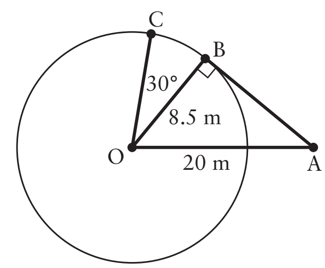
7.6 Solving Problems Involving Properties of Circles

KEY CONCEPTS

- There are a number of real-life applications that involve the geometry of circles.
- There are many occupations in which circle properties are applied to solve problems.

Example

Martha is visiting Murney Tower in Kingston, Ontario. She is standing outside the tower at point A, which is 20 m from the centre, O, of the building. She walks along the path of the tangent to the tower from point A to point B at the tower, and then walks around the building along an arc that subtends a central angle of 30° to point C. The distance BO is 8.5 m. How far did Martha walk, in total, from point A, to point B, to point C? Express your answer to the nearest tenth of a metre.



Solution

Determine the length of tangent AB.

$$AB^2 + BO^2 = AO^2$$
$$AB^2 + 8.5^2 = 20^2$$
$$AB^2 = 20^2 - 8.5^2$$
$$AB = \sqrt{20^2 - 8.5^2}$$
$$= 18.103\ldots$$

Determine the arc length BC.

$$a = \frac{\theta}{360°}(2\pi r)$$
$$= \left(\frac{30°}{360°}\right)2\pi(8.5)$$
$$= 4.450...$$

Determine the total distance.

AC = AB + BC

$\doteq 18.103 + 4.450$

$\doteq 22.553$

Martha walked a total distance of approximately 22.6 m.

A

1. A circle has radius 5 cm and centre C. From point A outside the circle, a tangent line that is 10 cm long is drawn to point B on the circumference of the circle.

a) Draw a diagram to represent this situation.

b) Determine the length of AC.

2. An inscribed angle, with vertex C on the circumference of a circle, is subtended by chord AB in the circle.

a) Draw a diagram to represent this situation.

b) If $AC^2 + BC^2 = AB^2$, what can you conclude about ∠ACB and line segment AB?

3. Three chords, GH, KL, and PQ, are equidistant from the centre, O, of a circle with radius 8 in. The length of GH is 3 in. What do you know about the lengths of chord KL and chord PQ? Explain.

4. Two inscribed angles, ∠CED and ∠CFD, are subtended by chord CD and are on the same side of CD.

a) What do you know about the measures of ∠CED and ∠CFD? Explain.

b) Are the areas of △CED and △CFD equal? If they are not equal, under what conditions would the areas be equal?

5. In a circle with centre O, chord AB = 6 cm, chord CD = 6 cm, and chord EF = 6 cm.

a) Show that the three chords are equidistant from the centre of the circle.

b) Show that the triangles formed by connecting the endpoints of each chord to the centre of the circle are congruent.

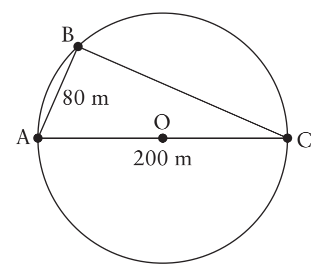
B

6. Kevin is standing at point A on the shore of a circular lake with a radius of 2 mi. The distance from the centre of the lake to point B, which is on the tangent line drawn from point A, is 5 mi.

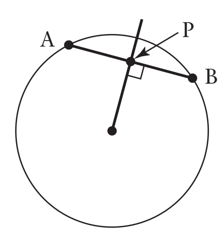
a) Draw a diagram to represent this situation.

b) How far would Kevin have to walk in a straight line to get from point A to point B?

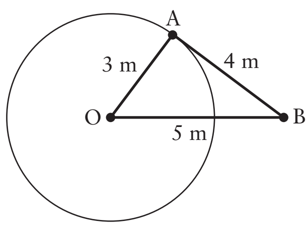
7. A circular area, with diameter 200 m, is part of an orienteering course. The distance AB is 80 m. Determine the distance BC.



