Answers

Mathematics for College Technology 12 Study Guide and Exercise Book

Chapter 1 Trigonometric Ratios

1.1 Sine, Cosine, and Tangent of Special Angles

1. a) $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

b) $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

c) $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

2. a)

θ	$\sin\theta$	
	Exact	**Calculator**
0°	0	0
30°	$\frac{1}{2}$	0.5
45°	$\frac{1}{\sqrt{2}}$	0.707…
60°	$\frac{\sqrt{3}}{2}$	0.866…
90°	1	1

b) Example: Some exact values are identical to the calculated values. Others are the same when evaluated.

3. a)

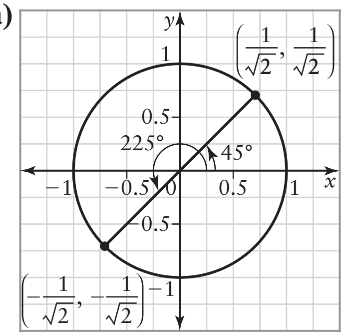
θ	$\cos\theta$		$\tan\theta$	
	Exact	**Calculator**	**Exact**	**Calculator**
0°	1	1	0	0
30°	$\frac{\sqrt{3}}{2}$	0.866…	$\frac{1}{\sqrt{3}}$	0.577…
45°	$\frac{1}{\sqrt{2}}$	0.707…	1	1
60°	$\frac{1}{2}$	0.5	$\sqrt{3}$	1.732…
90°	0	0	undefined	undefined

b) Example: Some exact values are identical to the calculated values. Others are the same when evaluated.

4. a) 30°

b) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$, $\tan 150° = -\frac{1}{\sqrt{3}}$

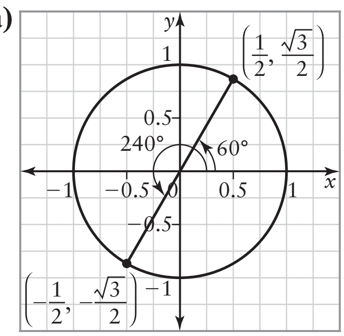
5. a)



b) 45°

c) $\sin 225° = -\frac{1}{\sqrt{2}}$, $\cos 225° = -\frac{1}{\sqrt{2}}$, $\tan 225° = 1$

6. a)



b) Example: 120°, 300°

c) $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\tan 240° = \sqrt{3}$

d) Example: $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$, $\tan 120° = -\sqrt{3}$; $\sin 300° = -\frac{\sqrt{3}}{2}$, $\cos 300° = \frac{1}{2}$, $\tan 300° = -\sqrt{3}$

7. Example:

b) B(0.87, 0.5); 1 unit; 30°

c) B′(0.87, −0.5); 1 unit

d) 1 unit; equilateral

e) OD: 0.87, BD: 0.5

f) OD is equal to the *x*-coordinates of B and B′; BD is equal to the *y*-coordinate of B; B′D is equal to the *y*-coordinate of B′.

g) The relationships in part f) remain.

8. a) $150\sqrt{3}$ m

b) 150 m

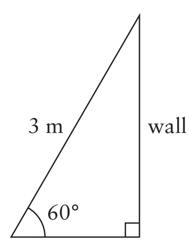
9. a) $6\sqrt{3}$ cm

b) 6 cm

10. a) $\frac{8}{\sqrt{2}}$ ft

b) $\frac{16}{\sqrt{2}}$ ft

11. a)



b) Let *h* represent the height, in metres, from the top of the ladder to the ground. We know the length of the hypotenuse and we want to find the length of the side opposite to 60°. Use the sine ratio.

$\sin 60° = \frac{h}{3}$

Substitute $\sin 60° = \frac{\sqrt{3}}{2}$.

$\frac{\sqrt{3}}{2} = \frac{h}{3}$

Solve for *h*.

$h = \frac{3\sqrt{3}}{2}$

The top of the brace is $\frac{3\sqrt{3}}{2}$ m up the wall.

12. $2\sqrt{3}$ m

13. a) $5\sqrt{2}$ km

b) Pythagorean theorem; let *d* represent the distance between the boats.

$$\begin{aligned} d &= \sqrt{5^2 + 5^2} \\ &= \sqrt{50} \\ &= \sqrt{25}\sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

The boats are $5\sqrt{2}$ km apart.

14. A regular hexagon can be divided into six equilateral triangles. Each equilateral triangle can be divided into two triangles that have angle measures 30°, 60°, 90°.

Determine the exact height of one of these smaller triangles.

$$\begin{aligned} \sin 60° &= \frac{h}{4} \\ h &= 4 \sin 60° \\ &= 4\left(\frac{\sqrt{3}}{2}\right) \\ &= 2\sqrt{3} \end{aligned}$$

Determine the area of the patio.

$$\begin{aligned} A &= 12\left(\tfrac{1}{2}bh\right) \\ &= 6(2)(2\sqrt{3}) \\ &= 24\sqrt{3} \end{aligned}$$

The area of the patio is $24\sqrt{3}$ m^2.

15. b)

Quadrant	Sign of Ratio		
	Sine	**Cosine**	**Tangent**
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative

16.

$$\begin{aligned} &\cos 30° \times \sin 240° + \sin 330° \\ &= \left(\frac{\sqrt{3}}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right) \\ &= -\frac{3}{4} - \frac{2}{4} \\ &= -\frac{5}{4} \end{aligned}$$

17. a) 30°, 330°

b) 30°, 150°

18. $8\sqrt{3}$ m

19.

$$\begin{aligned} \frac{\sin\theta}{\cos\theta} &= \frac{\frac{y}{r}}{\frac{x}{r}} \\ &= \frac{y}{r} \times \frac{r}{x} \\ &= \frac{y}{x} \\ &= \tan\theta \end{aligned}$$

20.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 \\ &= \frac{y^2 + x^2}{r^2} \\ &= \frac{r^2}{r^2} \\ &= 1 \end{aligned}$$

1.2 Sine, Cosine, and Tangent of Angles from 0° to 360°

1. **a)** 0.8480 **b)** −0.8098 **c)** 0.5878 **d)** 0.7193 **e)** 0.6691 **f)** 2.4751 **g)** −0.5317 **h)** −0.9848

2. **a)** $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

b) $\sin\theta = \frac{2}{\sqrt{29}}$, $\cos\theta = -\frac{5}{\sqrt{29}}$, $\tan\theta = -\frac{2}{5}$

c) $\sin\theta = -\frac{7}{\sqrt{65}}$, $\cos\theta = -\frac{4}{\sqrt{65}}$, $\tan\theta = \frac{7}{4}$

d) $\sin\theta = -\frac{5}{\sqrt{61}}$, $\cos\theta = \frac{6}{\sqrt{61}}$, $\tan\theta = -\frac{5}{6}$

e) $\sin\theta = -\frac{2}{\sqrt{68}}$, $\cos\theta = \frac{8}{\sqrt{68}}$, $\tan\theta = -\frac{1}{4}$

3. **a)** $\sin\theta = \frac{8}{\sqrt{89}}$, $\cos\theta = -\frac{5}{\sqrt{89}}$, $\tan\theta = -\frac{8}{5}$

b) $\sin\theta = -\frac{2}{\sqrt{53}}$, $\cos\theta = \frac{7}{\sqrt{53}}$, $\tan\theta = -\frac{2}{7}$

c) $\sin\theta = -\frac{5}{\sqrt{34}}$, $\cos\theta = -\frac{3}{\sqrt{34}}$, $\tan\theta = \frac{5}{3}$

d) $\sin\theta = \frac{3}{\sqrt{13}}$, $\cos\theta = \frac{2}{\sqrt{13}}$, $\tan\theta = \frac{3}{2}$

e) $\sin\theta = -\frac{1}{\sqrt{37}}$, $\cos\theta = \frac{6}{\sqrt{37}}$, $\tan\theta = -\frac{1}{6}$

f) $\sin\theta = -\frac{6}{\sqrt{52}}$, $\cos\theta = \frac{4}{\sqrt{52}}$, $\tan\theta = -\frac{3}{2}$

g) $\sin\theta = \frac{9}{\sqrt{130}}$, $\cos\theta = -\frac{7}{\sqrt{130}}$,
$\tan\theta = -\frac{9}{7}$

h) $\sin\theta = -\frac{3}{\sqrt{34}}$, $\cos\theta = \frac{5}{\sqrt{34}}$, $\tan\theta = -\frac{3}{5}$

4. **a)** $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$

b) Given $\cos B = -\frac{5}{8}$, $x = -5$, and $r = 8$.

$$x^2 + y^2 = r^2$$
$$(-5)^2 + y^2 = 8^2$$
$$25 + y^2 = 64$$
$$y^2 = 39$$
$$y = \pm\sqrt{39}$$

In quadrant III, y is negative.

$\sin B = -\frac{\sqrt{39}}{8}$, $\tan B = \frac{\sqrt{39}}{5}$

c) $\sin C = \frac{6}{\sqrt{61}}$, $\cos C = -\frac{5}{\sqrt{61}}$

d) $\cos D = \frac{\sqrt{65}}{9}$, $\tan D = \frac{4}{\sqrt{65}}$

e) $\sin E = -\frac{\sqrt{105}}{11}$, $\tan E = \frac{\sqrt{105}}{4}$

f) $\sin F = -\frac{6}{\sqrt{61}}$, $\cos F = \frac{5}{\sqrt{61}}$

g) $\sin G = -\frac{10}{\sqrt{109}}$, $\cos G = \frac{3}{\sqrt{109}}$

h) $\sin H = -\frac{\sqrt{56}}{9}$, $\tan H = -\frac{\sqrt{56}}{5}$

5. **a)** $\sin A = \frac{1}{2}$, $\cos A = -\frac{\sqrt{3}}{2}$, $\tan A = -\frac{1}{\sqrt{3}}$

b) $\sin B = 1$, $\cos B = 0$, $\tan B =$ undefined

c) $\sin C = \frac{1}{\sqrt{2}}$, $\cos C = -\frac{1}{\sqrt{2}}$, $\tan C = -1$

d) $\sin D = -\frac{\sqrt{3}}{2}$, $\cos D = \frac{1}{2}$, $\tan D = -\sqrt{3}$

e) $\sin E = -\frac{1}{2}$, $\cos E = \frac{\sqrt{3}}{2}$, $\tan E = -\frac{1}{\sqrt{3}}$

f) $\sin F = -\frac{\sqrt{3}}{2}$, $\cos F = -\frac{1}{2}$, $\tan F = \sqrt{3}$

g) $\sin G = -\frac{1}{\sqrt{2}}$, $\cos G = -\frac{1}{\sqrt{2}}$, $\tan G = 1$

h) $\sin H = 0$, $\cos H = -1$, $\tan H = 0$

6. **a)** $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

b) **i)** (−8, 6)
ii) (−8, −6)
iii) (8, −6)

c) **i)** $\sin(180° - \theta) = \frac{3}{5}$, $\cos(180° - \theta) = -\frac{4}{5}$,
$\tan(180° - \theta) = -\frac{3}{4}$

ii) $\sin(180° + \theta) = -\frac{3}{5}$, $\cos(180° + \theta) = -\frac{4}{5}$,
$\tan(180° + \theta) = \frac{3}{4}$

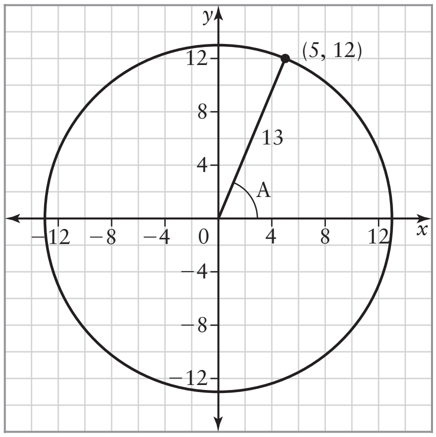
iii) $\sin(360° - \theta) = -\frac{3}{5}$, $\cos(360° - \theta) = \frac{4}{5}$,
$\tan(360° - \theta) = -\frac{3}{4}$

d) The ratios are the same except for the signs. The sine ratio is negative in quadrants III and IV, the cosine ratio is negative in quadrants II and III, and the tangent ratio is negative in quadrants II and IV.

