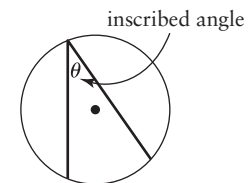
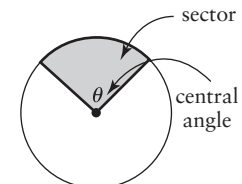
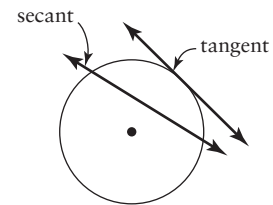
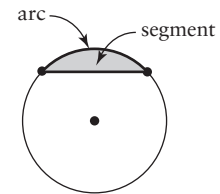
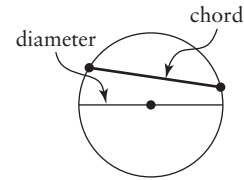


7.4 Properties of Circles

KEY CONCEPTS

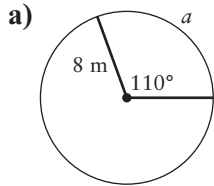
- A circle is a set of points that are a fixed distance (the radius) from a given point (the centre of the circle).
- A chord is a line segment inside a circle that joins two points on the circumference of the circle. The diameter is a chord that passes through the centre of the circle.
- An arc is a portion of the circumference of a circle that joins two points on the circle.
- A segment is a part of a circle bounded by a chord and an arc.
- A tangent to a circle is a line segment that intersects the circle at one point.
- A secant to a circle is a line segment that intersects the circle at two points.
- A sector is the area enclosed by two radii of a circle and the arc that connects the endpoints of the radii.
- A central angle is formed by two radii of a circle.
- An inscribed angle is an angle in a circle with its vertex on the circle.
- The length of an arc, a , given central angle θ and radius r , is given by $a = \frac{\theta}{360^\circ} (2\pi r)$.
- Given central angle θ and radius r , the area of a sector, A , is $A = \frac{\theta}{360^\circ} (\pi r^2)$.
- The area of a segment, A , is $A = \frac{1}{2} r^2 \left(\frac{\pi}{180^\circ} \theta - \sin \theta \right)$, given central angle θ and radius r .



Example

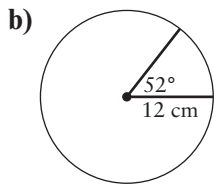
Express your answers to the following questions to the nearest tenth of a unit.

- Determine the length of the arc of a circle with radius 8 m and a central angle of 110° .
- A circle has radius 12 cm. The central angle of a sector of the circle is 52° . Calculate the area of the sector.
- A circle has radius 4 in. Determine the area of the segment if the central angle is 62.5° .

Solution

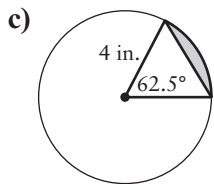
$$\begin{aligned} a &= \frac{\theta}{360^\circ} (2\pi r) \\ &= \left(\frac{110^\circ}{360^\circ}\right) 2\pi(8) \\ &= 15.358\dots \end{aligned}$$

The length of the arc is approximately 15.4 m.



$$\begin{aligned} A &= \frac{\theta}{360^\circ} \pi r^2 \\ &= \frac{52^\circ}{360^\circ} \pi(12)^2 \\ &= 65.645\dots \end{aligned}$$

The area of the sector is approximately 65.3 cm^2 .



$$\begin{aligned} A &= \frac{1}{2} r^2 \left(\frac{\pi}{180^\circ} \theta - \sin \theta \right) \\ &= \frac{1}{2} (4)^2 \left(\frac{\pi}{180^\circ} (62.5^\circ) - \sin 62.5^\circ \right) \\ &= 1.630\dots \end{aligned}$$

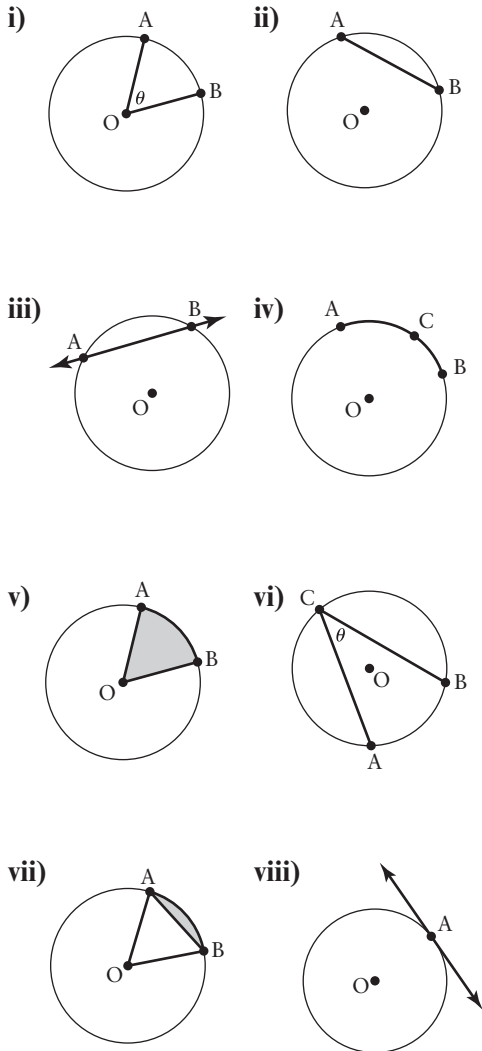
The area of the segment is approximately 1.6 in.^2 .