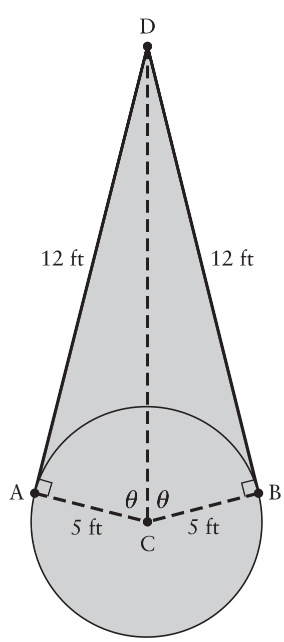
8. The flight path of an aircraft on a radar screen passes through the centre of the screen and is perpendicular to the line segment connecting two towns, Town A and Town B, on the edge of the circular area displayed on the radar screen. The distance, as represented on the radar screen, between the two towns is 8 in. What is the distance, AP, between the plane and Town A on the radar screen when the flight path intersects the line segment joining Town A and Town B? Explain.



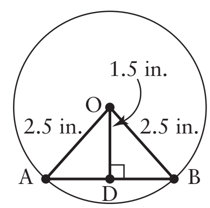
☆**9.** Alvin is at point A on the circumference of a circular area on a field, 3 m from the centre of the circular area. Benjamin is at point B outside the circle, 5 m from the centre of the circle and 4 m from Alvin. Explain how you can determine whether the line segment AB is a tangent to the circular area on the field at point A.



☆**10.** Philip designed a garden as shown. From point D outside the circle with centre C, two tangents, DA and DB, each 12 ft in length, meet the radius of the circle at points A and B respectively. Determine the total area of Phil's garden.

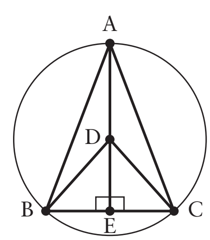


11. Alanna created the design below for the cover for her art project. The design is a circle with radius 2.5 in. She drew a line, 1.5 in. long, from the centre of the circle that is perpendicular to the chord AB and meets chord AB at point D. She plans to shade in the triangular area △OAD with a blue marker and the triangular area △OBD with a green marker. Determine the areas of △OAD and △OBD. Explain how you determined the areas.



C

12. A chord BC in a circle with centre D subtends a central angle, ∠BDC, and an inscribed angle, ∠BAC, on the same side of the chord. A line segment through D and A, and perpendicular to chord BC, intersects BC at E. Is △ABE congruent to △ACE? Justify your answer.



13. A runner starts at the 9 o'clock position on a 1000-m circular track and runs counterclockwise. She leaves the circle on a tangent and arrives at a point outside the circle, 100 m due west of her starting point. How far did the runner travel, in total?

Chapter 7 Checklist

By the end of this chapter, I will be able to:

- solve problems involving two-dimensional shapes and three-dimensional figures, and arising from real-world applications
- determine circle properties and solve related problems, including those arising from real-world applications
- gather and interpret information about real-world applications of geometric shapes and figures in a variety of contexts in technology-related fields, and explain these applications
- perform required conversions between the imperial system and the metric system using a variety of tools, as necessary within applications
- solve problems involving the areas of rectangles, parallelograms, trapezoids, triangles, and circles, and of related composite shapes, in situations arising from real-world applications
- solve problems involving the volumes and surface areas of spheres, right prisms, and cylinders, and of related composite figures, in situations arising from real-world applications
- recognize and describe arcs, tangents, secants, chords, segments, sectors, central angles, and inscribed angles of circles, and some of their real-world applications
- determine the length of an arc and the area of a sector or segment of a circle, and solve related problems
- determine, through investigation using a variety of tools, properties of the circle associated with chords, central angles, inscribed angles, and tangents
- solve problems involving properties of circles, including problems arising from real-world applications