KEY CONCEPTS

- The x-intercepts of a polynomial function are sometimes called the zeros of the function or the roots of the corresponding polynomial equation.
- The x-intercepts of a function are the x-values of the ordered pairs $(x, 0)$ where the graph of the function intersects the x-axis.
- If the graph of a polynomial function intersects the x-axis, the corresponding polynomial equation will have real roots.
- To find the x-intercepts of a function graphically, graph the function and determine the x-values of the points where the graph intersects the x-axis.
- Since the x-intercepts occur when $y = 0$, they can be found algebraically by setting $y = 0$, or $f(x) = 0$, and solving for x.
- The graph of a polynomial function changes sign only at x-intercepts that correspond to zeros of odd order. At x-intercepts that correspond to zeros of even order, the graph touches but does not cross the x-axis.

Example

Consider the polynomial function $f(x) = x(x + 2)(x - 2)$.

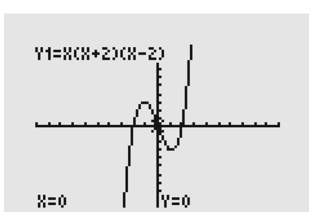
a) Use a graphing calculator to graph the polynomial function.

b) Use the graph of the function to determine the x-intercepts, or zeros, of the function. State whether each zero is of odd or even order.

c) Determine the zeros of the function $f(x) = x(x + 2)(x - 2)$ algebraically.

Solution

a) Use the standard window settings.



b) Use the **Zero** operation on a graphing calculator. The x-intercepts of the polynomial function are −2, 0, and 2. Notice that the graph of the polynomial function crosses the x-axis at all three x-intercepts, causing a change in sign of the function. All three zeros of the function are of odd order.

c) The zeros of the polynomial function will occur at the points where $f(x) = 0$.

Substitute 0 for $f(x)$ in $f(x) = x(x + 2)(x - 2)$, and then solve for x.

$$f(x) = x(x + 2)(x - 2)$$
$$0 = x(x + 2)(x - 2)$$
$$x = 0 \text{ or } x + 2 = 0 \text{ or } x - 2 = 0$$
$$x = -2 \qquad x = 2$$

Therefore, the zeros of the polynomial function are −2, 0, and 2.

A

1. Express each polynomial function in a factored form. Then, use the factored form to determine the x-intercepts of the function.

a) $f(x) = x^2 + 2x - 8$

b) $f(x) = x^2 - 9$

c) $f(x) = x^2 - 3x - 10$

d) $f(x) = 4x^2 - 25$

2. Express each polynomial function in a factored form. Then, use the factored form to determine the x-intercepts of the function.

a) $f(x) = 12x^2 + 5x - 2$

b) $f(x) = 8x^2 - 22x + 15$

c) $f(x) = x^3 + 5x^2 - 14x$

d) $f(x) = x^3 - x$

3. Graph each polynomial function. Explain how the number of x-intercepts is related to the function.

a) $f(x) = x^2 + 4x + 4$

b) $f(x) = x^2 - 6x + 9$

c) $f(x) = 4x^2 - 12x + 9$

d) $f(x) = 9x^2 + 24x + 16$

4. a) Determine the x-intercepts for the functions using an algebraic method.

i) $f(x) = 2x^2 - 14x + 24$

ii) $f(x) = -2x^2 - 14x + 36$

b) How does a numerical common factor affect the x-intercepts?

5. i) Graph each polynomial function.

ii) Use the graph of the function to determine the x-intercepts, or zeros, of the function. State whether each zero is of odd or even order.

a) $f(x) = 2x^3 - 8x$

b) $f(x) = 3x^3 - 9x^2$

B

☆ **6.** Consider the polynomial function $f(x) = x^3 - 2x^2 - 5x + 6$.

a) Use Technology Use a graphing calculator to graph the function and determine the x-intercepts.

b) Use the x-intercepts from part a) to express the function in factored form. Use the factored form to determine the x-intercept(s) algebraically.

c) Determine the y-intercept(s).

