# Chapter 7 Solve Problems Involving Geometry

## 7.1 Area of Two-Dimensional Objects

## KEY CONCEPTS

- The imperial system is a collection of units that includes inches, feet, and miles for lengths; cups, quarts, and gallons for volume; and ounces and pounds for weight.
- In the 1970s, the metric system and the International system of units (SI) were introduced in Canada to replace the imperial system. However, there are still individuals and industries, such as construction, that work with the imperial system.
- Conversions can be made between the imperial system and the metric system using tools such as tables, calculators, and online conversion tools.
- To calculate the area of an object, all measures must be in the same units.
- To calculate the area of a composite figure, add the areas of the component figures.

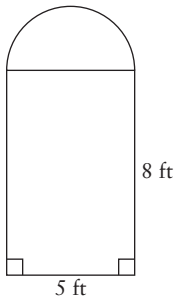
Imperial units can be represented in different ways. For example:

| | |
|---|---|
| 3 in. | 3″ |
| 5 in.$^2$ | 5 sq. in. |
| 12 ft | 12′ |
| 8 ft$^2$ | 8 sq. ft |

In this Study Guide and Exercise Book, the following notation is used:
3 in. 5 in.$^2$ 12 ft 8 ft$^2$

**Example**

The Norman window in the hallway of a house is in the shape of a rectangle with a semicircular top. The rectangular portion has width 5 ft and height 8 ft.




**a)** Determine the area of the glass in the window to the nearest tenth of a square foot.

**b)** Determine the perimeter of the window to the nearest tenth of a foot.

**Solution**

**a)** Calculate the area of the rectangular portion of the window.

$A = \ell \times w$
$= 8 \times 5$
$= 40$

Calculate the area of the semicircular portion of the window.
The radius of the semicircle is half the width of the window, or 2.5 ft.

$A = 0.5\pi r^2$
$= 0.5 \times \pi \times (2.5)^2$
$= 9.817\ldots$

Calculate the total area of the glass in the window.

$A = A_{\text{rectangle}} + A_{\text{semicircle}}$
$\doteq 40 + 9.8$
$\doteq 49.8$

The area of glass in the window is approximately 49.8 ft$^2$.

**b)** Calculate the perimeter of the rectangular portion of the window.

$P = \ell + \ell + w$
$= 8 + 8 + 5$
$= 21$

Calculate the perimeter of the semicircular portion of the window.

$P = \pi r$
$= \pi \times 2.5$
$= 7.853\ldots$

Determine the perimeter of the window.

$P = P_{\text{rectangle}} + P_{\text{semicircle}}$
$\doteq 21 + 7.9$
$\doteq 28.9$

The perimeter of the window is approximately 28.9 ft.

## A

*Unless otherwise specified, round all measures to the nearest tenth of a unit.*

**1.** **Use Technology** Use the Internet to find three online conversion calculators that you can use to convert metres to feet.

**a)** How many decimal places do each of the online conversion calculators round to?

**b)** Why do you think that the particular degree of accuracy is used for each of the three online conversion calculators?

**2.** Convert each measure from imperial units to metric units as indicated.

| | |
|---|---|
| **a)** 25 ft | metres |
| **b)** 38 in. | centimetres |
| **c)** 300 yd | metres |
| **d)** 12 miles | kilometres |

**3.** Convert each measure from metric units to imperial units as indicated.

| | |
|---|---|
| **a)** 72 m | feet |
| **b)** 92 m | yards |
| **c)** 45 cm | inches |
| **d)** 260 km | miles |

**4.** Convert each measure from imperial units to metric units as indicated.

| | |
|---|---|
| **a)** 412 $ft^2$ | square metres |
| **b)** 15 $in.^2$ | square centimetres |
| **c)** 120 $yd^2$ | square metres |
| **d)** 28 $mi^2$ | square kilometres |

**5.** Convert each measure from imperial units to metric units as indicated.

| | |
|---|---|
| **a)** 200 $in.^2$ | square centimetres |
| **b)** 58 $ft^2$ | square metres |
| **c)** 123 $yd^2$ | square metres |
| **d)** 18 acres | hectares |
| **e)** 95 $mi^2$ | square kilometres |

**6.** **a)** Copy and complete the table. Use an online conversion calculator.

| Square Inches | Square Centimetres |
|---|---|
| 10 | |
| 20 | |
| 30 | |
| 40 | |
| 50 | |

**b)** How is the number of square centimetres related to the number of square inches?

**7.** **a)** Copy and complete the table. Use an online conversion calculator.

| Metres | Feet | Square Metres | Square Feet |
|---|---|---|---|
| 1 | | 1 | |
| 2 | | 2 | |
| 3 | | 3 | |
| 4 | | 4 | |
| 5 | | 5 | |

**b)** Can you determine a relationship between the number of feet per metre and the number of square feet per square metre?

**c)** Use the relationship in part b) to determine the number of square feet in 6 $m^2$.

**d)** Could you use the relationship from part b) to determine the number of square feet in any number of square metres?

**8.** Use the Internet to learn more about the history of the imperial system, the metric system, and the International system of units (SI).Write a summary of your findings.

