq a\}$	$\{y \in \mathbb{R}, y \leq a\}$
Maximum/ Minimum Value	neither a maximum value nor a minimum value		minimum value is a	maximum value is a

Example

Consider the polynomial function $f(x) = x^3 + 3x^2 - 4x + 1$.

a) Is the polynomial function an odd-degree polynomial or an even-degree polynomial? Justify your answer.

b) Is the leading coefficient positive or negative?

c) Describe the end behaviour of the graph of the polynomial function.

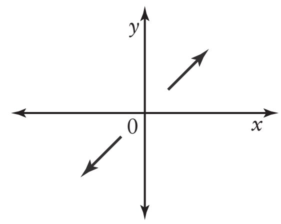
d) State the domain and range of the polynomial function.

Solution

a) The polynomial function is an odd-degree polynomial. The degree of the polynomial is 3.

b) The leading coefficient, 1, is positive.

c) The polynomial function is an odd-degree polynomial with a leading coefficient that is positive. Therefore, the end behaviour of the graph of the function will be as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.



d) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}\}$

A

1. State the leading coefficient for each of the following polynomial functions.

a) $f(x) = 7x^2$

b) $f(x) = -5x^4 + 2x^3 - 3x^2$

c) $f(x) = 0.4x^3 - 4x$

d) $f(x) = \frac{1}{4}x$

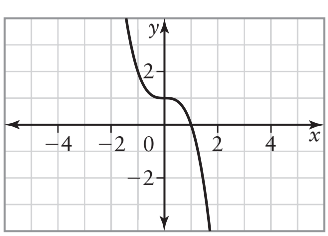
2. Consider each of the following graphs.

i) State the sign of the leading coefficient. Justify your answer.

ii) State the domain and range.

iii) Describe the end behaviour.

a)



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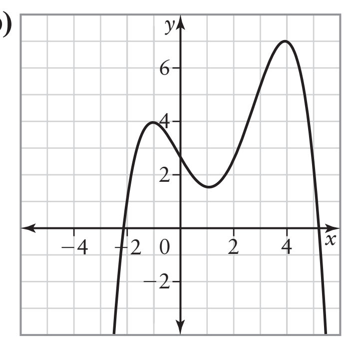

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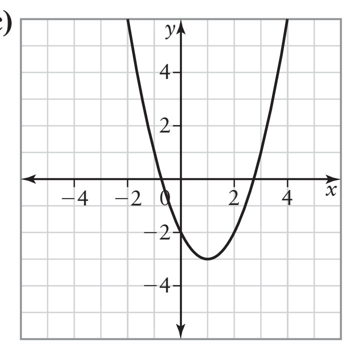


3. Use the degree and the sign of the leading coefficient to describe the end behaviour of each of the following polynomial functions.

a) $f(x) = 3x^4 + 4x^3 - 5x^2$

b) $f(x) = 2x^3 - 3x^2 + 2x - 1$

c) $f(x) = -5x^2 - 2x + 3$

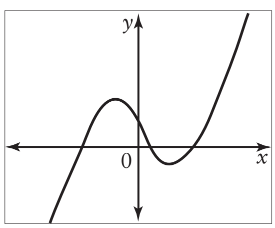
d) $f(x) = -\frac{1}{4}x^5 - 8x^3 + 5x$

e) $f(x) = 7 - 4x$

f) $f(x) = 6 + 3x^2$

4. Each of the graphs represents a polynomial function of degree 1, 2, 3, or 4. Determine the least possible degree of the function corresponding to the graph. Justify your answer.

a)



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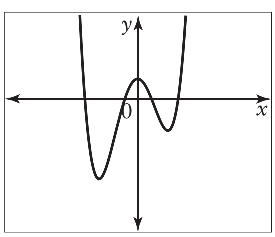

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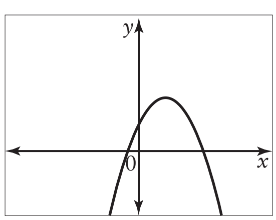
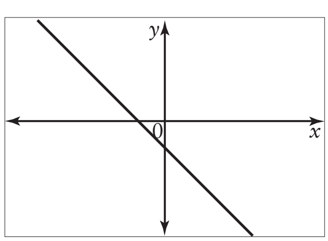


d)



5. Refer to question 4. For each graph, state the sign of the leading coefficient. Justify your answer.

B

☆6. **a)** Copy and complete a table similar to the one shown for n = 1, 2, 3, and 4.

	$f(x) = x^n$
Sketch	
Degree	
Type of Function	
Domain	
Range	
End Behaviour	

b) Use the tables to compare the key features of the graphs of the polynomial functions $f(x) = x$ and $f(x) = x^3$. Describe similarities and differences.

c) Use the tables to compare the key features of the graphs of the polynomial functions $f(x) = x^2$ and $f(x) = x^4$. Describe similarities and differences.

