KEY CONCEPTS

- If two factors have a product of zero, then one or both of the factors must equal zero. If $a \times b = 0$, then $a = 0$ or $b = 0$.
- One method of solving a polynomial equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is to factor the expression and then set each factor equal to zero. The solutions are the roots of the equation.
- The real solutions, or real roots, of a polynomial equation are the x-intercepts, or zeros, of the corresponding polynomial function. For example, the solution to $x^2 + 3x + 2 = 0$ is the same as the x-intercepts of the graph, or zeros, of the function $y = x^2 + 3x + 2$.

Example

Consider the polynomial equation $x^3 + 2x^2 - 3x = 0$.

a) Describe a strategy that you can apply to solve the polynomial equation algebraically.

b) Use your strategy from part a) to solve the polynomial equation.

c) Write the corresponding polynomial function for the polynomial equation.

d) Use a graphing calculator to graph the polynomial function from part c) and determine the x-intercepts, or zeros, of the polynomial function.

e) Consider the table of values for the graphing calculator graph in part d). For what values of x are the y-values of the function equal to zero?

f) What conclusion can you make about the relationship between the roots of a polynomial equation and the x-intercepts of the graph, or zeros, of the corresponding polynomial function?

Solution

a) Factor completely the left side of the polynomial equation by applying common factoring, and then trinomial factoring. Then, solve the factored form of the polynomial equation to determine the solution, or roots, of the equation.

b) $x^3 + 2x^2 - 3x = 0$

$x(x^2 + 2x - 3) = 0$

$x(x + 3)(x - 1) = 0$

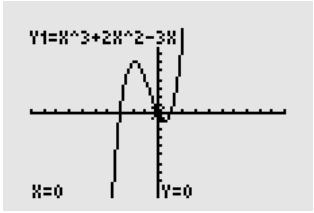
$x = 0$ or $x + 3 = 0$ or $x - 1 = 0$

$x = -3 \qquad x = 1$

The roots are -3, 0, and 1.

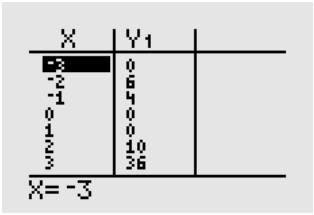
c) The corresponding polynomial function is $f(x) = x^3 + 2x^2 - 3$.

d) Graph the function. Use the **Zero** operation to determine the x-intercepts.



The x-intercepts are -3, 0, and 1.

e)



The y-value is 0 for x-values -3, 0, and 1.

f) The roots of a polynomial equation are the same as the x-intercepts of the graph, or zeros, of the corresponding polynomial function.

A

1. Solve each of the following polynomial equations.

a) $x^2 + x = 0$ **b)** $x^2 - 2x = 0$

c) $x^2 + 5x = 0$ **d)** $x^2 - 7x = 0$

e) $2x^2 + 5x = 0$ **f)** $3x^2 - 4x = 0$

2. a) Solve the polynomial equation $x^2 - 5x - 6 = 0$ by factoring.

b) Use Technology Use a graphing calculator to graph the polynomial function $f(x) = x^2 - 5x - 6$ and determine the x-intercepts of the polynomial function.

c) What do you notice about the roots of the polynomial equation $x^2 - 5x - 6 = 0$ and the x-intercepts of the graph of the corresponding polynomial function $f(x) = x^2 - 5x - 6$?

3. Solve each of the following polynomial equations by factoring.

a) $x^2 + x - 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 + 3x + 2 = 0$

d) $x^2 - 5x + 6 = 0$

e) $x^2 + 5x - 14 = 0$

f) $x^2 - 2x - 63 = 0$

4. Use Technology Use a graphing calculator to graph and solve each of the following polynomial equations.

a) $x^2 - 5x - 24 = 0$

b) $x^2 + 3x - 10 = 0$

c) $x^2 + 7x + 10 = 0$

d) $x^2 - 8x + 15 = 0$

e) $x^2 - 4x - 12 = 0$

f) $x^2 - 2x - 35 = 0$

5. a) Use Technology Use a graphing calculator to graph the polynomial functions $f(x) = 2x^2 - 2x - 12$ and $g(x) = x^2 - x - 6$ on the same set of axes. Determine the x-intercepts of the polynomial functions.

b) What do you notice about the x-intercepts for the two functions?

c) Solve, algebraically, the polynomial equation $2x^2 - 2x - 12 = 0$.

d) What effect does a common factor have on the solution?

B

☆**6.** Sandy says the solution to $x^2 = 4$ is $x = 2$ because you take the square root of both sides. What is wrong with this logic? What should be done in order to avoid this mistake?

7. Solve each of the following polynomial equations. Write your answers as fractions in lowest terms, if necessary.

a) $6x^2 - x - 15 = 0$

b) $6x^2 - 23x + 20 = 0$

c) $6x^2 - 19x + 15 = 0$

d) $5x^2 + 18x + 9 = 0$

8. Solve each polynomial equation. Write your answers as fractions in lowest terms, if necessary.

a) $8k^2 - 16k + 6 = 0$

b) $9p^2 + 15p - 6 = 0$

c) $10r^2 - 22r + 4 = 0$

d) $8y^2 - 22y + 12 = 0$

9. a) How is the polynomial equation $8x^2 = 22x + 21$ different from those in questions 7 and 8? What should be done in order to use the strategy of solving by factoring?

b) Solve the polynomial equation in part a).

c) Solve each polynomial equation.

i) $24x^2 + 4 = -22x$

ii) $3(x^2 + 1) + 4x = 2(x + 4)$

☆**10.** Consider the functions $f(x) = x^2 - 4x$, $g(x) = x^3 - 4x^2$, and $h(x) = x^4 - 4x^3$.

a) Express each function in factored form.

b) Use Technology Use a graphing calculator to graph each function.

c) How are the exponents on factors related to the function's behaviour at the corresponding x-intercept?

d) Predict what the graph of $y = 2x(x - 1)^2$ would look like. Consider the degree, factors, and end behaviour in your prediction.

☆**11. a)** Explain algebraically and graphically how you can solve the polynomial equation $2x^3 + 6x^2 - 20x = 0$.

b) Describe situations where one method might work better than the other. What do you assume about all equations that are to be solved algebraically?

12. Determine the real roots of each of the following polynomial functions. Verify your answers using a graphing calculator.

a) $x^4 - 1 = 0$

b) $x^3 + 2x^2 + 2x + 4 = 0$

c) $x^3 + 7x^2 + 12x = 0$

d) $x^4 - 81 = 0$

C

13. If the inner radius of a metal washer is 5 mm and the surface area of the washer is 39π mm^2, determine the outer radius of the washer.

14. The height, h, in metres, of a professional diver t seconds after diving off a cliff is given by $h(t) = -5t^2 + 5t + 10$.

a) Write a polynomial equation that can be used to determine when the diver hits the water.

b) Solve your equation from part a). How many answers are there? Are they all admissible? Explain.