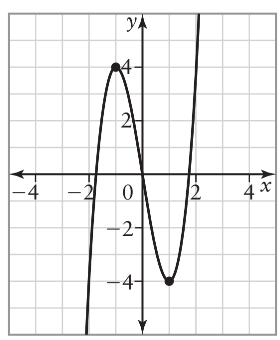
KEY CONCEPTS

- A polynomial function can be represented in four different ways: algebraically, numerically, graphically, and verbally.
- An odd-degree function has at least one x-intercept, up to a maximum of n x-intercepts, where n is the degree of the function.
- An even-degree function has from zero to a maximum of n x-intercepts, where n is the degree of the function.
- The x-intercepts of the graph of a polynomial function are the zeros of the function.
- The graph of a polynomial function changes sign only at x-intercepts that correspond to zeros of odd order. At x-intercepts that correspond to zeros of even order, the graph touches but does not cross the x-axis.
- A polynomial function of degree n, where n is a whole number greater than 1, may have at most $n - 1$ local minimum and local maximum points. For example, on the graph shown, the point $(-1, 4)$ is a local maximum, and the point $(1, -4)$ is a local minimum. Notice that the point $(-1, 4)$ is not a maximum point of the function, since other points on the graph of the function are greater.



- An even function satisfies the property that $f(-x) = f(x)$ for all values of x in its domain. Also, it is symmetric about the y-axis, which means it has line symmetry about the line $x = 0$.
- An odd function satisfies the property $f(-x) = -f(x)$ for all values of x in its domain. Also, it is rotationally symmetric about the origin, which means it has point symmetry about the point $(0, 0)$.

Example

Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Verify your response.

a) $f(x) = x^4 - x^2 + 2$

b) $f(x) = x^3 + 3x$

c) $f(x) = x^3 + 2x^2 - 3$

Solution

a) Since the exponent of each term is even, $f(x) = x^4 - x^2 + 2$ is an even function and has line symmetry about the y-axis.

Verify that $f(-x) = f(x)$.

$$\begin{aligned} f(-x) &= (-x)^4 - (-x)^2 + 2 \\ &= x^4 - x^2 + 2 \\ &= f(x) \end{aligned}$$

b) Since the exponent of each term is odd, $f(x) = x^3 + 3x$ is an odd function and has point symmetry about the origin.

Verify that $f(-x) = -f(x)$.

$$\begin{aligned} f(-x) &= (-x)^3 + 3(-x) \\ &= -x^3 - 3x \\ &= -(x^3 + 3x) \\ &= -f(x) \end{aligned}$$

c) Since $x^3 + 2x^2 - 3$ is a cubic function, it may be odd and have point symmetry about the origin.

$$\begin{aligned} f(-x) &= (-x)^3 + 2(-x)^2 - 3 \\ &= -x^3 + 2x^2 - 3 \\ -f(x) &= -(x^3 + 2x^2 - 3) \\ &= -x^3 - 2x^2 + 3 \end{aligned}$$

Since the two expressions are not equal, the function is not odd and does not have point symmetry about the origin.

A

1. **Use Technology**

   i) Use a graphing calculator to graph each function.

   ii) Identify the number of x-intercepts, the number of maximum and minimum points, and the number of local maximum and local minimum points.

   iii) Determine the x-intercepts. Explain how the factors of the function equation are related to the x-intercepts.

   a) $f(x) = (x + 2)(x + 1)(x - 3)$

   b) $f(x) = -(x + 3)(x + 2)(x - 1)^2$

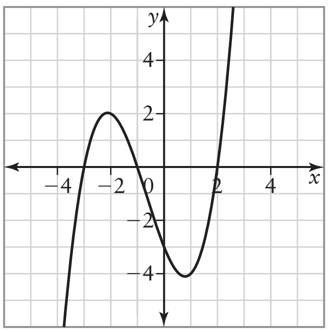
   c) $f(x) = (x + 4)^2(x - 1)(x - 3)$

   d) $f(x) = -(x + 1)^2(x - 1)^2$

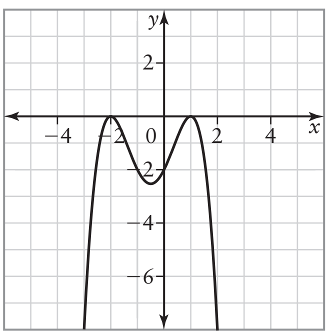
2. For each polynomial function in question 1, state the degree and the sign of the leading coefficient.

