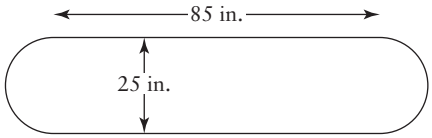
KEY CONCEPTS

- There are a number of real-world applications involving the volume of three-dimensional objects.
- For prisms and cylinders, the volume can be found by multiplying the base area by the height.

Example

A propane tank is in the shape of a cylinder with two hemispherical ends. The length of the cylindrical portion is 85 in. and the inside diameter of the tank is 25 in. Determine the volume of propane, in gallons, that will fit inside the propane tank.



Solution

Calculate the volume of the cylindrical portion of the tank.
Since $d = 25$, $r = 12.5$.

$$V_{\text{cylinder}} = \pi r^2 h$$
$$= \pi(12.5)^2(85)$$
$$= 41\,724.277\ldots$$

Calculate the volume of the hemispherical ends of the tank.
The two hemispheres form a sphere with $r = 12.5$.

$$V_{\text{two hemispheres}} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(12.5)^3$$
$$= 8181.230...$$

Calculate the total volume of the propane tank.

$$V_{\text{tank}} = V_{\text{cylinder}} + V_{\text{two hemispheres}}$$
$$\doteq 41\,724.277 + 8181.230$$
$$\doteq 49\,905.51$$

The volume of the propane tank is approximately 49 905.51 in.^{3}.

Use an online conversion tool to calculate the volume of propane, in gallons, that the tank will hold.
The tank will hold approximately 180 gallons of propane.

A

Unless otherwise specified, round all measures to the nearest tenth of a unit.

1. Convert each measure from imperial units to metric units as indicated.

a) 200 ft^3 cubic metres

b) 112 $in.^3$ cubic centimetres

c) 7 yd^3 cubic metres

d) 2 ft^3 cubic centimetres

2. Convert each measure from metric units to imperial units as indicated.

a) 85 m^3 cubic feet

b) 315 m^3 cubic yards

c) 208 cm^3 cubic inches

d) 300 000 cm^3 cubic feet

3. a) Copy and complete the table by converting from cubic centimetres to cubic metres.

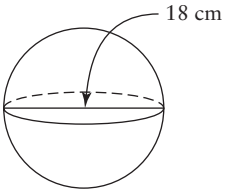
Cubic Centimetres	Cubic Metres
1	
2	
3	
4	
5	

b) What is the relationship between cubic centimetres and cubic metres? Explain how to convert from cubic centimetres to cubic metres, and from cubic metres to cubic centimetres.

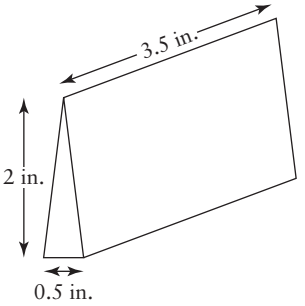
4. An advertisement for an older vehicle indicates it has an engine with displacement 400 $in.^3$. A potential buyer figures that since there are roughly 2.5 cm per inch, the displacement is about 1000 cm^3, or 1 L. Explain the error in the buyer's thinking. Determine the correct engine displacement, in litres.

5. Calculate the volume of each shape.

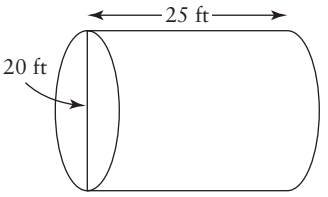
a)



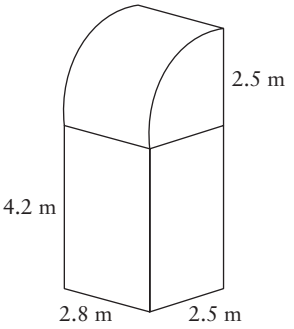
b)



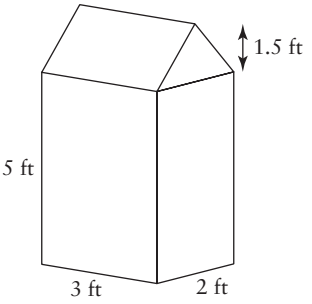
c)



d)



e)



6. A cylindrical container has diameter 8 cm and height 18 cm. What volume of flour can the container hold?

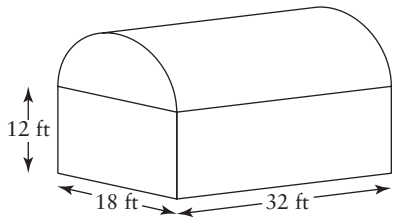
7. A coin box is in the shape of a cube. The volume of the box is 551.4 cm^3. Determine the side lengths of the box.