7. A graph has line symmetry if there is a line $x = a$ that divides the graph into two parts so that each part is a reflection of the other in the line $x = a$. A graph has point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b). For each function in question 6, decide which type of symmetry it has. Explain.

☆8. Consider the functions $f(x) = -x$, $f(x) = -x^2$, $f(x) = -x^3$, and $f(x) = -x^4$. How are these functions different from those in question 6? What parts of the chart in question 6 would be different?

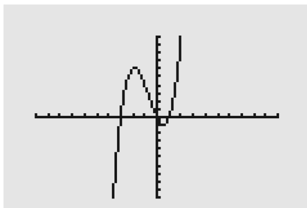
9. Each of the polynomial functions is represented by one of the graphs. Determine the appropriate graph for each polynomial function.

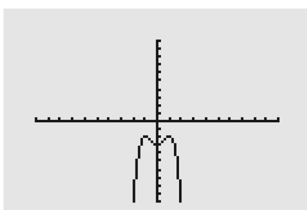
   a) $y = -x^4 + 2x^2 - 3$

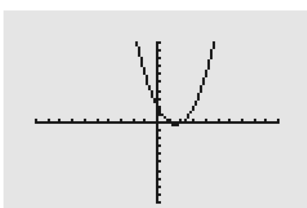
   b) $y = -x^3 - 3x^2 + 2x - 1$

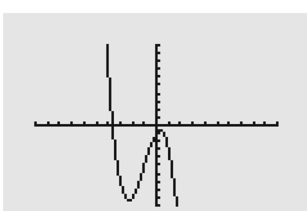
   c) $y = x^3 + 2x^2 - 3x$

   d) $y = x^2 - 3x + 2$

   i) 

   ii) 

   iii) 

   iv) 

10. **a)** Create a table of values for $y = 2x + 1$. Calculate the first differences.

   b) Create a table of values for $y = x^2$. Calculate the first and second differences.

   c) Create a table of values for $y = x^3$. Calculate the first, second, and third differences.

   d) Create a table of values for $y = -2x^3$. Calculate the first, second, and third differences.

   e) What is true about the first differences of a linear function? second differences of a quadratic function? third differences of a cubic function?

   f) How is the sign of the leading coefficient related to the sign of the constant value of the finite differences?

   g) How is the value of the leading coefficient related to the constant value of the finite differences?

☆**11.** Consider the table of values shown.

x	y
−3	94
−2	37
−1	10
0	1
1	−2
2	−11
3	−38

Use finite differences to determine

a) the degree of the polynomial function

b) the sign of the leading coefficient

c) the value of the leading coefficient

12. Consider the table of values shown.

x	y
−3	184
−2	54
−1	12
0	4
1	0
2	−6
3	4

Use finite differences to determine

a) the degree of the polynomial function

b) the sign of the leading coefficient

c) the value of the leading coefficient

13. Consider the polynomial function $f(x) = x^3 + 2x^2 - 4x + 5$.

a) State the degree of the function.

b) State the sign of the leading coefficient of the function.

c) What can you determine about the end behaviour of the function from the degree of the function and the sign of the leading coefficient of the function?

d) What do you know about the values of the third differences for this function?

e) How do you know that there are no maximum or minimum values on the graph of this function?

14. Consider the polynomial function $f(x) = -x^4 + x^2 - 1$.

a) State the degree of the function.

b) State the sign of the leading coefficient of the function.

c) What can you determine about the end behaviour of the function from the degree of the function and the sign of the leading coefficient of the function?

d) What do you know about the values of the fourth differences for this function?

e) Does the graph of this function have a maximum or a minimum? How do you know without graphing?

15. Navin has opened a new store to sell digital cameras. He determines that the monthly profit, P, in thousands of dollars, for the sale of digital cameras can be modelled by the function $P(x) = -x^2 + 9x - 8$, where x represents the number, in hundreds, of cameras sold.

a) What type of function is $P(x)$?

b) Determine which of the finite differences for this function will be constant.

c) Determine the value of the constant finite differences.

d) State the restrictions on the domain in this situation.

e) What do the x-intercepts of the graph represent for this situation?

f) If 200 cameras are sold, what will be the profit?

16. A glass designer wants to construct a cylindrical vase so that the ratio of the radius to the height is 1 : 7.

a) Write a polynomial function to represent the surface area, S, of the vase (not including the top) in terms of the radius, r. Describe the domain, range, and end behaviour of the function.

b) Write a polynomial function to represent the volume of the vase, in terms of the radius, r. Describe the domain, range, and end behaviour of the function.

C

17. Consider the function $f(x) = x^3 + 4x^2 - 5x + 12$. Explain why the x^3-term defines the end behaviour of $f(x)$ much more than the other three terms do.

18. Explain why odd-degree polynomial functions must have at least one x-intercept.

19. a) Explain why even-degree polynomial functions must have either a maximum or a minimum.

b) Explain the situations in which a function has no x-intercepts.