7. **a)** Since $\sin A = \frac{12}{13}$ and ∠A lies in quadrant I, $y = 12$ and $r = 13$.
Determine x.

$$r^2 = x^2 + y^2$$
$$13^2 = x^2 + (12)^2$$
$$169 = x^2 + 144$$
$$25 = x^2$$
$$x = \pm 5$$

Since ∠A lies in quadrant I, $x = 5$.



b) $\cos A = \frac{x}{r} = \frac{5}{13}$ $\tan A = \frac{y}{x} = \frac{12}{5}$

c) If the quadrant was not specified, x could be +5 or −5, so two cosine ratios and two tangent ratios would be possible.

8. $\sin B = \frac{24}{25}$, $\tan B = -\frac{24}{7}$

9. $\sin C = -\frac{7}{\sqrt{65}}$, $\cos C = -\frac{4}{\sqrt{65}}$

10. a) Example: Since $\sin A > 0$ and $\tan A < 0$, ∠A is located in quadrant II.
    b) $\cos A = -\frac{40}{41}$
    c) quadrant III; $\cos A = -\frac{40}{41}$
    d) quadrant IV; $\cos A = \frac{40}{41}$

11. $\cos D = \frac{3}{\sqrt{34}}$, $\tan D = -\frac{5}{3}$

12. a) 135°
    b) 60°
    c) 45°
    d) 310°
    e) 245°
    f) 130°

13. a) Yes; 180° − 135° = 360° − 315°
    b) Yes; 360° − 320° = 40°
    c) No; 215° − 180° ≠ 45°
    d) Yes; 180° − 150° = 210° − 180°
    e) Yes; 210° − 180° = 360° − 330°
    f) No; 180° − 125° ≠ 245° − 180°

14. c) Example: $\sin A = \frac{y}{r}$, $\cos A = \frac{x}{r}$, $\tan A = \frac{y}{x}$

15. a) $\sin\theta = -\frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$, $\tan\theta = 1$
    b) $\sin\theta = -\frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = 2$
    c) $\sin\theta = -\frac{4}{\sqrt{17}}$, $\cos\theta = -\frac{1}{\sqrt{17}}$, $\tan\theta = 4$

16. The tangent ratio is equal to the slope of the line through the terminal arm.

17. a) 8.7 m b) 5.0 m

1.3 Trigonometry of Angles

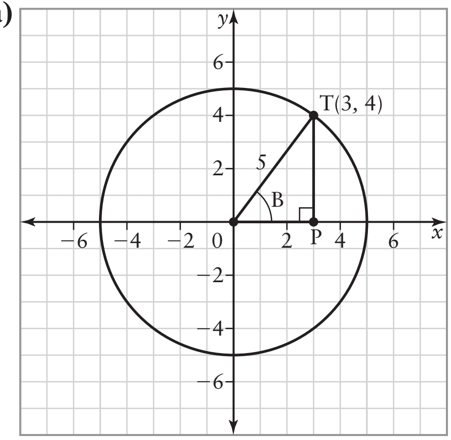
1. a) 0.8480, 0.8480
   b) −0.4540, −0.4540
   c) 1.6003, 1.6003
   d) −0.7660, −0.7660
   e) 0.1392, 0.1392
   f) 14.3007, 14.3007
   g) 0.9063, 0.9063
   h) −2.9042, −2.9042

2. They are equal; the angles are related angles in quadrants in which the relevant trigonometric ratio has the same sign.

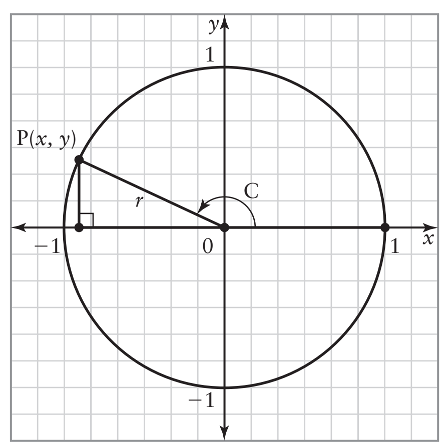
3. a) 0.9998; 91°
   b) 0.9063; 25°
   c) 0.7431; 48°
   d) −0.8391; 320°
   e) −0.9063; 205°
   f) −1.4281; 125°
   g) 0.6018; 53°
   h) 0.2250; 167°

4. a) $\sin\theta = 0.5145$, $\cos\theta = 0.8575$, $\tan\theta = 0.6$
   b) $\sin\theta = 0.8682$, $\cos\theta = -0.4961$, $\tan\theta = -1.75$
   c) $\sin\theta = -0.3162$, $\cos\theta = -0.9487$, $\tan\theta = 0.3333$
   d) $\sin\theta = -0.4472$, $\cos\theta = 0.8944$, $\tan\theta = -0.5$
   e) $\sin\theta = 0.2873$, $\cos\theta = 0.9578$, $\tan\theta = 0.3$
   f) $\sin\theta = -0.8137$, $\cos\theta = -0.5812$, $\tan\theta = 1.4$
   g) $\sin\theta = 0.6$, $\cos\theta = -0.8$, $\tan\theta = -0.75$
   h) $\sin\theta = -0.8944$, $\cos\theta = -0.4472$, $\tan\theta = 2$

5. **a)** sin 80° = 0.9848, cos 80° = 0.1736, tan 80° = 5.6713
   b) sin 110° = 0.9397, cos 110° = −0.3420, tan 110° = −2.7475
   c) sin 200° = −0.3420, cos 200° = −0.9397, tan 200° = 0.3640
   d) sin 324° = −0.5878, cos 324° = 0.8090, tan 324° = −0.7265
   e) sin 47° = 0.7314, cos 47° = 0.6820, tan 47° = 1.0724
   f) sin 192° = −0.2079, cos 192° = −0.9781, tan 192° = 0.2126
   g) sin 217° = −0.6018, cos 217° = −0.7986, tan 217° = 0.7536
   h) sin 345° = −0.2588, cos 345° = 0.9659, tan 345° = −0.2679
   i) sin 13° = 0.2250, cos 13° = 0.9744, tan 13° = 0.2309
   j) sin 270° = −1, cos 270° = 0, tan 270° = undefined
6. **a)** $\theta = 60°, 120°$
   b) $\theta = 45°, 315°$
   c) $\theta = 60°, 240°$
   d) $\theta = 90°$
   e) $\theta = 30°, 330°$
   f) $\theta = 45°, 225°$
7. Determine the reference angle in quadrant I for which $\sin \theta = \frac{\sqrt{3}}{2}$.
   Use the special triangles.
   $\sin \theta = \frac{\sqrt{3}}{2}$ when $\angle\theta = 60°$.
   Since the sine ratio is positive in quadrants I and II, there is another angle in quadrant II for which $\sin \theta = \frac{\sqrt{3}}{2}$.
   $\angle\theta = 180° - 60°$
   $= 120°$
   The angles are 60° and 120°.
8. 120°, 240°
9. 150°, 330°
10. 135°, 315°
11. **a)**



   b) Example: Substitute the values $x = 3$ and $y = 4$ into $x^2 + y^2 = r^2$ to determine that $r = 5$. Then, use the values of x, y, and r to determine the three primary trigonometric ratios for ∠B.
   c) $\sin B = \frac{4}{5}$, $\cos B = \frac{3}{5}$, $\tan B = \frac{3}{4}$
   d) Example: Use the inverse of one of the primary trigonometric ratios, for example, $\angle B = \tan^{-1}\left(\frac{3}{4}\right)$, and the fact that ∠B is in quadrant I to find the measure of ∠B to the nearest degree.
   e) ∠B = 53°
   f) $\sin B = -\frac{4}{5}$, $\cos B = \frac{3}{5}$, $\tan B = -\frac{4}{3}$
   g) $\sin B = \frac{4}{5}$, $\cos B = -\frac{3}{5}$, $\tan B = -\frac{4}{3}$
12. **a)**



   b) $\tan^{-1}(0.4663) = 25°$, so in quadrant III, ∠C = 205º.

13. a) $\theta = 47°, 133°$ **b)** $\theta = 63°, 297°$
c) $\theta = 120°, 300°$ **d)** $\theta = 72°, 108°$
e) $\theta = 28°, 332°$ **f)** $\theta = 45°, 225°$
g) $\theta = 227°, 313°$ **h)** $\theta = 111°, 249°$
i) $\theta = 68°, 248°$ **j)** $\theta = 252°, 288°$
k) $\theta = 79°, 281°$ **l)** $\theta = 3°, 183°$

14. a) 105° **b)** 170° **c)** 175° **d)** 288°

15. a) cos A = 0.4062, tan A = 2.2498
b) sin B = 0.9767, tan B = −4.5535
c) sin C = −0.9925, cos C = 0.1219

16. a) 19°, 161° **b)** 55°, 235° **c)** 138°, 222°

17. a) Since $\cos\theta = \frac{1}{3}$, $x = 1$ and $r = 3$.
Determine y.

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 3^2$$
$$y^2 = 8$$
$$y = \pm\sqrt{8}$$

Therefore, $\sin\theta = \frac{\sqrt{8}}{3}$ or $\sin\theta = -\frac{\sqrt{8}}{3}$.

b) If $\sin\theta = \frac{\sqrt{8}}{3}$,

$$\theta = \sin^{-1}\left(\frac{\sqrt{8}}{3}\right) = 70.528...° \doteq 71°$$

The sine ratio is positive in quadrants I and II. When $\sin\theta = \frac{\sqrt{8}}{3}$, $\theta = 71°$ or $109°$.

If $\sin\theta = -\frac{\sqrt{8}}{3}$,

$$\theta = \sin^{-1}\left(-\frac{\sqrt{8}}{3}\right) = 289.471...° \doteq 289°$$

The sine ratio is negative in quadrants III and IV. When $\sin\theta = -\frac{\sqrt{8}}{3}$, $\theta = 251°$ or $289°$.

18. a) Since S is a point on the terminal arm of $\angle A$, $x = -5$ and $y = -6$. Determine r.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (-6)^2} = \sqrt{61}$$

$$\sin A = \frac{y}{r} = \frac{-6}{\sqrt{61}} = -\frac{6}{\sqrt{61}}, \quad \cos A = \frac{x}{r} = \frac{-5}{\sqrt{61}} = -\frac{5}{\sqrt{61}}, \quad \tan A = \frac{y}{x} = \frac{-6}{-5} = \frac{6}{5}$$

b) Determine the reference angle, θ, in quadrant I.

$$\theta = \tan^{-1}\left(\frac{6}{5}\right) = 50.194...° \doteq 50°$$

The point S(−5, −6) is in quadrant III.

$$\angle A = 180° + \angle\theta = 180° + 50° = 230°$$

c) From part a), $\sin A = -\frac{6}{\sqrt{61}}$. The sine ratio is negative in quadrants III and IV. Since the point S(−5, −6) is in quadrant III, $\angle B$ must be in quadrant IV. The point T(5, −6) is on the terminal arm of $\angle B$ in quadrant IV. Given T(5, −6), $x = 5$, and $y = -6$. From part a), $r = \sqrt{61}$.

$$\sin B = \frac{y}{r} = \frac{-6}{\sqrt{61}} = -\frac{6}{\sqrt{61}}, \quad \cos B = \frac{x}{r} = \frac{5}{\sqrt{61}}, \quad \tan B = \frac{y}{x} = \frac{-6}{5} = -\frac{6}{5}$$

d) Determine the reference angle, θ, in quadrant I.

$$\angle\theta = \tan^{-1}\left(\frac{6}{5}\right) = 50.194...° \doteq 50°$$

The point T(5, −6) is in quadrant IV.

$$\angle B = 360° - \angle\theta = 360° - 50° = 310°$$

19. a)

$$2x^2 - x - 1 = 0$$
$$(2x + 1)(x - 1) = 0$$
$$x = -\frac{1}{2} \text{ or } 1$$

b) If $x = \sin\theta$ is substituted into $2x^2 - x - 1 = 0$, the result is $2\sin^2\theta - \sin\theta - 1 = 0$.

c)

$$2\sin^2\theta - \sin\theta - 1 = 0$$
$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$
$$\sin\theta = -\frac{1}{2} \text{ or } 1$$

When $\sin\theta = -\frac{1}{2}$, $\theta = 210°$ or $330°$.
When $\sin\theta = 1$, $\theta = 90°$.

