

6.5 Solving Multi-Step Problems Using Polynomial Equations

KEY CONCEPTS

- Formulas can be used to solve multi-step problems that arise from real-world applications of mathematical modelling.
- There are many occupations that require applications of mathematical modelling with polynomial functions.
- There are many college programs that explore the applications of mathematical modelling in occupations.

Example

Winson has decided to paint his living room. The length of the living room is 9 m, the width is 6 m, and the height from the floor to the ceiling is 3 m.

- Determine the total surface area that Winson will paint if he uses two coats of paint.
- How many 4-L cans of paint should Winson purchase if 1 L of paint will cover approximately 12 m^2 ?
- Winson has researched on the Internet and has found a suitable paint that will cost \$30 for a 4-L can before tax. Determine the total cost, before taxes, to paint his living room.
- What are some factors that Winson should consider before he makes his final decision on the amount of paint that he purchases?

Solution

- a) The area of the ceiling to be painted is 9×6 , or 54 m^2 .

The area of the longer wall is 9×3 , or 27 m^2 .

There are two longer walls for a total area of 27×2 , or 54 m^2 .

The area of the shorter wall is 6×3 , or 18 m^2 .

There are two shorter walls for a total area of 18×2 , or 36 m^2 .

Total area that Winson will be painting:

$$\begin{aligned}\text{Area} &= \text{area of the ceiling} + \text{area of the two longer walls} + \text{area of the two shorter walls} \\ &= 54 + 54 + 36 \\ &= 144\end{aligned}$$

Therefore, the total surface area to be painted if Winson uses two coats of paint is 2×144 , or 288 m^2 .

- b) Since each litre of paint will cover approximately 12 m^2 , the number of litres of paint needed is equal to the total surface area divided by 12.

$$\frac{288}{12} = 24$$

Since the paint can be purchased in 4-L cans, Winson should purchase $\frac{24}{4}$, or a minimum of 6 cans. Winson may wish to buy one extra can of paint for a total of 7 cans, to account for any spillage, absorption rates, and any future touch-ups.

- c) The total cost to purchase the paint is equal to the cost per 4-L can times the number of cans of paint.

$$30 \times 7 = 210$$

Therefore, the total cost, before taxes, to purchase enough paint is \$210.

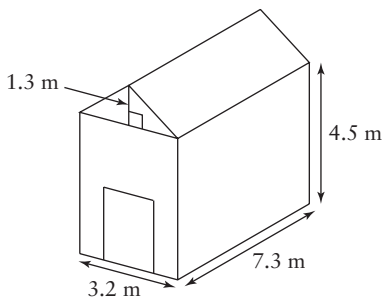
- d) Winson should take into consideration that the first coat may require more paint. He should also check with the store to see if he can bring any leftover paint back to the store.

A

- Janna is building a deck in her backyard that is in the shape of a triangle with a base length of 3 m and a height of 2 m. She wants to build the deck out of wood.
 - What is the minimum number of square metres of wood that Janna will require to build the deck?
 - Suggest a minimum amount of wood that Janna should purchase to build the deck.
- Jasmine is building a cedar chest to store her quilts in. The length of the chest is to be 1.5 m, the width of the chest is to be 0.8 m, and the height of the chest is to be 1 m.
 - Determine the volume of the cedar chest.
 - Determine the surface area of the cedar chest.
 - What materials other than cedar will Jasmine require to build the chest?
- An air conditioner at the back of a house is in the shape of a cylinder with a radius of 0.75 m and a height of 1.5 m. Suggest a minimum amount of material, in square metres, that would be needed to make a cover for the air conditioner.
- Mei plans to tile the foyer of her house with slate tiles that measure 8 in. by 10 in. The dimensions of the foyer are $5\frac{1}{2}$ ft by $7\frac{3}{4}$ ft.
 - Determine the total number of slate tiles that you recommend Mei should purchase to tile the foyer of her house.
 - If the tiles come 10 to a box, how many boxes will Mei need?
- Wilson is going to replace his countertop with a granite countertop. The dimensions of the countertop that he will be replacing are length 7 ft and width 2 ft. If the granite countertop costs $\$90/\text{ft}^2$, before taxes, how much will it cost, without taxes, for Wilson to purchase the granite countertop?

B

- ★6. Vaughn is building a cedar deck. The length of the deck is 15 ft and the width of the deck is 8 ft. Vaughn can purchase 12-ft lengths of cedar decking that have a width of 6 in. and a height of 2 in. for \$18.75 per piece, before taxes.
- What is the minimum number of 12-ft lengths that Vaughn should purchase to build the deck?
 - What is the total cost, before taxes, that Vaughn must pay for the cedar decking?
7. Brandon is hiring a contractor to replace the roof of his house. The roof is made up of two sides that are each 40 ft in length and 22.5 ft in width. The cost of the shingles is \$1.50/ft².
- Determine the minimum cost for the shingles.
 - If he purchases an extra 10% for wastage, what will be the cost of the shingles?
 - Use the Internet to research other costs that are associated with replacing the roof on a house.
- ★8. Karen is going to use white vinyl siding on the ends and sides of her boathouse, excluding the entrance, which is 2 m by 3 m.