**9.** Janet designed a circular piece of art for her living room. The diameter of the piece is 30 cm. She would like to make calculations using the formulas $2\pi r$ and $\pi r^2$.

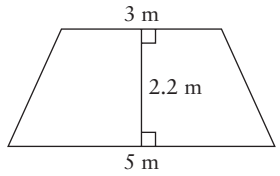
**a)** Which formula should Janet use to calculate the area of the piece of art? Explain how you know.

**b)** Which formula should Janet use to calculate the circumference of the piece? Explain how you know.

**c)** Calculate the area and the circumference of the piece.

**10.** Calculate the area of each shape.

**a)**

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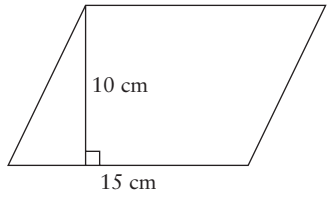


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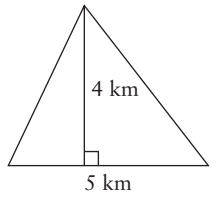
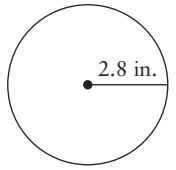
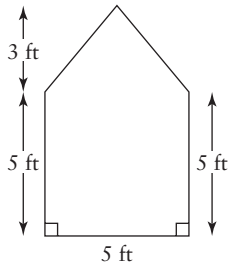


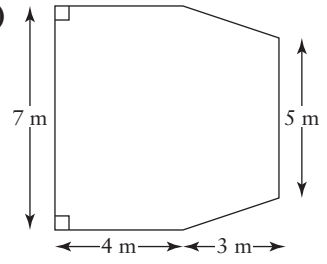

**d)**




**e)**




**f)**



## B

**11. a)** Cut out each shape from a piece of cardboard.

**i)** a triangle

**ii)** a square

**iii)** a rectangle

**iv)** a circle

**b)** For each shape, see if you can put the cutout through its hole without touching the edge of the hole.

**c)** What shape are sewer access hole covers? Why do you think this is?

**d)** Use the Internet to research the reason for round sewer access hole covers.

**12.** Laura is going to replace the floor in her garden shed with new vinyl flooring that costs \$3.09 per $ft^2$. The floor of the shed is 4.5 m long and 3.5 m wide.

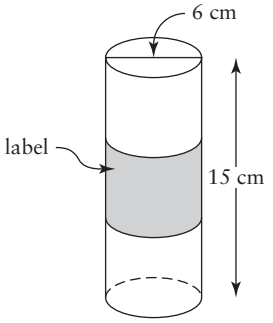
**a)** Is it better to convert the dimensions to feet and then calculate the number of square feet? Is it better to find the number of square metres and then convert to square feet? Does it matter? Explain your reasoning.

**b)** Determine the total cost, before taxes, for Laura to purchase the vinyl flooring.

**c)** Would you advise Laura to buy the exact amount of flooring, or another amount? Explain.

**13.** Kees is designing a table for his brother. The top of the table will be a circle with diameter 3 ft, 10 in. He has a square piece of maple with area 15 129 $cm^2$. Will Kees be able to use the piece of maple to make the tabletop? Explain.

**14.** Alyssa is designing a label for a cylindrical shampoo bottle that has diameter 6 cm and height 15 cm. The label will be placed around the middle third of the bottle. Determine the dimensions and the area of the label.




☆**15.** Bob is working on a project at college to design a deck in the shape of an isosceles trapezoid. The lengths of the parallel sides of the deck are 14 ft and 20 ft. Bob would like the total area of the deck to be 102 $ft^2$.

**a)** Draw a diagram to represent this situation.

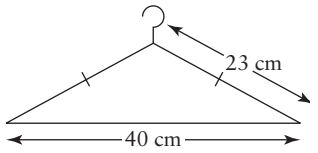
**b)** Determine the distance between the two parallel sides of the deck.

**c)** How is a trapezoid similar to a rectangle? How is a trapezoid different from a rectangle?

**d)** The formula to calculate the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$. How does this formula relate to the formula to calculate the area of a rectangle, $A = \ell \times w$, and the formula to calculate the area of a parallelogram, $A = b \times h$?

**e)** Bob is planning to stain the deck. He has determined that 1 gallon of stain will cover 8 $m^2$. How many gallons of stain does Bob need?

**16.** Steve and Phyllis own a dry cleaning business. They cover the hangers with white paper that is folded up from the base. The main part of each hanger is in the shape of an isosceles triangle with base length 40 cm and equal sides that are each 23 cm long.




**a)** Describe the shape of a sheet of white paper used to cover a hanger.

**b)** Determine the minimum area of a sheet of white paper used to cover a hanger.

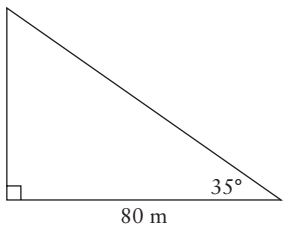
**c)** Determine the minimum area, in square inches, of a sheet of white paper used to cover a hanger.