20. a) 0°, 180°, 360° **b)** 60°, 120°, 240°, 300°

21. $\sin \theta = \frac{a + b}{\sqrt{2a^2 + 2b^2}}$, $\cos \theta = \frac{a - b}{\sqrt{2a^2 + 2b^2}}$; $a \neq b$

1.4 Solving Problems Using Primary Trigonometric Ratios

1. a) 9.4 m **b)** 15.1 m **c)** 13.1 km
d) 9.4 m **e)** 32.3 m **f)** 13.6 km

2. a) $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$; $\theta = 37°$

b) $\sin \theta = -\frac{4}{\sqrt{65}}$, $\cos \theta = \frac{7}{\sqrt{65}}$, $\tan \theta = \frac{4}{7}$; $\theta = 30°$

c) $\sin \theta = \frac{7}{\sqrt{58}}$, $\cos \theta = \frac{3}{\sqrt{58}}$, $\tan \theta = \frac{7}{3}$; $\theta = 67°$

d) $\sin \theta = \frac{2}{\sqrt{29}}$, $\cos \theta = \frac{5}{\sqrt{29}}$, $\tan \theta = \frac{2}{5}$; $\theta = 22°$

e) $\sin \theta = \frac{\sqrt{133}}{13}$, $\cos \theta = \frac{6}{13}$, $\tan \theta = \frac{\sqrt{133}}{6}$; $\theta = 63°$

f) $\sin \theta = \frac{15}{\sqrt{369}}$, $\cos \theta = \frac{12}{\sqrt{369}}$, $\tan \theta = \frac{5}{4}$; $\theta = 51°$

3. a) ∠C = 54°, a = 26.0 m, c = 21.1 m
b) ∠F = 42°, d = 12.9 cm, f = 8.6 cm
c) ∠L = 28°, k = 6.4 km, l = 3.4 km
d) ∠R = 90°, q = 10.2 m, r = 16.9 m

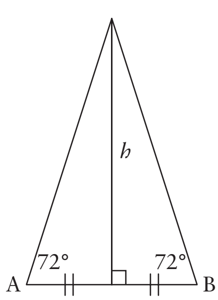
4. 3.9 m

5. 54°

6. The lengths of any two sides *or* the measure of one other angle and the length of any side.

7. a) 20 cm **b)** 37°

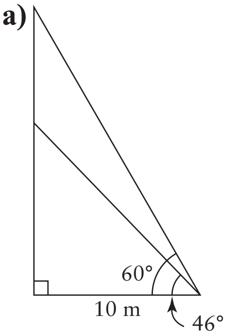
8. a)



b) 31 ft **c)** 65 ft

9. 5.4 m

10. a)



b) $\tan 46° = \frac{h_1}{10}$ $\tan 60° = \frac{h_2}{10}$

$h_1 = 10 \tan 46°$ $h_2 = 10 \tan 60°$
$= 10.355...$ $= 17.320...$
$\doteq 10.4$ $\doteq 17.3$

When the angle of inclination was 46°, the height of the water was approximately 10.4 m. When the angle of inclination was 60°, the height of the water was approximately 17.3 m.

c) 17.3 – 10.4 = 6.9
The water climbed 6.9 m in 15 s, so its average speed was $\frac{6.9}{15}$, or 0.46 m/s.

11. Example: Measure a distance from the base of the tower and measure the angle of inclination to the top of the antenna. Use the tangent ratio to calculate the height to the top of the antenna.

12. $\sin 62° = \frac{w_1}{18.9}$ $\sin 37° = \frac{w_2}{12.6}$

$w_1 = 18.9 \sin 62°$ $w_2 = 12.6 \sin 37°$

$w = w_1 + w_2$
$= 18.9 \sin 62° + 12.6 \sin 37°$
$= 24.270…$
$\doteq 24.3$

The patio is approximately 24.3 ft wide.

13. a) 8.6 km
b) 90°

14. a) 11.3 cm
b) 70°

15. a) 1.3 m
b) 3.9 m^3

16. 8.1 ft

17. 23.7 m

18. 57.3 m

19. a) Example: Since, △ABD is isosceles, ∠BAD = ∠BDA, which is 45°. Use this information to determine the length of AB and then the height, BC, of the cliff.

b) $\frac{30\sqrt{3}}{\sqrt{2}}$ ft

20. 11.0 m

21. 14.0 ft

1.5 Solving Problems Using the Sine Law

1. a) 15.2 cm **b)** 11.3 m
c) 43.5 km **d)** 14.7 m

2. a) $\theta = 23°$ **b)** $\theta = 61°$

3. a) 10.4 m
b) 19.0 cm

4. a) ∠C = 61°, b = 11.5 km, c = 13.8 km
b) ∠E = 64°, e = 7.9 m, f = 6.9 m
c) ∠J = 66°, ∠L = 62°, j = 29.0 cm
d) ∠P = 28°, ∠Q = 117°, q = 57.5 m

5. a) 7.8
b) no solution

6. a) 3.1
b) two solutions
c) ∠B = 44°, ∠C = 99°, c = 7.4 cm or ∠B = 136°, ∠C = 7°, c = 0.9 cm

7. a) one solution
b) ∠Q = 90°, ∠R = 60°, r = 9.2 km

8. Since two sides and one angle are known, and since $m < n$, check for the ambiguous case.

$$\begin{aligned} LS &= m & RS &= n \sin M \\ &= 6.7 & &= 12.4 \sin 26° \\ & & &= 5.435\ldots \end{aligned}$$

LS > RS, so two triangles are possible.

Use the sine law to determine acute $\angle N_1$.

$$\begin{aligned} \frac{\sin N_1}{n} &= \frac{\sin M}{m} \\ \frac{\sin N_1}{12.4} &= \frac{\sin 26°}{6.7} \\ \sin N_1 &= \frac{12.4 \sin 26°}{6.7} \\ \angle N_1 &= 54.224\ldots° \\ &\doteq 54.2\ ° \end{aligned}$$

If $\angle N_1 = 54.2°$, determine $\angle P_1$.

$$\begin{aligned} \angle P_1 &= 180° - (26° + 54.2°) \\ &= 99.8° \end{aligned}$$

Determine p_1.

$$\begin{aligned} \frac{p_1}{\sin P_1} &= \frac{m}{\sin M} \\ \frac{p_1}{\sin 99.8°} &= \frac{6.7}{\sin 26°} \\ p_1 &= 15.060\ldots \\ &\doteq 15.1 \end{aligned}$$

Determine obtuse $\angle N_2$.

$$\begin{aligned} \angle N_2 &= 180° - \angle N_1 \\ &= 180° - 54.2° \\ &= 125.8° \end{aligned}$$

If $\angle N_2 = 125.8°$, determine $\angle P_2$.

$$\begin{aligned} \angle P_2 &= 180° - (26° + 125.8°) \\ &= 28.2° \end{aligned}$$

Determine p_2.

$$\begin{aligned} \frac{p_2}{\sin P_2} &= \frac{m}{\sin M} \\ \frac{p_2}{\sin 28.2°} &= \frac{6.7}{\sin 26°} \\ p_2 &= 7.223\ldots \\ &\doteq 7.2 \end{aligned}$$

In △MNP, the length of side p could be approximately 7.2 cm or 15.1 cm.

9. a) Since $b > c$, only one triangle can be constructed.
a = 17.0 cm

b) Since $r < s$ and $r > s \sin R$, two triangles can be constructed.
t = 9.8 m or t = 2.8 m

10. Determine the measure of ∠C.

$$\begin{aligned} \angle C &= 180° - 38° - 42° \\ &= 100° \end{aligned}$$

Determine the length of side b.

$$\begin{aligned} \frac{b}{\sin B} &= \frac{c}{\sin C} \\ \frac{b}{\sin 42°} &= \frac{10\ 560}{\sin 100°} \\ b &= \frac{10\ 560 \sin 42°}{\sin 100°} \\ b &= 7175.023\ldots \\ b &\doteq 7175.0 \end{aligned}$$

The length of side b is 7175 ft.

Determine the height, h, of the mountain.

$$\begin{aligned} \frac{h}{7175} &= \sin 38° \\ h &= 7175 \sin 38° \\ &= 4417.371\ldots \\ &\doteq 4417.4 \end{aligned}$$

The height of the mountain is approximately 4417.4 ft.

11. Determine the angle at the beginning of the delta.

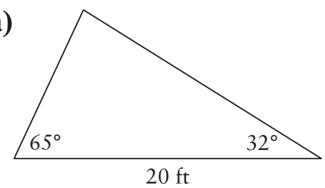
$\angle\theta = 180° - 58° - 47°$
$= 75°$

Determine the length of the delta along the coastline.

$$\frac{x}{\sin 75°} = \frac{80}{\sin 47°}$$
$$x = \frac{80 \sin 75°}{\sin 47°}$$
$$= 105.658...$$
$$\doteq 105.7$$

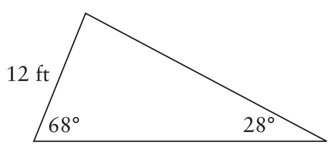
The length of the delta along the coastline is approximately 105.7 km.

12. a)



b) 10.7 ft

13. a)



b) 23.7 ft

14. a) 126.4 m **b)** 126.6 m

c) Example: The calculated heights of the rocket are different; they depend on the accuracy of the measurements of the angles of elevation of the rocket.

15. 51.1 ft

1.6 Solving Problems Using the Cosine Law

1. **a)** 14.0 cm **b)** 19.6 km **c)** 14.2 m **d)** 7.4 cm **e)** 10.4 cm

2. **a)** 86° **b)** 80° **c)** 131° **d)** 38°

3. **a)** 12.2 cm **b)** 8.8 m

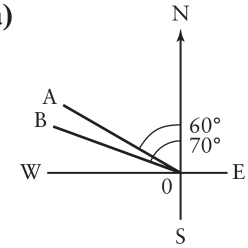
4. **a)** $\angle B = 90°$, $\angle C = 42°$, $a = 13.4$ cm

b) $\angle J = 52.3°$, $\angle K = 59.4°$, $\angle L = 68.3°$

c) $\angle P = 47°$, $\angle Q = 56°$, $\angle R = 77°$

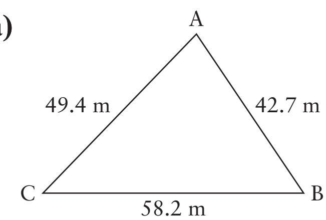
d) $\angle E = 68°$, $\angle F = 50°$, $d = 11.5$ m

5. **a)**



b) 4.5 km

6. **a)**



b) Determine $\angle A$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{49.4^2 + 42.7^2 - 58.2^2}{2(49.4)(42.7)}$$
$$\angle A = 78°$$

Determine $\angle B$.

$$\frac{\sin B}{49.4} = \frac{\sin 78°}{58.2}$$
$$\sin B = \frac{49.4 \sin 78°}{58.2}$$
$$\angle B = 56°$$

Determine $\angle C$.

$$\angle C = 180° - 78° - 56°$$
$$= 46°$$

7. 3.7 in.

8. Ship A is N30°W and Ship B is S40°W.

Determine $\angle ACB$.

$$\angle C = (90° - 30°) + (90° - 40°)$$
$$= 60° + 50°$$
$$= 110°$$

Determine the distance AB.