A

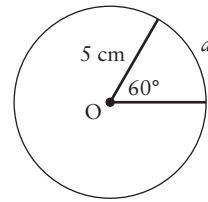
Unless otherwise specified, round all measures to the nearest tenth of a unit.

1. Match each term with the correct diagram.

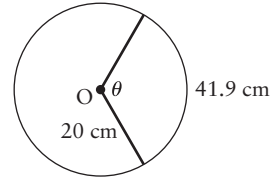
- a) chord
- b) arc
- c) central angle
- d) secant
- e) tangent
- f) sector
- g) inscribed angle
- h) segment



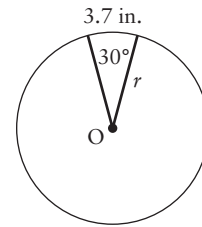
2. Determine the arc length, a .



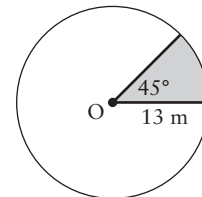
3. Determine the measure of $\angle\theta$.



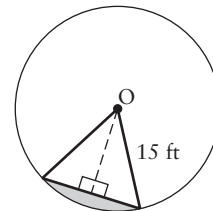
4. Determine the length of the radius, r .



5. Calculate the area of the shaded sector.



6. This circle has radius 15 ft and central angle 60° .



- a) Calculate the area of the sector with central angle 60° .
- b) Calculate the area of the triangle formed by the radii and the chord.
- c) Use your answers to parts a) and b) to calculate the area of the segment.

7. An alternative equation relating an arc length, a , to the central angle, θ , and the radius, r , is $\frac{\theta}{360^\circ} = \frac{a}{2\pi r}$.

- a) Explain what the fraction on each side of the equation represents.
 b) How does the equation $\frac{\theta}{360^\circ} = \frac{a}{2\pi r}$ relate to the equation $a = \frac{\theta}{360^\circ} (2\pi r)$?
 c) Write an alternative equation for the area of a sector with central angle θ .

8. A sector angle (or central angle) is 60° . What fraction of the circle does the sector represent? Explain.

B

9. A sector of a circle has central angle 50° and area 141.4 cm^2 . What is the radius of the circle?

10. A segment of a circle has central angle 40° and area 5.4 cm^2 . What is the diameter of the circle?

- ★11. An indoor track is in the shape of a circle with radius 32 m. Quinn ran $\frac{3}{4}$ of the way around the track.

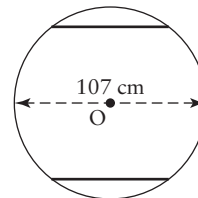
- a) What distance did Quinn run?
 b) What is the central angle subtended by the path Quinn ran around the track? Explain how you determined the angle.
 c) Isha ran a path that subtends a central angle of 200° . What distance did she run?

12. Dan, Nadine, and Vidak shared an apple pie with diameter 12 in. Dan had a piece with a central angle of 120° , Nadine had a piece with a central angle of 100° , and Vidak had a piece with a central angle of 140° . Determine the area of each person's piece.

- ★13. Kristen is designing a circular patio, with radius 12 ft. Two thirds of the patio will be concrete that is 20 in. thick. The rest of the patio will be flagstone that is 2 in. thick.

- a) Sketch a diagram of the patio.
 b) What volume of concrete is needed?
 c) What volume of flagstone is needed?

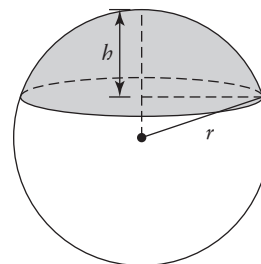
- ★14. A circular tabletop has diameter 107 cm. Two segments, each with a central angle 90° , are hinged to fold down. What percent of the tabletop can be folded down?



15. A medicine wheel is a spiritual symbol that represents harmony among all living beings on Earth. Research the history and construction of a medicine wheel. Use circle properties to explain the construction of a medicine wheel.

C

16. A spherical cap is the region of a sphere that lies above a given plane. The volume of a spherical cap is $V_{\text{cap}} = \frac{1}{3} \pi h^2 (3r - h)$. Determine the volume of a spherical cap with height 20 cm in a sphere with radius 40 cm.



17. Determine a formula for the area of a sector that is $\frac{1}{6}$ of a circle. Explain how you determined the formula.