The cost of the siding that she is going to use is \$10.25/m². Assuming that she purchases an extra 10% of siding for wastage, determine how much it will cost Karen, before taxes, to purchase the siding for her boathouse.

9. Niklos has decided to insulate the area under the back part of his cottage as part of a government tax incentive. The area to be insulated is 25 ft by 17 ft.
- Determine the total area that needs to be insulated.
 - Use the Internet to research the different types of insulation that Niklos can use to insulate the area.
 - List the advantages and disadvantages of using each type of insulation.
 - Research the prices to purchase the different types of insulation at different suppliers.
 - What other costs are there that Niklos must consider when he is making his plan to insulate under the back part of his cottage?
 - Based on your research, prepare an estimate of how much it will cost Niklos to insulate the area.

10. Use Technology

- Design a spreadsheet that a painter might use to input the dimensions of a room, the cost per litre of paint, and the coverage of the paint, and that then calculates the total cost of paint required to cover the room.
- What other features might be included in the spreadsheet?

- 11. a)** Determine how many litres of varnish are required to varnish the top of a circular wooden table if the diameter of the table is 5.2 ft and the varnish that will be purchased covers approximately $12 \text{ ft}^2/\text{L}$.
- b)** If the varnish costs \$23.09 per litre, what is the total cost of the varnish, before taxes?
- 12.** An annuity is an investment in which regular payments are deposited into an account. The amount, A , of an annuity can be calculated using the formula $A = \frac{R[(1+i)^n - 1]}{i}$, where R represents the regular withdrawal, i represents the interest rate per compounding period, as a decimal, and n represents the number of compounding periods.
- a)** Dennis deposits \$300 every month into an account paying 3% per year compounded monthly for 25 years. Determine the amount in his account at the end of the 25 years.
- b)** The present value, PV , of an annuity is the total amount that can finance a series of regular withdrawals over a specific period of time. The regular withdrawal, R , can be calculated using the formula $R = \frac{i(PV)}{1 - (1+i)^{-n}}$, where i represents the interest rate per compounding period, as a decimal, and n represents the number of compounding periods. Suppose that Dennis then receives monthly payments from his account in part a) for the next 25 years, at the same interest rate. Determine how much Dennis can withdraw each month.
- 13.** The dimensions of a bedroom floor are $12\frac{1}{2}$ ft by $9\frac{1}{4}$ ft. The bedroom is being renovated, and the wood trim moulding that is going to be used around the base of the walls is sold in pieces that are $4\frac{1}{2}$ in. by $1\frac{1}{16}$ in. by 8 ft in length.
- a)** Determine how many 8-ft long pieces of moulding must be purchased to renovate the room.
- b)** If the price for each piece of moulding is \$12.50, determine the cost, before taxes, to purchase the moulding.
- 14.** For a circular log of radius 40 cm, determine the maximum cross-sectional area of a rectangular beam cut from this log.
- C**
- 15.** Suppose a backyard is in the shape of a parabola defined by $y = -x^2 + 25$, where the back of the house is defined by the x -axis. Determine the area of the largest rectangle that can be enclosed for a pool while maintaining a 1-m clearance from the edge of the yard.
- 16.** Work with a partner and use the Internet to research occupations that require applications of mathematical modelling. Choose one career and prepare a report on how applications of mathematical modelling are used in this occupation.
- 17.** Work with a partner and use the Internet to research college programs that explore applications of mathematical modelling. Choose one of the college programs and prepare a report on how applications of mathematical modelling are explored in this college program.

Chapter 6: Checklist

By the end of this chapter, I will be able to:

- expand and simplify polynomial expressions involving more than one variable, including expressions arising from real-world applications
- solve polynomial equations in one variable, of degree no higher than four, by selecting and applying strategies, and verify solutions using technology
- solve problems algebraically that involve polynomial functions and equations of degree no higher than four, including those arising from real-world applications
- solve equations of the form $x^n = a$ using rational exponents
- identify and explain the roles of constants and variables in a given formula
- determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values
- make connections between formulas and linear, quadratic, and exponential functions, using a variety of tools and strategies
- solve multi-step problems requiring formulas arising from real-world applications
- gather, interpret, and describe information about applications of mathematical modelling in occupations, and about college programs that explore these applications