7. Explain how graphing a function can be used to determine its factors. Is it possible for a graph to have an x-intercept where it will not help to factor the function? Explain your reasoning.

8. Consider the function $f(x) = (x - 1)(x + 2)(x + 4)$.

a) Determine the zeros of the function. Explain how these are related to the test intervals in the table.

b) Copy and complete the table. Indicate whether the function is positive (above the x-axis) or negative (below the x-axis) for the corresponding interval.

Test Interval	Sign of $f(x)$
$x < -4$	
$x = -4$	
$-4 < x < -2$	
$x = -2$	
$-2 < x < 1$	
$x = 1$	
$x > 1$	

c) Use your completed table to sketch a graph of $f(x) = (x - 1)(x + 2)(x + 4)$.

9. Consider the function $f(x) = -(x - 2)(x + 1)^2(x + 3)$.

a) Determine the zeros of the function.

b) Create a table similar to the one in question 8. Use the zeros from part a) to write the intervals, and then indicate whether the function is positive or negative for the corresponding interval.

c) Use your completed table to sketch a graph of $f(x) = -(x - 2)(x + 1)^2(x + 3)$.

☆ 10. Consider the function $f(x) = x^3 + 2x^2 - 3x$.

a) State the degree of the polynomial function and the value and the sign of the leading coefficient.

b) Determine the end behaviour of the function.

c) Determine the intercepts of the function algebraically.

d) Sketch a graph of the function.

11. a) Graph the function $y = x^3 - 4x^2$.

b) Determine the x-intercept(s) of the graph of the function. State whether each zero is of odd or even order.

c) State the value of the real root(s) of the corresponding polynomial equation.

d) What conclusion can you make about the relationship between the x-intercepts of the graph of a function and the real root(s) of the corresponding polynomial equation?

12. a) Graph the function $y = -x^3 + 16x$.

b) Is this an even-degree or odd-degree polynomial?

c) Determine the x-intercept(s) of the function.

d) State the value of the real root(s) of the corresponding polynomial equation.

e) What conclusion can you make about the relationship between the x-intercepts of the graph of a function and the real root(s) of the corresponding polynomial equation?

13. Sketch a graph of each polynomial function.

a) $f(x) = (x - 2)(x + 3)^2$

b) $y = -2(x - 1)(x + 2)(x + 3)$

C

14. a) Determine the real roots of the polynomial equation $0 = x^4 - 34x^2 + 225$.

b) How are the real roots of the polynomial equation related to the x-intercepts of the graph of the corresponding polynomial function?

15. Determine the equation for the polynomial function with zeros of -2 (order 1), -1 (order 1), and -3 (order 1) that passes through the point $(1, -24)$.

Chapter 5: Checklist

By the end of this chapter, I will be able to:

- recognize and evaluate polynomial functions, describe key features of their graphs, and solve problems using graphs of polynomial functions
- make connections between the numeric, graphical, and algebraic representations of polynomial functions
- solve polynomial equations by factoring, make connections between functions and formulas, and solve problems involving polynomial expressions arising from a variety of applications
- recognize a polynomial expression as a series of terms where each term is the product of a constant and a power of x with a non-negative integral exponent
- recognize the equation of a polynomial function and give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions
- distinguish polynomial functions from sinusoidal and exponential functions, and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions
- describe key features of the graphs of polynomial functions (e.g., the domain and range, the shape of the graphs, the end behaviour of the functions for very large positive or negative x-values)
- compare, through investigation using graphing technology, the graphical and algebraic representations of polynomial (i.e., linear, quadratic, cubic, quartic) functions
- substitute into and evaluate polynomial functions expressed in function notation, including functions arising from real-world applications
- pose problems based on real-world applications that can be modelled with polynomial functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation
- recognize, using graphs, the limitations of modelling a real-world relationship using a polynomial function, and identify and explain any restrictions on the domain and range
- factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring)
- make connections, through investigation using graphing technology, between a polynomial function given in factored form and the x-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features
- determine, through investigation using technology, and describe the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function