8. Enid plans to add topsoil to the area between her house and her fence. The area is 28 ft long and 6 ft wide. How many cubic yards of topsoil must she purchase if she wishes to raise the ground level by 6 in.?

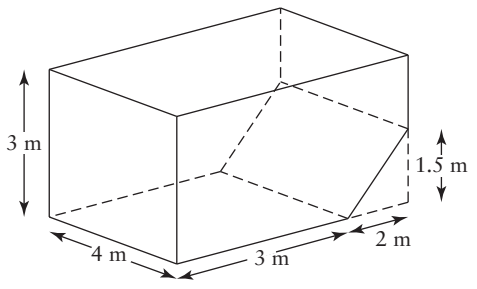
9. A box in the shape of a square-based prism has volume 1152 cm^3 and height 18 cm. Determine the side length of the square base.

B

10. A building is in the shape of a rectangular prism topped by half a cylinder. Determine the volume of the building.



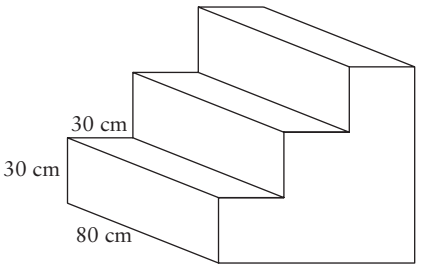
11. Doug's swimming pool is designed as shown.



a) Determine the maximum volume of water that the pool could hold.

b) Determine the volume of water that the pool could hold if it were filled to a point that is $\frac{1}{4}$ m below the top of the pool.

12. John is adding three steps to his back porch as shown. Each step is 80 cm long, 30 cm wide, and 30 cm high.



a) Determine the amount of space under the steps in cubic centimetres.

b) Determine the amount of space under the steps in cubic metres.

☆**13. Use Technology** Jenna has a piece of cardboard with dimensions 40 cm by 30 cm. She plans to make an open box by cutting congruent squares out of the corners.

a) Draw a diagram to represent this situation.

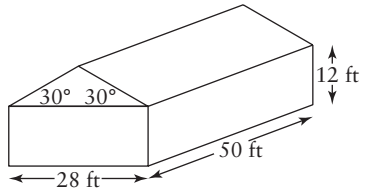
b) Write an equation expressing the volume, V, of the box in terms of the length, x, of the squares that were cut out.

c) Describe how you could use a graphing calculator to determine the side lengths of the square that Jenna should cut from each corner if she wants to construct a box with maximum volume.

d) Determine the maximum volume of the box.

e) What are the dimensions of the box with maximum volume?

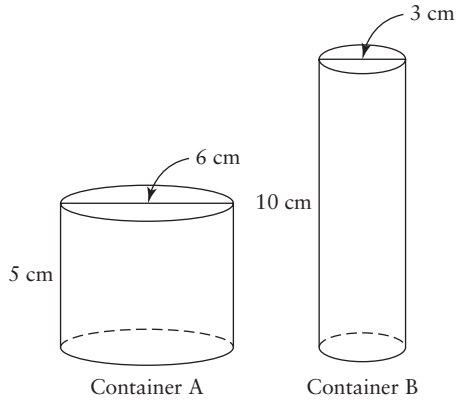
☆**14.** For this office building, the air must be exchanged every 40 min.



a) Would a ventilation system that exchanges air at a rate of 500 ft^3/min meet the criteria? Explain.