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$= 60^2 + 40^2 - 2(60)(40) \cos 110°$$
$$= 6841.7$$
$$c \doteq 82.7$$

The distance is approximately 82.7 km.

9. Determine the distance BC.

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 3^2 + 5^2 - 2(3)(5) \cos 25°$

$= 6.810\ldots$

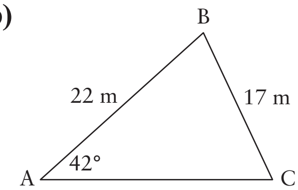
$a \doteq 2.6$

The distance is approximately 2.6 km.

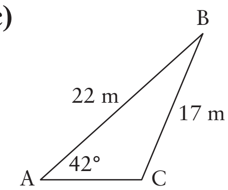
10. 10°

11. a) Example: Both calculations are correct.

b)



c)



d) Example: This is an example of the ambiguous case. Since $a < b$, calculate $b \sin A$, which is 14.7. Since $a > b \sin A$, two triangles are possible. Leslie used one triangle for her calculations; Kelly used the other triangle.

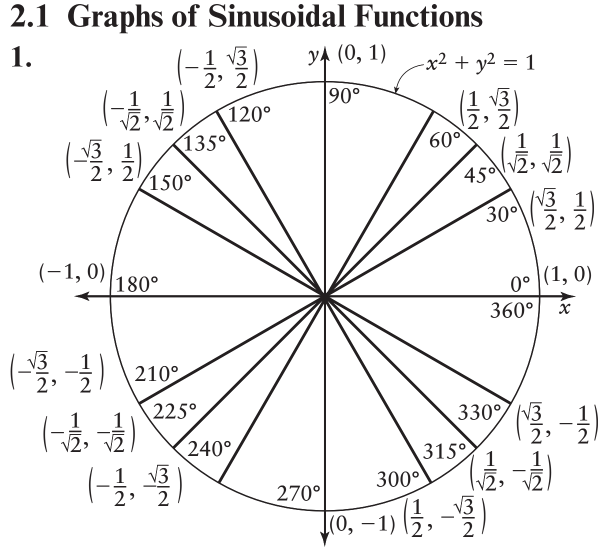
12. a) 47.3 m **b)** 77.0 m

13. 3.4 mm **14.** 120.4 m **15.** 98.6 km

Chapter 2 Sinusoidal Functions

2.1 Graphs of Sinusoidal Functions

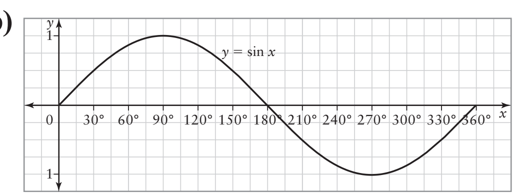
1.



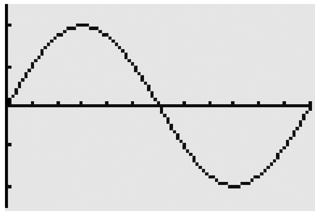
2. a)

x	$y = \sin x$ Exact	$y = \sin x$ Approximate
0°	0	0
30°	$\frac{1}{2}$	0.500
45°	$\frac{1}{\sqrt{2}}$	0.707
60°	$\frac{\sqrt{3}}{2}$	0.866
90°	1	1
120°	$\frac{\sqrt{3}}{2}$	0.866
135°	$\frac{1}{\sqrt{2}}$	0.707
150°	$\frac{1}{2}$	0.500
180°	0	0
210°	$-\frac{1}{2}$	−0.500
225°	$-\frac{1}{\sqrt{2}}$	−0.707
240°	$-\frac{\sqrt{3}}{2}$	−0.866
270°	−1	−1
300°	$-\frac{\sqrt{3}}{2}$	−0.866
315°	$-\frac{1}{\sqrt{2}}$	−0.707
330°	$-\frac{1}{2}$	−0.500
360°	0	0

b)



c) Example: The graph will have the same shape past 360°.

3. a) 

b) Example: Press **TRACE** and use the cursor keys to move along the graph. Or, press **2nd**, **GRAPH** to view a table of values. Or, press **2nd**, **TRACE**, 1, enter an x-value, and then press **ENTER**.

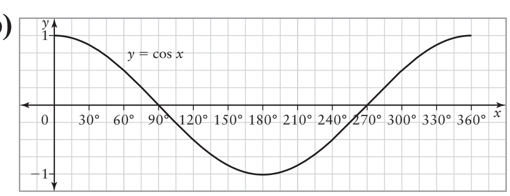
c)

x	$y = \sin x$ Approximate
0°	0
30°	0.5
45°	0.7071
60°	0.8660
90°	1
120°	0.8660
135°	0.7071
150°	0.5
180°	0
210°	–0.5
225°	–0.7071
240°	–0.8660
270°	–1
300°	–0.8660
315°	–0.7071
330°	–0.5
360°	0

d) The values are the same.

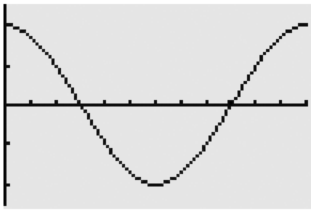
4. a)

x	$y = \cos x$ Exact	Approximate
0°	1	1
30°	$\frac{\sqrt{3}}{2}$	0.866
45°	$\frac{1}{\sqrt{2}}$	0.707
60°	$\frac{1}{2}$	0.500
90°	0	0
120°	$-\frac{1}{2}$	–0.500
135°	$-\frac{1}{\sqrt{2}}$	–0.707
150°	$-\frac{\sqrt{3}}{2}$	–0.866
180°	–1	–1
210°	$-\frac{\sqrt{3}}{2}$	–0.866
225°	$-\frac{1}{\sqrt{2}}$	–0.707
240°	$-\frac{1}{2}$	–0.500
270°	0	0
300°	$\frac{1}{2}$	0.500
315°	$\frac{1}{\sqrt{2}}$	0.707
330°	$\frac{\sqrt{3}}{2}$	0.866
360°	1	1

b) 

c) Example: The graph will have the same shape past 360°.

5. a)



b) Example: Press **TRACE** and use the cursor keys to move along the graph. Or, press **2nd**, **GRAPH** to view a table of values. Or, press **2nd**, **TRACE**, 1, enter an x-value, and then press **ENTER**.

c)

x	$y = \cos x$ Approximate
0°	1
30°	0.8660
45°	0.7071
60°	0.5
90°	0
120°	–0.5
135°	–0.7071
150°	–0.8660
180°	–1
210°	–0.8660
225°	–0.7071
240°	–0.5
270°	0
300°	0.5
315°	0.7071
330°	0.8660
360°	1

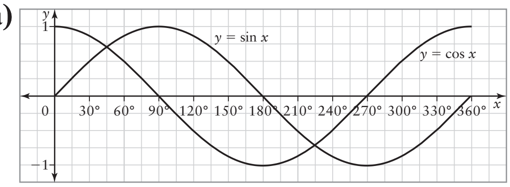
d) The values are the same.

6. a) Example: The graphs of $y = \sin x$ and $y = \cos x$ have the same period, amplitude, and maximum/minimum values, but they have different x- and y-intercepts.

b) Yes; each x-value corresponds with exactly one y-value.

c) Yes; each x-value corresponds with exactly one y-value.

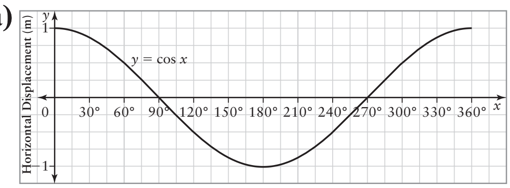
7. a)



b) 2

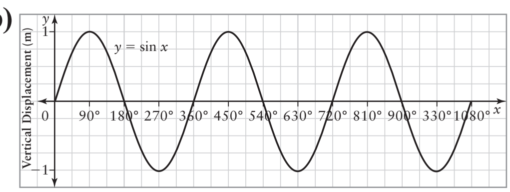
c) The functions have the same values at 45° and 225° because the x-coordinates are the same as the y-coordinates at these points.

8. a)

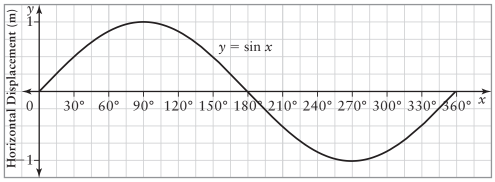


A cosine function models the horizontal displacement, because the horizontal displacement starts at 1 m and decreases to 0 m at 90°, a characteristic of the cosine function.

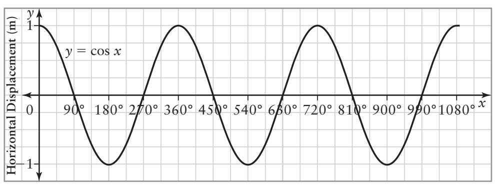
b)



A sine function models the vertical displacement, because the vertical displacement starts at 0 m and increases to 1 m at 90°, a characteristic of the sine function.

9. a) 

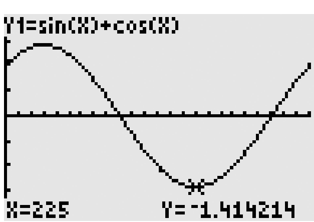
A sine function models the horizontal displacement, because the horizontal displacement starts at 0 and increases to 1, a characteristic of the sine function.

b) 

A cosine function models the vertical displacement, because the vertical displacement starts at 1 and decreases to 0, a characteristic of the cosine function.

c) 3

10. In question 8, the horizontal displacement was modelled by a cosine function. In this case, the rotation was counterclockwise and the angle was measured relative to the horizontal. In question 9, the horizontal displacement was modelled by a sine function. In this case, the rotation was clockwise and the angle was measured relative to the vertical.

11. 

a) sinusoidal

b) There is a maximum of $\sqrt{2}$, approximately 1.414, when $x = 45°$; minimum of $-\sqrt{2}$, approximately –1.414, when $x = 225°$.

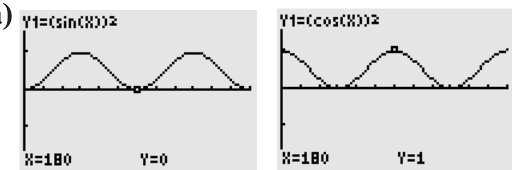
c) $\sqrt{2}$

d) 1

e) 135° and 315°

f) Each x-intercept is the mean of the x-coordinates of the maximum/minimum on either side.

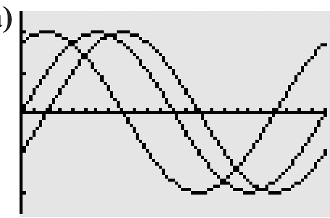
g) Add the coordinates of each point.

12. a) 

b) Example: Both graphs have the same amplitude, maximum, minimum, period, domain, and range, but they have different x- and y-intercepts.

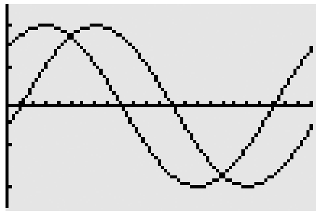
c) Example: The relations $y = (\sin x)^2$ and $y = (\cos x)^2$ are functions. Use the vertical line test to show that a vertical line moved from the left of the graph to the right of either of the graphs will not intersect the graph more than once for any of the x-values.

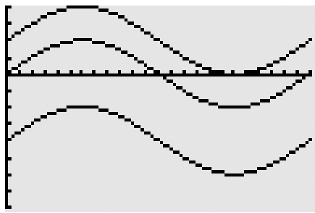
2.2 Translations of Sinusoidal Functions

1. a) 

b) The graph of $g(x)$ is the image of $f(x)$ after a translation of 30° right, and $h(x)$ is the image of $f(x)$ after a translation of 60° left. The function $g(x)$ has a y-intercept of –0.5 and x-intercepts at 30° and 210°, whereas $h(x)$ has a y-intercept of $\frac{\sqrt{3}}{2}$ and x-intercepts at 120° and 300°.

