KEY CONCEPTS

- A polynomial function can be evaluated graphically, with or without technology, by using the graph of the function to determine the y-value for the given x-value.
- A polynomial function can be evaluated numerically, with or without technology, by using a table of values to determine the y-value of the function for the given x-value.
- Polynomial functions can be evaluated by substituting the given x-value into the polynomial expression and then performing the required operations to determine the y-value.
- For a given value $x = a$, the value $f(a)$ is the y-value for the function when $x = a$, and can be represented by the ordered pair $(a, f(a))$ on the graph of the given function.
- The value $f(0)$ for a polynomial function represents the value of the y-intercept of a polynomial function, $f(x)$.

Example

Consider the polynomial function $f(x) = x^3 + 2x^2 - 3x + 1$.

a) Use a graphing calculator for each of the following:

i) Graph the function.

ii) Use the **Value** operation to determine the value of y when $x = 2$.

iii) Use the **TABLE** feature to determine the value of y when $x = 2$.

iv) Evaluate the polynomial expression for $x = 2$.

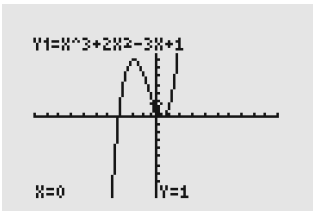
b) Evaluate the polynomial expression by substituting $x = 2$ into $f(x)$.

c) What do you notice about the results for parts ii), iii), iv), and b)? How do they relate to the graph of $f(x)$?

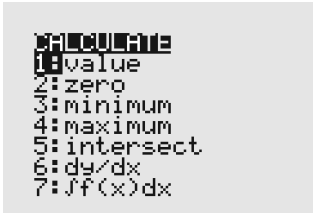
Solution

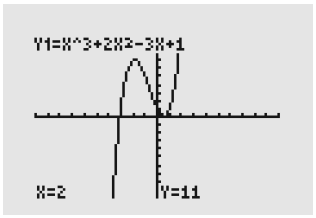
a) Use the standard window settings.

i)



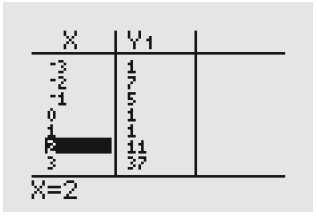
ii) From the **CALCULATE** menu, select **1:value**. Type 2 and press **ENTER**.





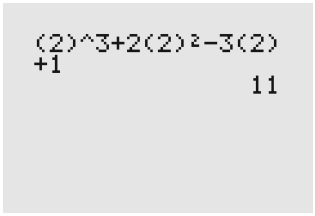
When $x = 2$, $y = 11$.

iii) Enter the desired starting x-value and x-increment for the table using the **TBLSET** feature.



When $x = 2$, $y = 11$.

iv) Enter the calculation.



When $x = 2$, $y = 11$.

b) $f(x) = x^3 + 2x^2 - 3x + 1$

$f(2) = (2)^3 + 2(2)^2 - 3(2) + 1$

$f(2) = 8 + 8 - 6 + 1$

$f(2) = 11$

c) The results are the same. When $x = 2$, the value of y is equal to 11. Also, $f(2) = 11$ represents the ordered pair (2, 11) on the graph of the polynomial function $f(x) = x^3 + 2x^2 - 3x + 1$.

A

1. Consider the polynomial function $f(x) = 2x^3 + 7x^2 + 3x - 4$. Evaluate the polynomial function for each of the following values.

a) $x = 4$ **b)** $x = -3$

c) $x = 1$ **d)** $x = 0$

e) $x = -2$ **f)** $x = -1$

2. Consider the polynomial function $f(x) = 3x^4 + 2x^3 - x^2 + 2x - 3$. Evaluate the polynomial function for each of the following values.

a) $x = 3$ **b)** $x = -5$

c) $x = 2$ **d)** $x = -4$

e) $x = -2$ **f)** $x = 0$

3. **Use Technology** Consider the polynomial function $f(x) = \frac{1}{2}x^3 - \frac{1}{3}x^2$. Use a graphing calculator and the **Value** operation to evaluate the polynomial function for each of the following values. Round your answers to two decimal places, if necessary.

a) $x = 2$

b) $x = -1$

c) $x = 3$

d) $x = -5$

e) $x = 0$

f) $x = 1$

4. **Use Technology** Consider the polynomial function $f(x) = -\frac{1}{4}x^3 + \frac{2}{3}x^2 - \frac{1}{2}x$. Use a graphing calculator and the **TABLE** feature to evaluate the polynomial function for each of the following values. Round your answers to two decimal places, if necessary.

