5.5 Solving Problems Involving Polynomial Functions

KEY CONCEPTS

- Real-world applications can be modelled with polynomial functions.
- Key features of polynomial functions can be used to determine solutions to real-world problems.
- Certain aspects of real-world applications may result in restrictions on the domain of the polynomial functions that represent them.

Example

The forces acting on a horizontal support beam in a garage cause it to sag by d centimetres, x metres from one end of the beam. The relationship between d and x can be represented by the polynomial function $d(x) = \frac{1}{1800}\left(950x - 22x^3 + x^4\right)$.

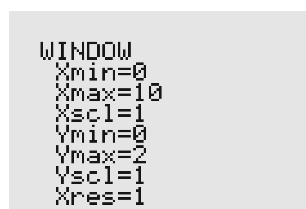
a) Graph the function using technology.

b) Determine the domain over which the function models the relationship between d and x. What do you think causes the restriction on the domain?

c) Determine the maximum deflection of the beam.

Solution

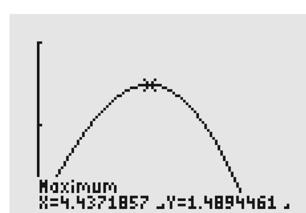
a) Graph the function using the window setting shown.



b) The **Zero** operation can be used to determine that the right-most x-intercept of the graph is approximately 8.3. The domain of $d(x)$ is $\{x \in \mathbb{R}, 0 \le x \le 8.3\}$. The deflection of the beam cannot be less than zero. The domain is determined by the length of the beam, which appears to be approximately 8.3 m.

c) Use the **Maximum** operation.

The maximum deflection of the beam is approximately 1.5 m.



A

1. A golf ball is hit from the ground and its height above the ground can be modelled by the function $h(t) = -4.9t^2 + 30t$, where $h(t)$ is the height, in feet, and t is time, in seconds, since the ball was hit.

a) Determine the height of the golf ball after 2 s.

b) Determine the height of the golf ball after 5 s.

c) Determine the ball's initial height.

d) **Use Technology** Use a graphing calculator to determine how long the ball is in the air.

2. The function $d(t) = 4.9t^2$ may be used to model the approximate distance, $d(t)$, in metres, travelled by a skydiver after jumping out of an airplane, where t is the time, in seconds.

a) Determine the distance travelled by the skydiver in the first 3 s after jumping out of the airplane.

b) Determine the distance travelled by the skydiver in the first 7 s after jumping out of the airplane.

c) How long does it take the skydiver to travel 510 m?

d) Part c) generates a quadratic equation that has two solutions. Why is only one of them valid?

3. The yearly profit, $P(n)$, in thousands of dollars, for the sale of n thousand tennis balls can be modelled by the function $P(n) = -0.08n^3 + 1.86n^2 + 8n$.

a) Determine the yearly profit on the sale of 13 000 tennis balls.

b) Determine the yearly profit on the sale of 20 000 tennis balls.

c) For what number of tennis balls is the profit a maximum?

d) What is the domain of the function? Explain your reasoning.

4. An oil tank is being drained. The volume, $V(t)$, in litres, of oil remaining in the tank after t minutes can be modelled by the function $V(t) = 0.28(28 - t)^3$.

a) How much oil was in the tank initially? after 10 min? after 18 min?

b) After how many minutes was the tank drained?

c) How can you determine the domain of the function by simply examining the function?

B

5. The profit, $P(x)$, in thousands of dollars, for the sale of a certain type of digital camera can be modelled by the function $P(x) = 0.001\,31x^4 + x - 2$, where x represents the number, in hundreds, of digital cameras sold.

a) What type of function is $P(x)$?

b) Which finite differences are constant for this polynomial function?

c) Describe the end behaviour of this function assuming there are no restrictions on the domain.

d) What are the restrictions on the domain of this function? Explain why there are restrictions.

☆ **6.** A patient's reaction time, $R(t)$, in minutes, to a small dose of a certain drug is $R(t) = -0.6d^3 + d^2$, where d is the amount of the drug, in millilitres, that is absorbed into the patient's bloodstream.

a) What type of function is $R(t)$?

b) Which finite differences are constant for this polynomial function?

c) Describe the end behaviour of this function assuming there are no restrictions on the domain.

d) What are the restrictions on the domain of this function?

7. The population, P, of a town can be modelled by the function $P(t) = 5t^4 - 4t^3 + 100t + 13\ 000$, where t is the time, in years, from the present time.

   a) What is the population of the town now?

   b) What will the population of the town be in 15 years?

☆8. Shirley is draining the water out of her swimming pool. The amount of water remaining in the pool as it is being drained is given by the following table.

Time (h)	Amount of Water Remaining (L)
0	17 750
1	16 280
2	14 870
3	13 520
4	12 230
5	11 000

   Use finite differences for the data in the table to determine the type of polynomial function that best models this situation.

9. A soccer ball is kicked into the air and the path that it follows can be modelled by the function $h(t) = -4.9t^2 + 12t + 1$, where t is in seconds and $h(t)$ is in metres.

   a) Describe the end behaviour of this function assuming there are no restrictions on the domain.

   b) Determine the height of the ball after 1 s.

10. The purchase price, $P(t)$, in dollars, to buy one share of stock in a company can be modelled by the function $P(t) = -0.3t^3 + 2t^2 + 5t + 1$, where t is the time, in years, from now.

   a) **Use Technology** Use a graphing calculator to graph the function $P(t)$.

   b) Determine the price to purchase one share of stock in the company in 3 years.

11. The height, $h(t)$, in metres, of a toy rocket above the ground can be modelled by the function $h(t) = -4.9t^2 + 25t$, where t is in seconds.

   a) **Use Technology** Use a graphing calculator to graph the function $h(t)$.

   b) Determine the height of the rocket after 2 s.

   c) Determine the height of the rocket after 4 s.

   d) When will the rocket hit the ground? Round your answer to one decimal place, if necessary.

12. The distance, $d(t)$, in metres, travelled by a windsurfer from shore can be modelled by the function $d(t) = 0.002t^3 + 0.04t^2 + 0.3t$, where t is the time, in seconds.

   a) **Use Technology** Use a graphing calculator to graph the function $d(t)$.

   b) Determine the distance travelled by the windsurfer in 10 s.

   c) When will the windsurfer have travelled 38 m?

C

13. A ball's height, h, in metres, t seconds after being thrown off a cliff is given by the function $h(t) = -5t^2 + 15t + 50$.

   a) Determine the ball's initial height.

   b) **Use Technology** Use graphing technology to determine the maximum height of the ball and when it occurs.

   c) Determine the average speed of the ball during the first second after it was thrown.

   d) **Use Technology** Use graphing technology to determine when the ball strikes the ground.

   e) What was the ball's height one second before hitting the ground?