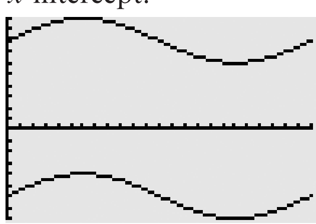
c) The graph of $p(x)$ will be the image of $f(x)$ after a translation of 15° right, and $q(x)$ will be the image of $f(x)$ after a translation of 45° left. The function $p(x)$ has a y-intercept of approximately –25.9 and x-intercepts at 15° and 195°, whereas $q(x)$ has a y-intercept of $\frac{1}{\sqrt{2}}$ and x-intercepts at 45° and 225°.

d) 

2. a) 

b) The graph of $c(x)$ is the image of $b(x)$ after a translation of 1 up. The graph of $d(x)$ is the image of $b(x)$ after a translation of 2 down. The function $c(x)$ has a y-intercept of 1 and an x-intercept at 270°, whereas the function $d(x)$ has a y-intercept of –2 and no x-intercept.

c) The graph of $f(x)$ will be the image of $b(x)$ after a translation of 3 down. The graph of $g(x)$ will be the image of $b(x)$ after a translation of 4 up. The function $f(x)$ has a y-intercept of –3 and no x-intercept, whereas the function $g(x)$ has a y-intercept of 4 and no x-intercept.

d) 

3 . a) 60° **b)** –90° **c)** –30°
d) –45° **e)** 90° **f)** 45°

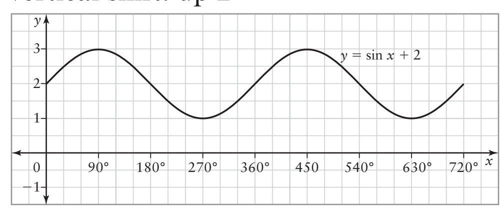
4. a) 3; $\{y \in \mathbb{R}, 2 \leq y \leq 4\}$
b) 4; $\{y \in \mathbb{R}, 3 \leq y \leq 5\}$
c) –5; $\{y \in \mathbb{R}, -6 \leq y \leq -4\}$
d) –6; $\{y \in \mathbb{R}, -7 \leq y \leq -5\}$
e) –2; $\{y \in \mathbb{R}, -3 \leq y \leq -1\}$
f) 1; $\{y \in \mathbb{R}, 0 \leq y \leq 2\}$

5. a) phase shift: left 46°; vertical shift: up 2
b) phase shift: left 72°; vertical shift: down 3
c) phase shift: right 65°; vertical shift: up 4
d) phase shift: right 41°; vertical shift: down 5
e) phase shift: left 27°; vertical shift: down 6
f) phase shift: right 80°; vertical shift: up 1

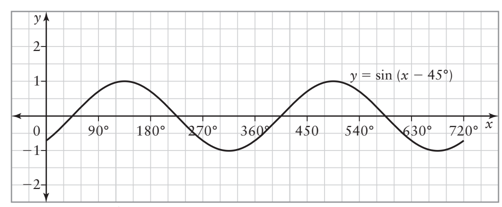
6. a) phase shift: right 32°; vertical shift: up 2
b) phase shift: left 55°; vertical shift: down 4
c) phase shift: left 73°; vertical shift: down 7
d) phase shift: right 42°; vertical shift: up 3
e) phase shift: right 18°; vertical shift: down 1
f) phase shift: left 64°; vertical shift: down 2

7. Example: First, determine the phase shift, and then apply the phase shift to the x-intercepts, the maxima, and the minima of the basic function. Next, determine the vertical shift and apply it to the key points.

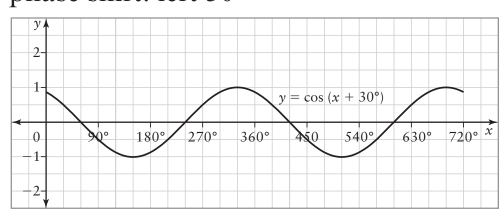
8. a) vertical shift: up 2



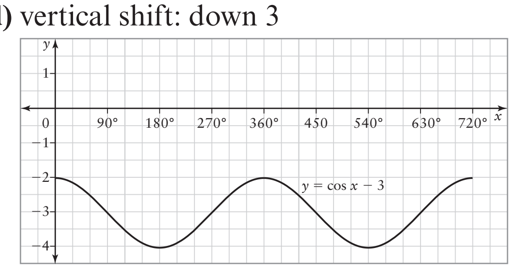
b) phase shift: right 45°



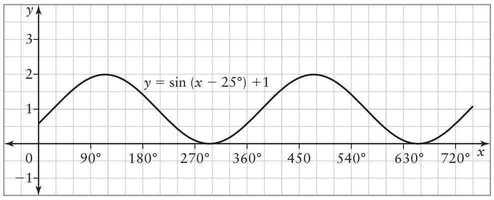
c) phase shift: left 30°



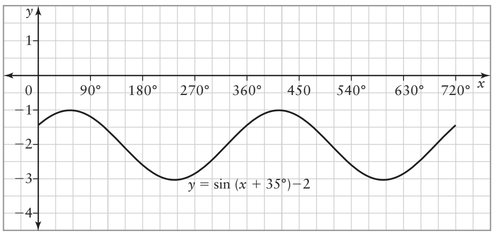
d) vertical shift: down 3



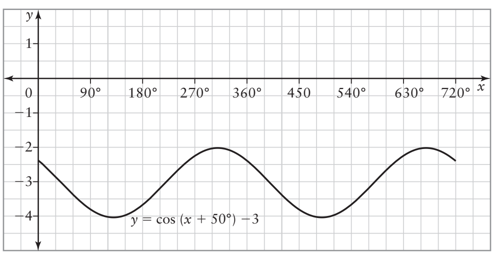
9. a) phase shift: right 25°; vertical shift: up 1



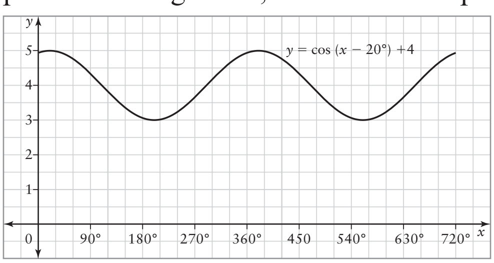
b) phase shift: left 35°; vertical shift: down 2



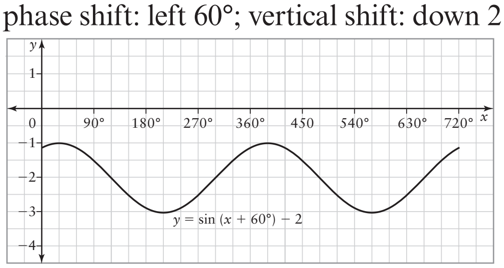
c) For function $y = \cos (x + 50°) - 3$, $d = -50°$, and $c = -3$. Therefore, the function has a phase shift left 50° and a vertical shift down 3.



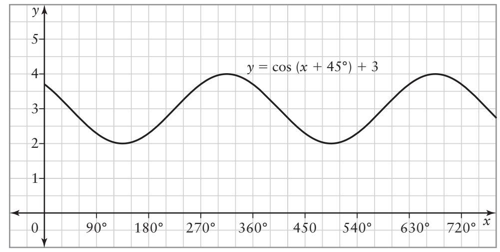
d) phase shift: right 20°; vertical shift: up 4



e) phase shift: left 60°; vertical shift: down 2



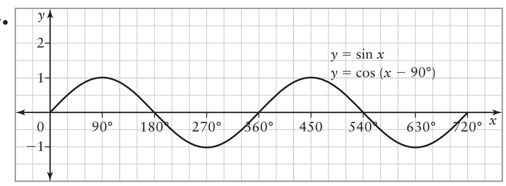
f) phase shift: left 45°; vertical shift: up 3



10. Example: If the constant is in brackets with the independent variable, it represents a phase shift. If the constant is outside the brackets, it represents a vertical shift.

11. a) $y = \sin (x + 58°) - 4$
b) $y = \cos (x - 67°) + 5$
c) $y = \cos (x + 41°) - 8$
d) $y = \sin (x - 15°) + 2$

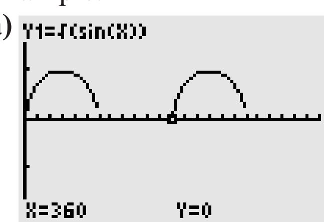
12.



The sine function is the same as a cosine function that has been shifted right 90°.

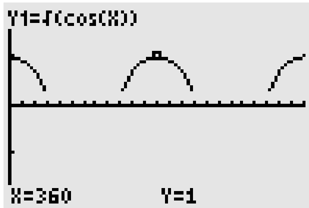
13. Example:
a) $y = \sin (x - 60°)$; $y = \cos (x - 150°)$
b) $y = \sin (x + 30°) + 1$; $y = \cos (x - 60°) + 1$
c) $y = \sin (x - 45°) - 2$; $y = \cos (x - 135°) - 2$

14. Example:
a)



b) i) The graph of $y = \sqrt{\sin x} + 2$ is the graph of $y = \sqrt{\sin x}$ translated up 2 units.
ii) The graph of $y = \sqrt{\sin (x - 60°)}$ is the graph of $y = \sqrt{\sin x}$ translated right 60°.

15. Example:

a) 

b) i) The graph of $y = \sqrt{\cos x} - 3$ is the graph of $y = \sqrt{\cos x}$ translated down 3 units.

ii) The graph of $y = \sqrt{\cos (x + 45°)}$ is the graph of $y = \sqrt{\cos x}$ translated left 45°.

2.3 Stretches, Compressions, and Reflections of Sinusoidal Functions

1. **a)** 3 **b)** 4 **c)** $\frac{1}{2}$
d) $\frac{2}{3}$ **e)** 3 **f)** 2

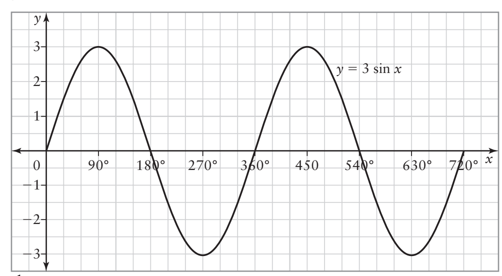
2. **a)** k: 2; period: 180°
b) k: 5; period: 72°
c) k: −4; period: 90°
d) k: −3; period: 120°
e) k: $\frac{1}{2}$; period: 720°
f) k: $\frac{3}{5}$; period: 600°

3. **a)** vertical stretch by a factor of 3; horizontal compression by a factor of $\frac{1}{4}$; amplitude of 3; period of 90°
b) vertical stretch by a factor of 2; horizontal compression by a factor of $\frac{1}{3}$; amplitude of 2; period of 120°
c) no vertical stretch; horizontal stretch by a factor of 2; amplitude of 1; period of 720°
d) vertical compression by a factor of $\frac{1}{3}$; horizontal compression by a factor of $\frac{1}{6}$; amplitude of $\frac{1}{3}$; period of 60°
e) vertical compression by a factor of $\frac{1}{4}$; horizontal compression by a factor of $\frac{1}{4}$; amplitude of $\frac{1}{4}$; period of 90°
f) vertical compression by a factor of $\frac{2}{5}$; horizontal stretch by a factor of 3; amplitude of $\frac{2}{5}$; period of 1080°
g) vertical stretch by a factor of 4; horizontal stretch by a factor of 5; amplitude of 4; period of 1800°
h) vertical compression by a factor of $\frac{1}{2}$; horizontal stretch by a factor of 4; amplitude of $\frac{1}{2}$; period of 1440°

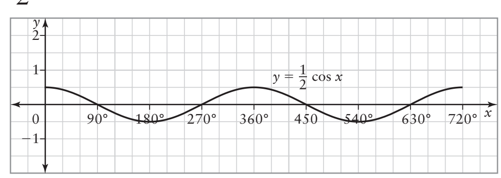
4. Example: If $0 < a < 1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are compressed vertically by a factor of $\frac{1}{a}$. If $a > 1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are stretched vertically by a factor of a. If $a = -1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are reflected in the x-axis.