a) $x = 4$

b) $x = -3$

c) $x = 6$

d) $x = -6$

e) $x = 2$

f) $x = -2$

5. Evaluate each of the following functions for the indicated x-value.

a) $f(x) = 3x^2 + 2$, for $x = -1$

b) $f(x) = -2x^3 + 4x$, for $x = 3$

c) $f(x) = x^4 - 3x^2$, for $x = 2$

d) $f(x) = 2x^5 - 5x^2$, for $x = -2$

B

6. **Use Technology** Evaluate each of the following polynomial functions for $x = 4$.

a) $f(x) = 4x^2 + 3x - 5$

b) $g(x) = 3x^4 - 7x^2 + 2$

c) $h(x) = -5x^3 + 4x^2 - 3x + 4$

d) $f(x) = 8$

e) $g(x) = -2x^4 + 3x^3 - 4x^2 + 5x - 1$

f) $h(x) = -\frac{1}{4}x^3 + x^2 - 2x + 3$

7. **a)** Evaluate each of the following polynomial functions for $x = 0$.

i) $f(x) = 5x^3 - 4x^2 + 3x$

ii) $f(x) = -2x^4 + 5x^2 - 5$

iii) $f(x) = x^3 + 4x^2 - 5x + 1$

iv) $f(x) = -5$

v) $f(x) = 3x^4 + 2x^3 - 5x^2 - 2x + 4$

b) How can you determine the y-intercept by inspection?

8. Consider the polynomial function $f(x) = x^4 - 2x^3 + 3x - 5$.

a) **Use Technology** Use a graphing calculator for each of the following:

i) Graph the function.

ii) Use the **Value** operation to determine the value of y when $x = -1$.

iii) Use the **TABLE** feature to determine the value of y when $x = -1$.

iv) Evaluate the polynomial expression for $x = -1$.

b) Evaluate the polynomial expression by substituting $x = -1$ into $f(x)$.

☆**9.** **Use Technology** During the first 20 h of an experiment, the growth rate of a bacterium population at time t hours is $r(t) = -0.02t^3 + 0.5t^2 + 0.5t + 2$.

a) Graph the function using a graphing calculator.

b) What is the growth rate at 4 h?

c) What is the growth rate at 15 h?

d) At what time is the growth rate 37 bacteria per hour?

☆**10. Use Technology** David is designing a rectangular storage box without a lid. He plans to construct the rectangular box from a 25-cm by 35-cm piece of cardboard by cutting equal squares of side length x from the corners and folding up the sides. The volume of the box can be modelled by the polynomial function $V(x) = x(25 - 2x)(35 - 2x)$.

a) Graph the function using a graphing calculator.

b) Use the graphing calculator to determine the volume of the rectangular box if the side length of each square is 10 cm.

c) Determine the volume of the rectangular box if the side length of each square is 10 cm by substituting $x = 10$ into $V(x)$.

d) Use the graph of the polynomial function $V(x)$ to determine the size of square that should be cut from the corners if the required volume of the box is 1875 cm^3.

11. a) Determine a function that models the volume of a cylinder whose height is double its radius.

b) Determine the volume when the radius is 5 cm.

c) Use Technology Use a graph of your function from part a) to determine the radius when the volume is 200 cm^3.

12. The length of a rectangular dog run is 3 m more than twice the width.

a) Show this information on a diagram.

b) Determine a function that expresses the area of the dog run in terms of its width.

c) Determine the area when the width is 4 m.

d) What width would produce an area of 50 m^2?

13. Consider the three functions $f(x) = 3x - 4$, $g(x) = x^2 - 2$, and $h(x) = x^3 - 3x$.

a) Determine the value of y for each of the three functions when $x = 2$.

b) What do you notice about the value of y when $x = 2$ for each of the three functions? What conclusion can you make about the graphs of the three functions?

c) Use Technology Use a graphing calculator to check your answer to part b) by graphing.

d) Use Technology Look at the row where $x = 2$ in the **TABLE** feature on a graphing calculator. How does this also verify your answer to part b)?

C

14. Evaluate each of the following functions. Provide exact answers. Do not round.

a) $f(x) = \sqrt{3x - 4}$, for $x = 3$

b) $f(x) = 4^x$, for $x = -2$

c) $f(x) = \left(\frac{2}{3}\right)^x$, for $x = -3$

d) $f(x) = \frac{1}{x^2 - 3}$, for $x = 5$

15. Consider the polynomial function $f(x) = x^2 + 2x$.

a) Determine an expression for $f(a)$.

b) Determine an expression in simplified form for $f(x + 2)$.

c) Determine an expression in simplified form for $f(x - 3)$.

d) Determine an expression in simplified form for $f(x^2 + 2x)$.