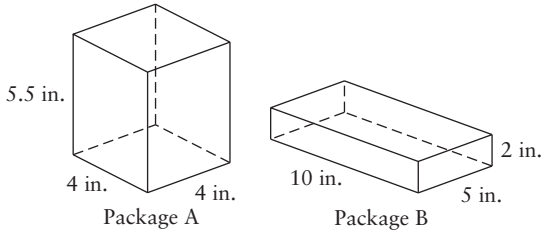
b) Determine the minimum acceptable air exchange rate.

15. a) Predict which container will hold more water, Container A or Container B. Justify your prediction.



b) Calculate the volume of each container. Which container will hold more water?

16. You have a company that sells green tea. Do you think that it would be better to sell the tea in Package A or in Package B? Justify your response.



17. Use Technology

a) Design a spreadsheet that accepts the length, width, and height of a rectangular prism, and then calculates the surface area and the volume of the prism.

b) Find the dimensions that minimize the surface area of a rectangular prism with volume 1000 cm^3.

c) Describe the shape of the prism in part b). Suggest reasons why companies do not always make packages in this shape.

18. A sphere and a cube each have volume 625 cm^3.

a) Determine the surface area of each object. Which has the smaller surface area?

b) Based on your answer to part a), suggest reasons why companies do not always use shapes that minimize the amount of packaging for a given volume.

c) Research the phrase *surface tension*. Describe how surface tension, along with the answer to part a), helps explain the shape of a drop of water.

19. Use Technology Use geometry software to construct a cylinder.

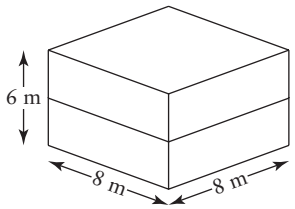
a) Use a slider to double the height of the cylinder. By what factor does the volume change?

b) Use a slider to double the radius of the cylinder. By what factor does the volume change?

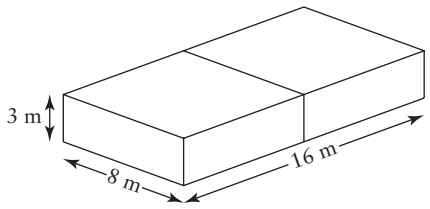
c) Explain why changing the radius has a different effect on the volume than changing the height.

20. Use Technology Use geometry software to construct a rectangular prism. Use a slider to double one dimension of the prism at a time. By what factor does the volume change each time?

☆**21.** When designing a house, an architect must consider the ratio comparing surface area to volume. In general, the greater the surface area of the building, the greater the heat gain or heat loss.



House A



House B

a) Copy and complete the table.

	House A	House B
Surface Area		
Volume		
Surface Area/ Volume		

b) Which house will have less heat gain or less heat loss? Explain.

c) Research other factors that influence the heat gain or heat loss of a building.

22. Research applications of surface area and volume in construction and architecture. Write a report for your classmates.

C

23. A pile of gravel is in the shape of a right circular cone with diameter 20 ft and height 4 ft.

a) Determine the volume of gravel.

b) If the radius was halved but the height remained the same, by what percent would the volume of gravel decrease? Explain your reasoning.

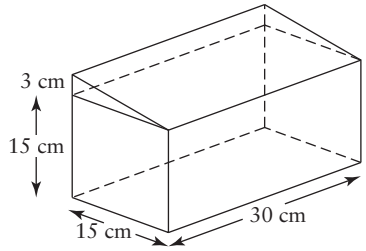
24. A box is to hold 24 cans. Each can has diameter 6.5 cm and height 10 cm. The cans will be packed in 1 layer and arranged in 4 rows with 6 cans in each row. Each can will touch the can beside it, the cans at the edge will touch the box, and the bottoms and tops of the cans will touch the box.

a) Draw a diagram to illustrate this situation.

b) Determine the dimensions of the box needed to hold the cans.

c) How much empty space will there be in the box?

25. The wooden mailbox by the front door of Mario's house is shown.



a) Determine the total volume of the mailbox.

b) Explain how you calculated the total volume of the mailbox. Describe another method to determine the total volume of the mailbox.

c) Determine the total surface area that will be painted if the outside of the box is painted.