3. Each graph represents a polynomial function. State the x-intercepts and indicate whether they are of even or odd order.

a)



b)



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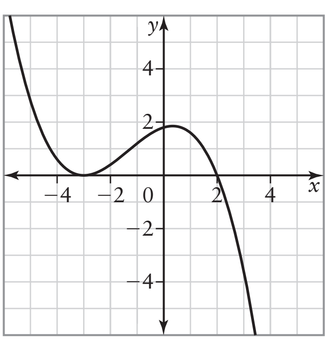

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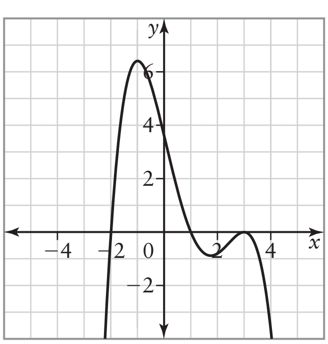

4. If a polynomial function has a factor $(x - a)$ that is repeated n times, then $x = a$ is a zero of order n. Determine the zeros of each polynomial function. Indicate whether they are of order 1, 2, or 3.

a) $f(x) = (x + 3)^2(x - 1)^2(x - 3)$

b) $f(x) = (x + 2)^3(x - 4)^2$

c) $f(x) = (x + 1)^2(x - 2)(x - 3)^2$

d) $f(x) = (x + 4)(x - 1)^3$

5. Use the results from question 4 to describe how you can tell whether a graph crosses the x-axis or just touches the x-axis.

6. Determine algebraically whether each of the following functions is even, odd, or neither. Verify your results using graphing technology.

a) $f(x) = x^4 + x^2 + 4$

b) $f(x) = x^3 - 5x$

c) $f(x) = x^3 + 3x^2 - 1$

d) $f(x) = x^2 + 5x + 4$

B

☆7. Consider the polynomial functions $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, and $f(x) = x^4$.

a) Use Technology Use a graphing calculator to graph the polynomial functions.

b) State the degree and end behaviour of each polynomial function.

c) What do you notice about the end behaviour of the odd-degree polynomial functions?

d) What do you notice about the end behaviour of the even-degree polynomial functions?

e) Determine the degree and predict the end behaviour of the polynomial functions $f(x) = x^5$ and $f(x) = x^6$.

f) Use Technology Verify your predictions using a graphing calculator.

8. **Use Technology** Graph each function using a graphing calculator. Determine its x-intercepts, maximum and minimum points, and the local maximum and local minimum points.

a) $f(x) = x^3 - x$

b) $f(x) = x^3 + x^2 - 2x$

c) $f(x) = x^3 + 5x^2 + 3x - 9$

d) $f(x) = -x^4 + 8x^2 - 16$

9. **Use Technology**

a) Use sliders in data software or parameters on a graphing calculator to experiment with graphs of the following functions. For each type of function (linear, quadratic, cubic), find and sketch as many significantly different shapes as possible. Compare your results to those of a classmate.

i) $y = ax + b$

ii) $y = ax^2 + bx + c$

iii) $y = ax^3 + bx^2 + cx + d$

b) Repeat the above for polynomials of degree 4, and then 5. Does the graph of a polynomial of degree 4 look similar to any of the graphs in part a)? How about the polynomial of degree 5? Describe what you notice.

10. **a)** **Use Technology** Graph the linear functions $f(x) = 2x - 1$, $g(x) = 2x$, and $h(x) = 2x + 1$ using a graphing calculator.

b) State the number of x-intercepts of each of the linear functions.

c) What is the maximum number of x-intercepts of a linear function?

11. **a)** **Use Technology** Graph the quadratic functions $f(x) = 2x^2 - 1$, $g(x) = 2x^2$, and $h(x) = 2x^2 + 1$ using a graphing calculator.

b) State the number of x-intercepts of each of the quadratic functions.

c) What is the maximum number of x-intercepts of a quadratic function?

12. **a)** **Use Technology** Graph the cubic functions $f(x) = x^3 - 2x^2 - 1$, $g(x) = x^3 - 2x^2$, and $h(x) = x^3 - 2x^2 + 1$ using a graphing calculator.

b) State the number of x-intercepts of each of the cubic functions.

c) What is the maximum number of x-intercepts that a cubic function can have?

13. **a)** **Use Technology** Graph the quartic functions $f(x) = x^4 - 2x^2 + 2$, $g(x) = x^4 - 2x^2 - 1$, $h(x) = x^4 - 2x^2$, and $k(x) = x^4 - 2x^2 + 0.5$ using a graphing calculator.

b) State the number of x-intercepts of each of the quartic functions.

c) What is the maximum number of x-intercepts that a quartic function can have?

14. **a)** **Use Technology** Use a graphing calculator to determine the x-intercepts of the polynomial function $f(x) = x^4 - 13x^2 + 36$.

b) Is the function $f(x) = x^4 - 13x^2 + 36$ even, odd, or neither? Justify your answer.

15. **Use Technology** Use a graphing calculator to determine the x-intercept of the cubic function $f(x) = x^3 - x^2 + 2x - 3$, to two decimal places.

C

16. Explain why an odd function cannot have a constant term.

17. You may have learned that even functions have variables whose exponents are even numbers. Explain why it makes sense that an even function can have a constant term.