5. Example: If $0 < k < 1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are stretched horizontally by a factor of k. If $k > 1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are compressed horizontally by a factor of $\frac{1}{k}$. If $k = -1$, the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are reflected in the y-axis.

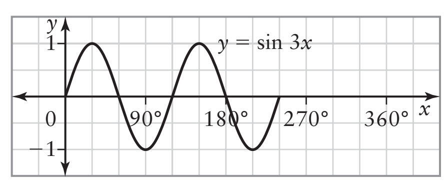
6. **a)** 3



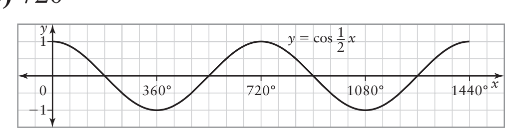
b) $\frac{1}{2}$



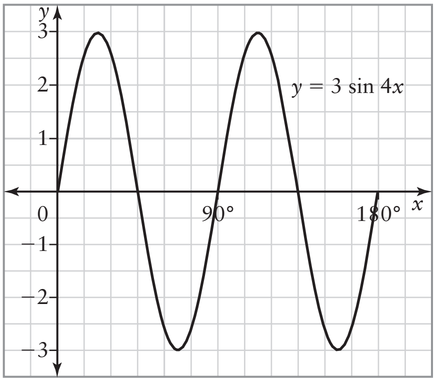
7. **a)** 120°



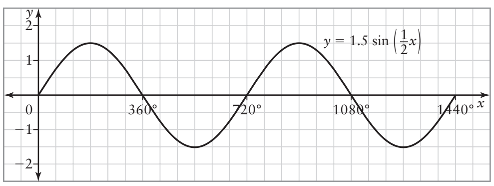
b) 720°



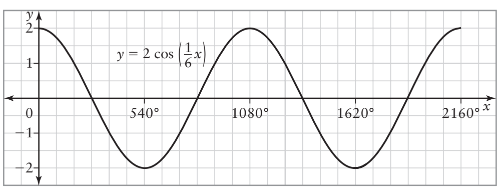
8. **a)** amplitude: 3; period: 90°



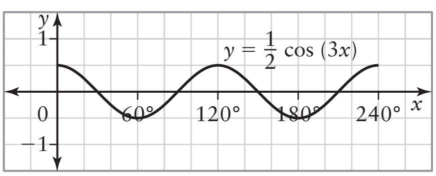
b) amplitude: 1.5; period: 720°



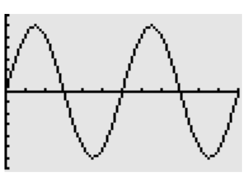
c) amplitude: 2; period: 2160°



d) amplitude: $\frac{1}{2}$; period: 120°

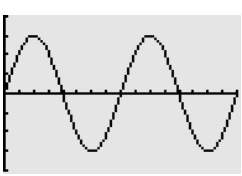


9. **a)**



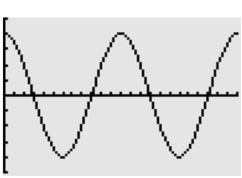
WINDOW
Xmin=0
Xmax=180
Xscl=15
Ymin=-3.5
Ymax=3.5
Yscl=.5
Xres=1

b)



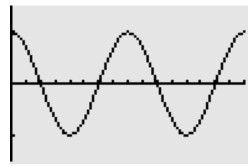
WINDOW
Xmin=0
Xmax=1440
Xscl=90
Ymin=-2
Ymax=2
Yscl=.5
Xres=1

c)



WINDOW
Xmin=0
Xmax=4320
Xscl=180
Ymin=-2.5
Ymax=2.5
Yscl=.5
Xres=1

d)



WINDOW
Xmin=0
Xmax=240
Xscl=15
Ymin=-.75
Ymax=.75
Yscl=.5
Xres=1

10. **a)** x-intercepts and period do not change; amplitude and maximum/minimum values change; y-intercept changes for $y = \cos x$, but does not change for $y = \sin x$

b) x-intercepts and period change; amplitude and maximum/minimum values do not change; y-intercept may change

11. $y = 6\sin\frac{1}{2}x$; there is also a cosine equation: $y = 6\cos\frac{1}{2}(x - 90°)$

12. **a)** $y = \frac{1}{4}\cos 2x$

b) $f(x) = -4\sin 2x$

c) $f(x) = \frac{1}{3}\left(\cos -\frac{1}{5}x\right)$

13. **a)** maximum: $\frac{1}{4}$; minimum: $-\frac{1}{4}$; amplitude: $\frac{1}{4}$; period: 180°; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -\frac{1}{4} \le y \le \frac{1}{4}\}$; x-intercept: 90°; y-intercept: $\frac{1}{4}$

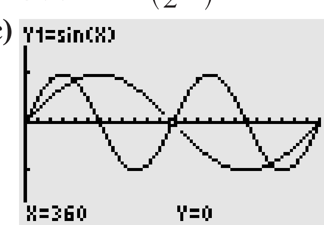
b) maximum: 4; minimum: −4; amplitude: 4; period: 180°; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -4 \le y \le 4\}$; x-intercepts: 0°, 90°, and 180°; y-intercept: 0

c) maximum: $\frac{1}{3}$; minimum: $-\frac{1}{3}$; amplitude: $\frac{1}{3}$; period: 1800°; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -\frac{1}{3} \le y \le \frac{1}{3}\}$; x-intercept: 900°; y-intercept: $\frac{1}{3}$

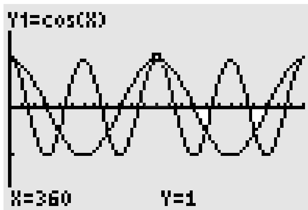
14. **a)** Example: The graphs of $f(x)$ and $g(x)$ will intersect five times, at $x = 0°$, 120°, 360°, 600°, and 720°.

b) $g(x) = \sin\left(\frac{1}{2}x\right)$

c)



15. Example: The graphs of $h(x)$ and $k(x)$ will intersect seven times, at $x = 0°$, 120°, 240°, 360°, 480°, 600°, and 720°.

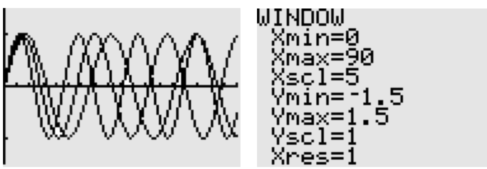


16. a) physical: $y = \sin\left(\frac{360°}{23}t\right)$;

emotional: $y = \sin\left(\frac{360°}{28}t\right)$;

intellectual: $y = \sin\left(\frac{360°}{33}t\right)$

b)



c) Example: days 6, 75

d) Example: days 20, 61, 89

2.4 Combining Transformations of Sinusoidal Functions

1. a) 3 **b)** 4 **c)** $\frac{1}{2}$ **d)** 1

2. a) 180° **b)** 1080° **c)** 1440° **d)** 120°

3. a) amplitude: 5; period: 90°

b) amplitude: 2; period: 720°

c) amplitude: $\frac{1}{4}$; period: 60°

d) amplitude: $\frac{2}{3}$; period: 480°

4. a) amplitude: 3; period: 180°; phase shift: 20° right; vertical shift: 5 units up

b) amplitude: 4; period: 120°; phase shift: 50° left; vertical shift: 2 units down

c) amplitude: 2; period: 40°; phase shift: 40° left; vertical shift: 4 units up

d) amplitude: $\frac{1}{3}$; period: 900°; phase shift: 30° right; vertical shift: 1 unit down

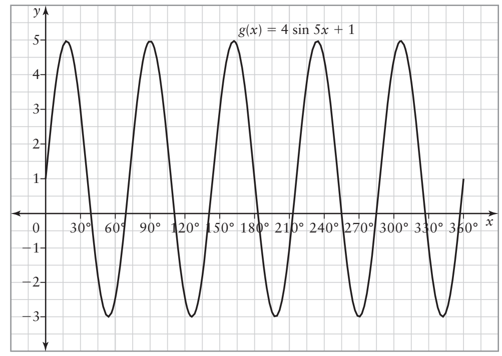
5. a) amplitude: 8; period: 90°; phase shift: 45° left; vertical shift: 3 units down

b) amplitude: 5; period: 60°; phase shift: 38° right; vertical shift: 7 units up

c) amplitude: 7; period: 12°; phase shift: 75° left; vertical shift: 5 units down

d) amplitude: $\frac{2}{7}$; period: 480°; phase shift: 67° right; vertical shift: 8 units up

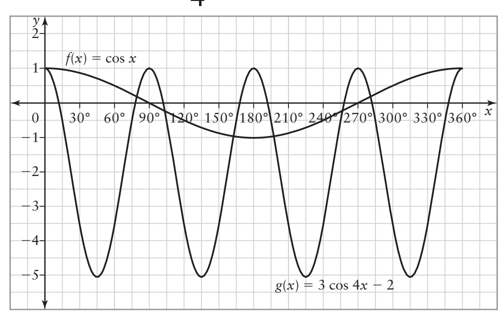
6. a) Example: Apply the amplitude of 4, apply the vertical shift of 1 unit up, and apply the horizontal compression by a factor of $\frac{1}{5}$.



b) $f(x)$: domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, -1 \le y \le 1\}$; $g(x)$: domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, -3 \le y \le 5\}$

c) $h(x) = 4 \sin [5(x + 30°)] + 1$

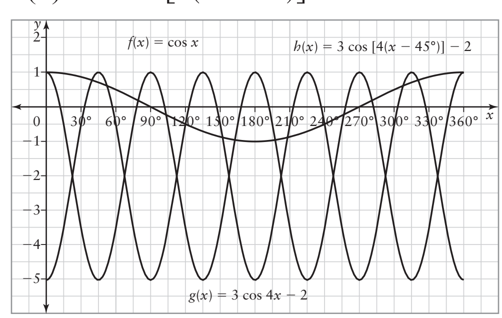
7. a) Example: Apply the amplitude of 3, apply the vertical shift of 2 units down, and apply the horizontal compression by a factor of $\frac{1}{4}$.



b) $f(x)$: domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, -1 \le y \le 1\}$

$g(x)$: domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, -5 \le y \le 1\}$

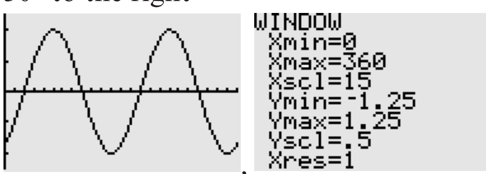
c) $h(x) = 3 \cos [4(x - 45°)] - 2$



8. Example: Apply the amplitude of 3, the vertical shift of 5 units up, and the horizontal compression by a factor of $\frac{1}{4}$.

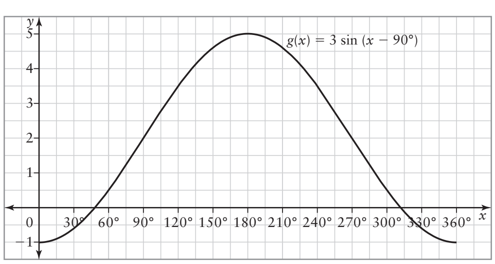
9. Example: Apply the amplitude of 5, the vertical shift of 2 units down, and the horizontal compression by a factor of $\frac{1}{3}$.

