

6.4 Functions and Formulas

KEY CONCEPTS

- Formulas in mathematics, sciences, and business may be represented mathematically by polynomial functions and other functions such as exponential functions.
- Formulas can be solved for an unknown by substituting known values directly into the formula and solving for the unknown, or by rewriting the formula in terms of the unknown and then substituting the known values into the formula.
- Formulas may contain variables and/or constants.
- When known values are substituted into a formula, polynomial or exponential functions in one variable may result.

Example

- a) Michael has \$10 000 to invest. If he invests the money at 4% per year compounded semi-annually, how much money will he have at the end of 5 years?
- b) Would it be a better choice for Michael to invest the \$10 000 at 3.5% per year compounded quarterly for 5 years? Explain.

Solution

The compound interest formula is $A = P(1 + i)^n$, where A represents the amount, in dollars, P represents the principal (or amount invested), in dollars, i represents the interest rate per compounding period, expressed as a decimal, and n represents the number of compounding periods for the term of the investment or loan.

a) $P = 10\,000$

The interest rate is 4% per annum compounded semi-annually.

$$i = \frac{0.04}{2}$$
$$= 0.02$$

The interest is compounded semi-annually, or twice a year.

$$n = 2 \times 5$$
$$= 10$$

Substitute into $A = P(1 + i)^n$.

$$A = 10\,000(1 + 0.02)^{10}$$
$$= 10\,000(1.02)^{10}$$
$$\doteq 12\,189.94$$

Therefore, Michael will have \$12 189.94 at the end of 5 years.

b) $P = 10\,000$

The interest rate is 3.5% per year compounded quarterly.

$$i = \frac{0.035}{4}$$
$$= 0.008\,75$$

The interest is compounded quarterly, or 4 times a year.

$$n = 4 \times 5$$
$$= 20$$

Substitute into $A = P(1 + i)^n$.

$$A = 10\,000(1 + 0.008\,75)^{20}$$
$$= 10\,000(1.008\,75)^{20}$$
$$\doteq 11\,903.40$$

Therefore, Michael would have \$11 903.40 at the end of 5 years.

It would be a better choice for Michael to invest the money at 4% per year compounded semi-annually for 5 years. That option will yield him \$286.54 more than the second investment option.

A

- The formula for the area of a circle is $A = \pi r^2$, where A represents the area of the circle and r represents the radius of the circle.
 - Identify the constants and the variables in this formula.
 - Calculate the area of a circle with radius 5 cm. Round your answer to one decimal place.
 - Calculate the radius of a circle with area 35 m^2 . Round your answer to one decimal place.
- The formula for simple interest is $I = Prt$, where I represents the interest, in dollars; P represents the principal, in dollars; r represents the annual interest, expressed as a decimal; and t represents the time, expressed in years.
 - Identify the constants and the variables in this formula.
 - Calculate the simple interest when \$5000 is invested at 3% per year for 1 year.
 - Calculate the simple interest when \$3000 is invested at 2.5% per year for 1 year and 6 months.
- The formula for the density of an object is $d = \frac{m}{v}$, where d represents the density, in kilograms per metre cubed; m represents the mass, in kilograms; and v represents the volume, in metres cubed.
 - Rewrite the formula to express m in terms of d and v .
 - Rewrite the formula to express v in terms of d and m .
 - Calculate the volume of an object with a density of 0.5 kg/m^3 and a mass of 10 kg.

