5.7 Difference of Squares of Polynomial Expressions

KEY CONCEPTS

- To factor a polynomial expression of the form $a^2 - b^2$ using the method of factoring a difference of squares, factor the expression as follows: $a^2 - b^2 = (a + b)(a - b)$.
- When you multiply factors of the form $(a + b)(a - b)$, the middle terms are opposites and therefore cancel each other.

Example

Factor each expression completely.

   a) $x^2 - 36$

   b) $49m^2 - 25n^2$

   c) $a^3 - 16a$

   d) $y^4 - 1$

   e) $(w - x)^2 - (y + z)^2$

Solution

   a) Use the pattern for a difference of squares.

$$x^2 - 36 = (x)^2 - 6^2$$
$$= (x + 6)(x - 6)$$

   b) Use the pattern for a difference of squares.

$$49m^2 - 25n^2 = (7m)^2 - (5n)^2$$
$$= (7m + 5n)(7m - 5n)$$

   c) First, factor out the common factor. Then, use the pattern for a difference of squares.

$$a^3 - 16a = a(a^2 - 16)$$
$$= a(a + 4)(a - 4)$$

   d) Use the pattern for a difference of squares twice.

$$y^4 - 1 = (y^2 + 1)(y^2 - 1)$$
$$= (y^2 + 1)(y + 1)(y - 1)$$

   e) $(w - x)^2 - (y + z)^2 = [(w - x) + (y + z)][(w - x) - (y + z)]$
$$= (w - x + y + z)(w - x - y - z)$$

A

1. Factor completely.

   a) $x^2 - 25$ b) $y^2 - 16$

   c) $a^2 - 9$ d) $m^2 - 49$

   e) $b^2 - 1$ f) $d^2 - 36$

2. Factor completely.

   a) $1 - e^2$ b) $25 - h^2$

   c) $81 - q^2$ d) $36 - n^2$

   e) $16 - s^2$ f) $49 - c^2$

3. Factor completely.

   a) $x^2 - y^2$ b) $a^2 - b^2$

   c) $m^2 - n^2$ d) $p^2 - q^2$

   e) $s^2 - t^2$ f) $d^2 - e^2$

4. Factor completely.

   a) $a^2 - 4c^2$ b) $h^2 - 144y^2$

   c) $d^2 - 9m^2$ d) $f^2 - 100e^2$

   e) $b^2 - 169w^2$ f) $z^2 - 49u^2$

5. Factor completely.

   a) $49r^2 - f^2$ b) $64y^2 - x^2$

   c) $256g^2 - k^2$ d) $25m^2 - c^2$

   e) $196j^2 - l^2$ f) $100m^2 - d^2$

B

6. Factor completely.

   a) $169x^2 - 9y^2$ b) $144m^2 - 49n^2$

   c) $289a^2 - 81b^2$ d) $16w^2 - 361v^2$

   e) $256k^2 - 225m^2$ f) $36h^2 - 25q^2$

7. a) Factor each expression completely.

   i) $m^4 - 1$

   ii) $m^4 - 16$

   iii) $m^4 - 81$

   iv) $m^4 - 256$

   b) Extend the pattern in part a) for two more steps.

⭐8. Factor.

   a) $18x^2 - 32y^2$

   b) $q^3 - 25q$

   c) $16m - 64m^3$

   d) $9m^2n^2 - 81m^4$

   e) $125a^4b^4 - 5a^4b^6$

   f) $36p^2q^4 - 4p^2$

⭐9. a) Assume $(82)(78)$ is equivalent to $(a + b)(a - b)$. Identify a and b, and evaluate $a^2 - b^2$.

   b) Use the pattern from part a) to evaluate each expression without using a calculator.

   i) $(54)(46)$

   ii) $(102)(98)$

   iii) $(76)(64)$

⭐10. Factor completely.

   a) $\frac{1}{4}a^4 - \frac{1}{16}b^2$

   b) $\frac{1}{8}m^2 - \frac{1}{2}n^2$

11. a) Two square sheets of cardboard have areas of $25x^2$ and $16y^2$. Express the difference in the areas of the two square sheets of cardboard in a factored form.

    b) Two square decks have been constructed that have areas of $9a^2$ and $4b^2$. Express the difference in the areas of the two square decks in a factored form.

C

12. Factor completely.

    a) $(a^2 + 2ab + b^2) - (x^2 + 2xy + y^2)$

    b) $m^2 + 4mn + 4n^2 - a^2 + 6ab - 9b^2$

13. a) Factor completely $x^2 - 4$ and $x^2 - 9$.

    b) Use your answer from part a) to factor each of the following expressions.

    i) $x^2 - 5$

    ii) $x^2 - 6$

    iii) $x^2 - 7$

    iv) $x^2 - 8$