**17.** Roland hired a contractor to tile his laundry room floor. The dimensions of the laundry room are $4\frac{1}{2}$ ft by $6\frac{3}{4}$ ft. He plans to use square tiles that are 14 in. by 14 in. and that cost $1.49 per square foot.

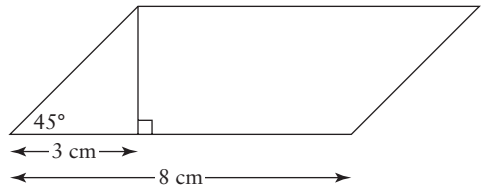
**a)** What is the minimum number of tiles that Roland should purchase, including 10% for wastage?

**b)** Determine the total cost, before taxes.

**18.** Leslie surveyed a property that is in the shape of a right triangle. She recorded her measurements on a sketch. Determine the area of the property.




**19.** Haroon designed a logo that is in the shape of a parallelogram. Determine the area of the logo.

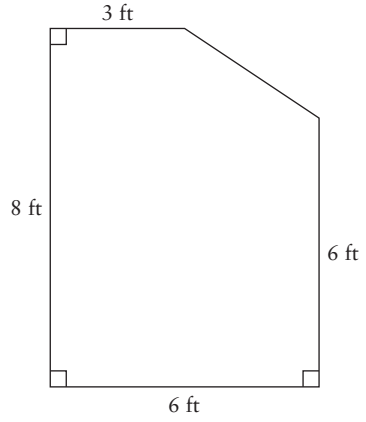



☆**20.** Amy was hired to design a new city park. She decided that the park will have a flower garden, 10 m long by 8 m wide, surrounded by a grassy area of uniform width. The area of grass will equal the area of the flower garden.

**a)** Draw a diagram to illustrate this situation.

**b)** Determine the width of the grassy area that will surround the flower garden.

**21.** Roger designed a dock as shown.



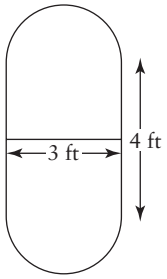

**a)** Explain how Roger could calculate the area of his dock.

**b)** Describe another method that Roger could use to calculate the area.

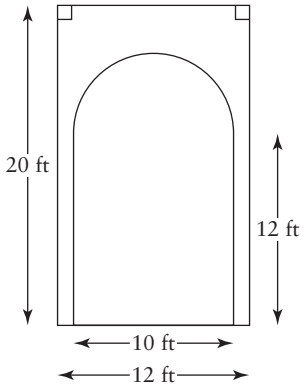
**c)** Describe a third method that Roger could use to calculate the area.

**d)** Calculate the area of Roger's dock.

**22.** The top of Nancy's dining room table is in the shape of a 3-ft by 4-ft rectangle with two semicircular ends. Calculate the area of the tabletop.




**23.** Grant has a rectangular piece of wood with dimensions 20 ft by 12 ft. He plans to cut a composite figure out of the centre of the wood—a 12-ft by 10-ft rectangle topped by a semicircle with radius 5 ft. He will use what remains to make an archway for his garden.




**a)** What is the area of the wooden part of the archway?

**b)** What percent of the original piece of wood can be used for another project?

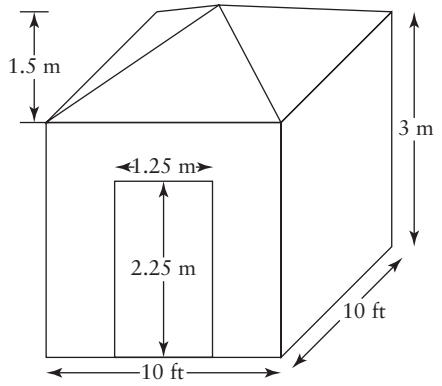
## C

☆**24.** Ashley is tiling the wall between the counter and the cupboards in her kitchen. The tiles are regular hexagons with side length 6 cm. The area to be tiled is 22 in. deep and 48 in. long.

**a)** Determine the area that Ashley plans to tile in square centimetres.

**b)** Determine the number of tiles that Ashley needs, including 10% extra for wastage.

**25.** Sam purchased a new shed in the shape of a rectangular prism topped by a square-based pyramid. The walls of the shed are 10 ft long, 10 ft wide, and 3 m high. The roof reaches 1.5 m above the walls at its highest point. The doorway, which is in the centre of the front wall, measures 1.25 m by 2.25 m. The walls and roof, excluding the doorway, will be covered with waterproof sheeting.




**a)** Determine the surface area, in square feet, to be covered with waterproof sheeting.

**b)** Express your answer to part a) in square metres.

**26.** Glen is building a scale model of the house he plans to build. The base of the house is a rectangular-based prism with walls that are 30 ft long, 15 ft wide, and 20 ft high. The roof of the house is a rectangular-based pyramid. The height of the roof measures 12 m above the walls at its highest point. Glen will use the scale $\frac{1}{4}$ in. : 1 ft.

**a)** Determine the dimensions of the scale model of Glen's house.

**b)** Determine the surface area of the scale model in square inches.

**c)** Draw a scale model of the house using the dimensions from part a).

**d)** Construct a scale model of the house that Glen is going to build.