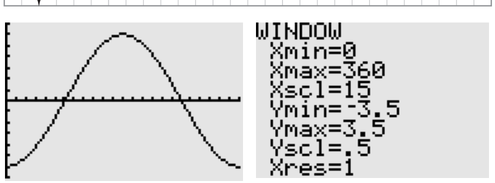
10. a) amplitude: 5; period: 120°; phase shift: 60° right; no vertical shift
 b) maximum 5; minimum −5
 c) 30°; 90°
 d) −5

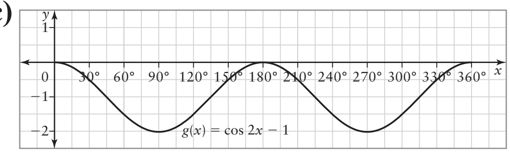
11. a) Previously, equations have been written with the number multiplying x outside brackets. For example, $y = \sin[3(x - 45°)]$.
 b) 30° to the right
 c) 

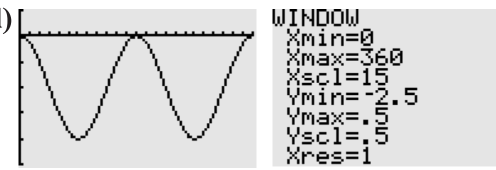
 d) Take a common factor of 2 from the expression in the brackets. $y = \sin[2(x - 30°)]$

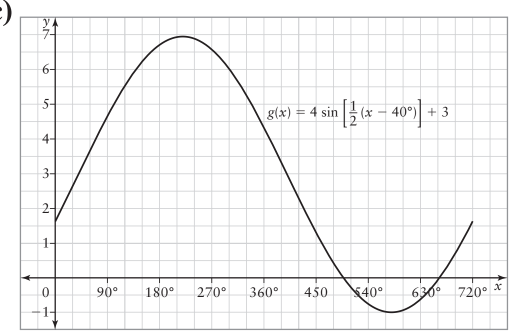
12. a) amplitude: 4; period: 180°; phase shift: 15° left; vertical shift: down 2 units
 b) maximum: 2; minimum: –6
 c) 60°; 180°
 d) 0

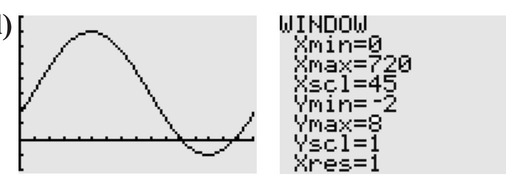
13. a) Apply the amplitude of 3, and translate the function 90° to the right.
 b) y-intercept: −3; x-intercepts: 90°, 270°; maximum: 3; minimum: −3; period: 360°; amplitude: 3; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -3 \le y \le 3\}$
 c) 

 d) 

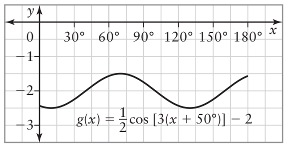
14. a) Apply the vertical shift of 1 unit down, and apply the horizontal compression factor of $\frac{1}{2}$.
 b) y-intercept: 0; x-intercepts: 0°, 180°, 360°; maximum: 0; minimum: –2; period: 180°; amplitude: 1; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -2 \le y \le 0\}$
 c) 

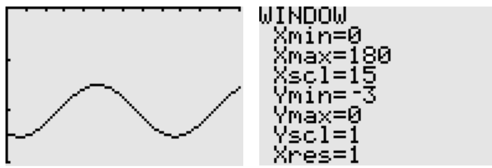
 d) 

15. a) Apply the amplitude of 4, the vertical shift of 3 units up, the horizontal stretch of factor 2, and translate the function 40° to the right.
 b) maximum: 7; minimum: –1; period: 720°; amplitude: 4; domain: $\{x \in \mathbb{R}\}$; range: $\{y \in \mathbb{R}, -1 \le y \le 7\}$
 c) 

 d) 

16. a) Apply the amplitude of $\frac{1}{2}$, the vertical shift of 2 units down, the horizontal compression of factor $\frac{1}{3}$, and translate 50° left.
 b) maximum: −1.5; minimum: −2.5; period: 120°; amplitude: $\frac{1}{2}$; domain: $\{x \in \mathbb{R}\}$; range: $\left\{y \in \mathbb{R}, -\frac{5}{2} \le y \le -\frac{3}{2}\right\}$

c) 

d) 

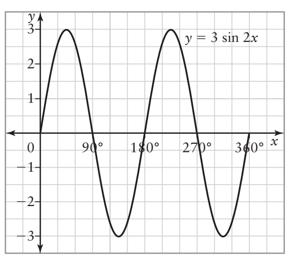
17. Example: Start with $y = \cos x$, then increase the amplitude to 15, shift the graph vertically 2 units, and compress the graph horizontally by a factor of $\frac{5}{18}$.

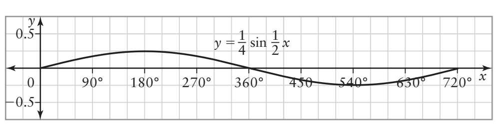
18. Answers may vary.

19. Answers may vary.

2.5 Representing Sinusoidal Functions

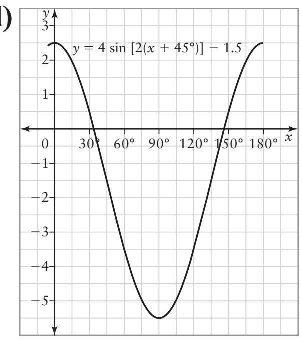
1. **a)** Example: The graphs are the same.
b) Example: The graphs are the same.

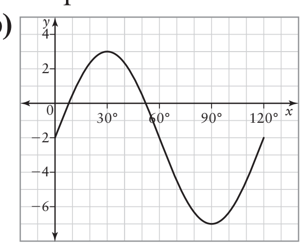
2. **a)** $y = 3 \sin 2x$; yes, any function of the form $y = 3 \sin [2(x - d)] + c$
b) amplitude: 3; period: 180°; maximum: $3 + c$; minimum: $-3 + c$; y-intercept: $0 + c$
c) 

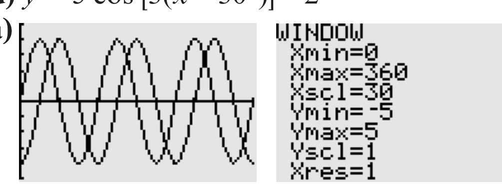
3. **a)** $y = \frac{1}{4} \sin \frac{1}{2}x$; no, only one equation is possible.
b) amplitude: $\frac{1}{4}$; period: 720°; maximum: $\frac{1}{4}$; minimum: $-\frac{1}{4}$; y-intercept: 0; x-intercepts: 0°, 360°, 720°
c) 

4. **a)** Example: $y = 6 \cos 3x$; any function of the form $y = 6 \cos [3(x - d)] + c$
b) amplitude: 6; period: 120°; maximum: $6 + c$; minimum: $-6 + c$; y-intercept: $6 + c$

5. **a)** Example: $y = \frac{1}{3} \cos \frac{1}{3} x$; any function of the form $y = \frac{1}{3} \cos \left[\frac{1}{3}(x - d)\right] + c$
b) amplitude: $\frac{1}{3}$; period: 1080°; maximum: $\frac{1}{3} + c$; minimum: $-\frac{1}{3} + c$; y-intercept: $\frac{1}{3} + c$

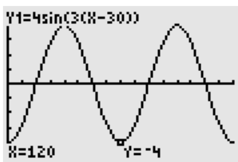
6. **a)** $y = 4 \sin[2(x + 45°)] - 1.5$
b) $y = 4 \cos 2x - 1.5$
c) Example: The cosine function is easier to use because there is no horizontal shift.
d) 

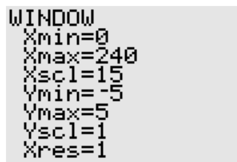
7. **a)** Example: Plot the maximum point. Since the period is 120°, there will be another maximum at (150°, 3). There is a minimum halfway between the maxima. Since the amplitude is 5 and the maximum is 3, the minimum is –7. Plot a minimum point at (90°, –7). Join the points with a smooth curve.
b) 
c) $y = 5 \sin 3x - 2$
d) $y = 5 \cos [3(x - 30°)] - 2$

8. **a)** 

The graph of $f(x)$ is the same as the graph of $g(x)$ after a phase shift of 30° right.
b) 30° right **c)** $g(x) = 4 \sin x$

9. The equation of the sine function is of the form $g(x) = a \sin [k(x - d)]$, since the sine function does not have a vertical shift. The cosine function will have the same amplitude and period as the sine function, so $a = 4$ and $k = 3$. Thus far the equation is $y = 4 \cos [3(x - d)]$. Determine the phase shift. Sketch a graph of the sine function to find d.





Since the first maximum occurs at 60°, the phase shift is 60° to the right, so $d = 60°$. A cosine function that models this function is $y = 4 \cos [3(x - 60°)]$.

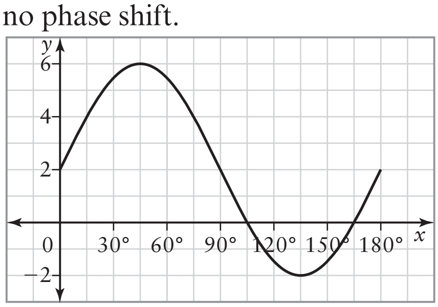
10. $f(x) = 5 \cos 2x$

11. a) amplitude: 2; maximum: 1; minimum: –3; vertical shift: 1 down; period: 720°; phase shift: 45° right

b) $y = 2 \sin \left[\frac{1}{2}(x - 45°)\right] - 1$

12. a) Example: If the maximum is 6 and the amplitude is 4, the minimum is –2 and the vertical shift is up 2. If the maximum is at 45° and the period is 180°, there is no phase shift.

b)



c) $y = 4 \sin 2x + 2$; $y = 4 \cos [2(x - 45°)] + 2$

13. $y = 3 \sin [2(x + 55°)] - 5$

14. $y = \frac{1}{5} \cos \left[\frac{1}{3}(x + 38°)\right] + 2$

15. $y = -7 \sin \left[-\frac{1}{2}(x + 20°)\right] - 3$

16. $y = -\frac{2}{3} \cos [-4(x - 49°)] + 2$

17. a) $y = 3 \sin (2x) + 2$

b) $y = 3 \cos [2(x - 45°)] + 2$

18. a) The minimum value is –2 and the maximum value is 4. Determine the amplitude, a.

$$a = \frac{4 - (-2)}{2}$$
$$= \frac{6}{2}$$
$$= 3$$

The amplitude is 3.

The period is 180° and the graph is horizontally compressed by a factor of $\frac{1}{2}$, so $k = 2$.

The graph is shifted 45° to the right, so $d = 45°$.

The vertical shift is 1 unit up, so $c = 1$.

The equation of the transformed graph is $y = 3 \sin[2(x - 45°)] + 1$.

b) The equation of the sine function is of the form $g(x) = a \sin [k(x - d)] + c$. The cosine function will have the same amplitude and period as the sine function, so $a = 3$ and $k = 2$. Therefore, the equation becomes $y = 3 \cos [2(x - d)] + c$. Since the first maximum occurs at 90°, the phase shift is 90° left, so $d = -90°$, and therefore the equation becomes $y = 3 \cos [2(x + 90°)] + c$. The vertical shift is 1 unit up, so $c = 1$. A cosine function that models this function is $y = 3 \cos [2(x + 90°)] + 1$.

19. a) $y = 5 \cos 30x + 13$

b) $y = 5 \sin [30(x + 3)] + 13$

20. a) $\left(d + \frac{90°(1 + 4n)}{k}, a + c\right)$, where n is an integer

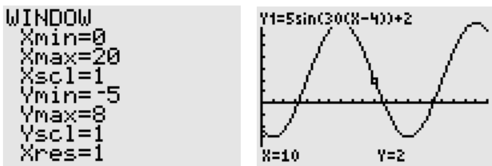
b) $\left(d + \frac{90°(3 + 4n)}{k}, c - a\right)$, where n is an integer

21. a) $\left(d + \frac{n360°}{k}, a + c\right)$, where n is an integer

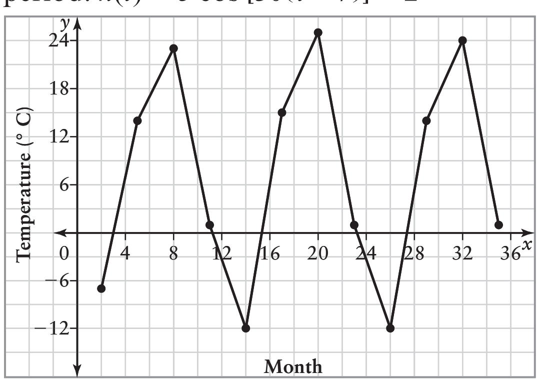
b) $\left(d + \frac{180°(1 + 2n)}{k}, c - \text{a}\right)$, where n is an integer

2.6 Solving Problems Involving Sinusoidal Functions

1. a) Example: The graph models a periodic function. The values along the vertical axis are in a repeating pattern.
b) 11.5 °C
c) 360 days; the period represents one year.
d) Example: The temperature is less variable.