4. The Pythagorean theorem states that the relationship between the three sides in a right triangle is $c^2 = a^2 + b^2$, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides.
- Rewrite the formula to determine the length of a in terms of b and c .
 - Use the formula to determine the length of side a if the length of side b is 4 cm and the length of the hypotenuse is 8 cm. Round your answer to one decimal place.
 - Rewrite the formula to determine the length of b in terms of a and c .
 - Use the formula to determine the length of side b if the length of side a is 5 cm and the length of the hypotenuse is 9 cm. Round your answer to one decimal place.
5. The formula $P = P_0 + kh$ is used to determine the pressure, P kilopascals (kPa), at a depth of h metres under water, where k kilopascals per metre is the rate of change of the pressure as the depth increases, and P_0 kilopascals is the pressure at the surface.
- Identify and describe the roles of P , P_0 , k , and h in this relationship, and explain your reasoning.
 - The pressure at sea level is approximately 101.3 kPa. The pressure at a depth of 100 m below sea level is approximately 1347.9 kPa. Determine the rate of change of pressure as the depth increases.
- B**
6. The formula $V = \frac{1}{3}\pi r^2 h$ is used to determine the volume, V , of a cone with radius r and height h .
- Determine the volume of a cone with radius 5 cm and height 7 cm, to one decimal place.
 - Determine the height of a cone with volume 35 cm³ and radius 4 cm, to one decimal place.
 - Determine the radius of a cone with volume 50 cm³ and height 3 cm, to one decimal place.
 - Which variable(s) in the formula $V = \frac{1}{3}\pi r^2 h$ would you need to set as a constant to generate a linear equation?
 - Which variable(s) in the formula $V = \frac{1}{3}\pi r^2 h$ would you need to set as a constant to generate a quadratic equation?
- ★7. The formula $s = ut + \frac{1}{2}at^2$ relates the distance, s , travelled by an object to its initial velocity, u , acceleration, a , and the elapsed time, t . A racing car travels 480 m from rest in 14 s.
- Determine the acceleration of the racing car by isolating a , and then substituting in the known values.
 - Determine the acceleration of the racing car by substituting in the known values first.
 - Which method do you prefer and why?
- ★8. **Use Technology** Consider the formula for compound interest, $A = P(1 + i)^n$, in the example at the beginning of this section.
- If the variables P and i are constant, $A(n)$ will be what type of function?
 - Use a graphing calculator to check your prediction by graphing the function $A(n) = 100(1.03)^n$. Is your prediction correct?
 - If the variables n and i are constant, $A(P)$ will be what type of function?
 - Use a graphing calculator to check your prediction by graphing the function $A(P) = P(1.025)^5$. Is your prediction correct?

9. Refer to question 8. If you double the amount of time over which money is invested, will the final amount double? Will the amount of interest gained double?
- ★10. Consider the formula $V = \pi r^2 h$, which is used to determine the volume of a cylinder, where the volume is V , the radius is r , and the height is h .
- Determine which variable(s) you would need to set as a constant to generate a linear function. Check your answer by graphing.
 - Determine which variable(s) you would need to set as a constant to generate a quadratic function. Check your answer by graphing.
11. The symbol π is used in many formulas involving round shapes. Ariel argues that π is a symbol that represents a number, and therefore it is a variable. Is Ariel correct? Explain your answer.
12. **Use Technology** A 10-mL dose of medicine, is administered to a patient. Every 2 h, the amount of medicine remaining in the patient's body is reduced by a factor of $\frac{1}{5}$.
- Create a table of values to show the amount of medicine remaining in the patient's body.
 - Use a graphing calculator to create a scatter plot of the data.
 - What type of function do the data appear to model?
 - Use the regression feature on the graphing calculator to determine an equation that models the data.
13. **Use Technology** Design a spreadsheet that will accept different radii and heights to produce a volume of 1000 cm^3 for a cylinder while producing the minimum surface area.
14. Due to rent controls, the price for renting a two-bedroom apartment has increased at the rate of 3% per year over the last 10 years. The rent controls are expected to continue for the next 5 years.
- Create the equation of a function that will model the data.
 - If the current price for renting a two-bedroom apartment is \$919.97, what is the expected price for renting the apartment 5 years from now?
 - State some factors that you should take into consideration when you decide to rent an apartment.
- C**
15. **a)** Use the Internet to research how much caffeine remains in a person's body after a certain number of hours.
- Create a graph to model the amount of caffeine remaining in a person's body after a certain number of hours.
 - What type of function does your graph in part b) appear to represent?
 - Use the Internet to determine the amount of caffeine in different substances and how it affects different individuals.
16. **a)** According to Statistics Canada, the population of Canada is growing at the rate of 1.163% per year. On December 28, 2009, the estimated population of Canada was 33 890 491. Estimate the population of Canada on December 28, 2012.
- Use the Internet to research the growth rate of the population in other countries, and compare this with the growth rate of the population in Canada.
 - Determine some of the factors that will cause other countries to have a growth rate of population different from that of Canada.