2. a)


b) maximum height: 7 m; minimum height: −3 m
c) high tide at 7 a.m. and 7 p.m.; low tide at 1 a.m. and 1 p.m.
d) 4.5 m
e) Example: The amplitude, vertical shift, and period are the same, but the phase shift is increased by 3, which is $\frac{1}{4}$ of the period. $h(t) = 5 \cos [30(t - 7)] + 2$

3. a)


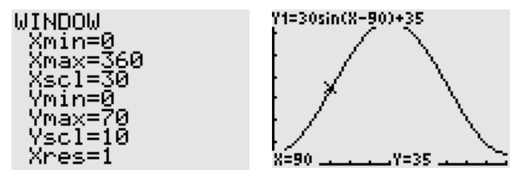
b) Example: The graph appears to model a periodic function. The values along the vertical axis are repeated.
c) Example: $y = 18 \sin[30(x - 2)] + 6$

4. a) maximum 700; minimum 300
b) maximum in February; minimum in August
c) 693
d) March, November
e) Example: There is low attendance during the summer when school is not in session.

5. a) Example: The range is $\{y \in \mathbb{R}, 1 \le y \le 4\}$, as shown by the maximum and minimum in the graph; the amplitude is 1.5, half of the difference between the maximum and minimum; the vertical shift is up 2.5.
b) 3.6; 100 **c)** $y = 1.5 \sin 100x + 2.5$

6. a)

x	$h(x)$
0°	5
30°	9
60°	20
90°	35
120°	50
150°	61
180°	65
210°	61
240°	50
270°	35
300°	20
330°	9
360°	5



The maximum height is 65 m and the minimum height is 5 m.

b) The start of the first cosine wave is at 180°, so the phase shift is 180° to the right. The period, amplitude, and vertical shift remain the same as in the given sine function. A cosine equation that models the height is $h(x) = 30 \cos (x - 180°) + 35$.

c) The period is 360°. The diameter of the Ferris wheel is 65 − 5, or 60 m.
Determine the distance travelled in one revolution.
$C = \pi d$
$= 60\pi$
There are 60 s in 1 min.
Determine the speed.
Speed = distance ÷ time
$= 60\pi \div 60$
$= \pi$
$\doteq 3.1$
The speed of the rider is approximately 3.1 m/s.

7. Example: The domain is $\{n \in \mathbb{R}, n \geq 0\}$ because measurements may not been taken prior to January 1, 2005. The range is $\{d \in \mathbb{R}, 7 \leq d \leq 19\}$ because the function cannot give a d-value outside of this range.

8. **a)** $y = 60 \sin 10\,800t + 10$, $k = 10\,800$
   Determine the period, p.
   $$p = \frac{360°}{k} = \frac{360°}{10\,800} = \frac{1}{30}$$
   The period of the motion is $\frac{1}{30}$ s.

   b) The period gives the time for one complete cycle, or revolution. The RPM is the number of complete cycles in one minute. Each cycle takes $\frac{1}{30}$ s, so there are 30 cycles per second. There are 60 s in 1 min, so there are 30 × 60, or 1800 RPM.

   c) If the speed increased, the RPM would increase. The period would decrease. The coefficient on t would increase.

   d) The maximum is 70 mm and the minimum is −50 mm. Determine the amplitude, a.
   $$a = \frac{70 - (-50)}{2} = \frac{120}{2} = 60$$
   The amplitude is 60.

9. Example:
   a) $y = 2.4 \cos [180(x - 1)] + 7$
   b) 1 s

10. Example:
   a) $y = 181 \sin [30(x - 3)] + 189$
   b) domain: $\{x \in \mathbb{R}, 1 \leq x \leq 12\}$ because the data only covers one year; range: $\{y \in \mathbb{R}, 8 \leq y \leq 370\}$

11. Example:
   a) $y = 8 \cos (x - 225°) + 9.2$
   b) $y = 8 \sin (x + 225°) + 9.2$
   c) The phase shift changes; in part a), the shift is now 180° either left or right; in part b), the shift is now 90° to the right.

12. Example:
   a) 12 breaths per minute
   b) $y = \sin (30x)$

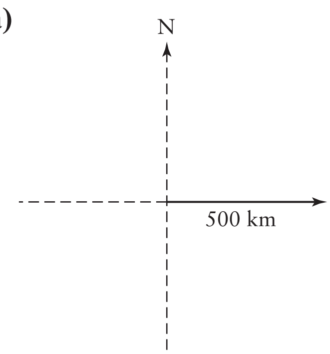
13. Answers may vary.
14. Answers may vary.
15. Answers may vary.
16. approximately 61 cm

Chapter 3 Model With Vectors

3.1 Vectors

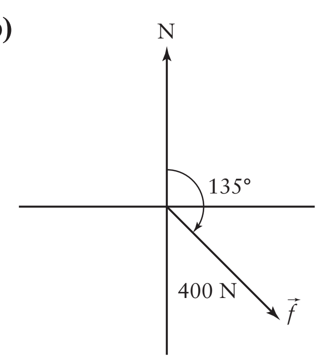
1. **a)** Scalar; time does not have a direction.
   b) Vector; gravity has both magnitude and direction.
   c) Scalar; temperature does not have a direction.
   d) Vector; velocity has both magnitude and direction.
   e) Scalar; there is no indication of direction.
   f) Vector; weight is a force downward due to gravity.
   g) Scalar; there is no indication of direction.

2. **a)**



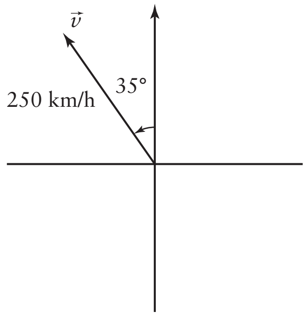
500 km west

b)



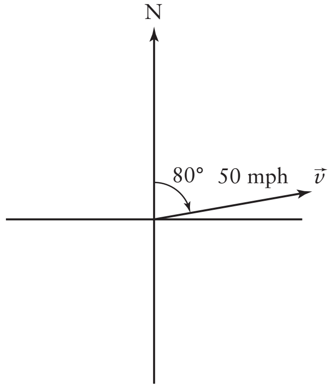
400 N on a bearing of 315°

c)



250 km/h on a quadrant bearing of S35°E

d)



50 mph on a bearing of 260°

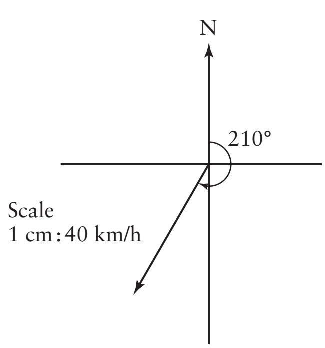
3. **a)** scalar **b)** vector
c) scalar **d)** vector
e) vector **f)** scalar

4. **a)** 25 N on a bearing N40°E; 25 N [N40°E]
b) 8 m/s up; $\vec{V}$ = 8 m/s [up]

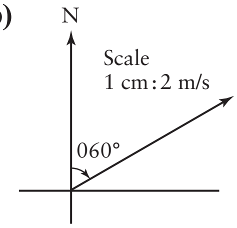
5. **a)** N30°E **b)** S30°E
c) S45°W **d)** N35°W

6. **a)** 140° **b)** 330°
c) 240° **d)** 070°

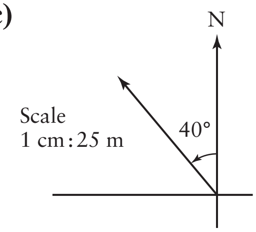
7. Examples:
a)



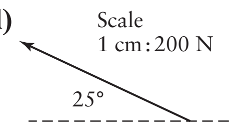
b)



c)



d)



8. **a)** No; speed does not have a direction.
b) Yes; force has both magnitude and direction.
c) No; sound does not have a direction.
d) Yes; velocity has both magnitude and direction.
e) Yes; displacement has both magnitude and direction.
f) Yes; force has both magnitude and direction.
g) No; mass does not have a direction.
h) No; time does not have a direction.

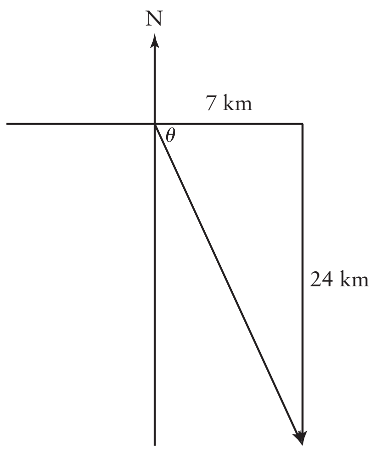
9. Example:
a) **i)** 200 m/s on a bearing of 310°
ii) 125 m/s on a bearing of 130°
iii) 200 m/s on a bearing of 180°
b) **i)** 25 km on a quadrant bearing of N35°W
ii) 10 km in the direction S35°E
iii) 25 km N35°

10. a) Determine the final position, x km, of the car:

$x^2 = 7^2 + 24^2$
$= 49 + 576$
$= 625$
$x = 25$

The magnitude of the displacement vector is 25 km.

b) Draw a diagram to represent the situation.



$\tan\theta = \frac{24}{7}$

$\theta = \tan^{-1}\left(\frac{24}{7}\right)$

$\doteq 74°$

The true bearing of the car's displacement vector is 90° + 74°, or 164°.

c) No, the distance is not the same as the magnitude of the car's displacement vector. The car travelled 7 km to the east and then 24 km south for a total distance of 31 km.

11. a) Yes; the magnitude of two vectors can be compared, and the magnitude of one vector can be larger than the magnitude of another vector.

b) No; the magnitude of two vectors can be compared, but not the vectors themselves.

c) No; magnitude is a positive measurement.

12. a) $\overrightarrow{DC}$

b) $\overrightarrow{CB}$

c) i) true **ii)** true
iii) true **iv)** false

13. Example:

a) i) $\overrightarrow{CD}$ **ii)** $\overrightarrow{AE}$
iii) $\overrightarrow{BD}$ **iv)** $\overrightarrow{BE}$

b) i) $\overrightarrow{DB}$ **ii)** $\overrightarrow{CB}$
iii) $\overrightarrow{GD}$ **iv)** $\overrightarrow{EB}$

14. (−5, −2), (−3, 4), (2, 2), (0, −7), (2, −1)

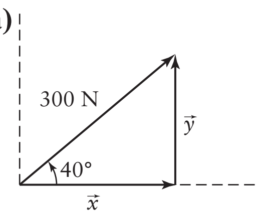
3.2 Components of Vectors

1. a) horizontal: 6.9 m/s; vertical: 9.8 m/s
b) horizontal: 459.6 N; vertical: 385.7 N
c) horizontal: 15.8 cm/s; vertical: 12.3 cm/s

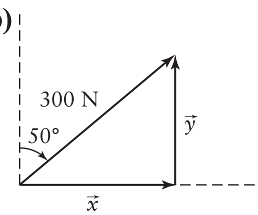
2. a) horizontal: 169.0 N; vertical: 362.5 N
b) $\vec{x} = 400\cos(90° - 25°)$

3. a) horizontal: 47.0 N; vertical: 17.1 N
b) horizontal: 19.3 m/s; vertical: 23.0 m/s
c) horizontal: 725.0 m; vertical: 338.1 m
d) horizontal: 75.2 N; vertical: 206.7 N

4. a)



b)



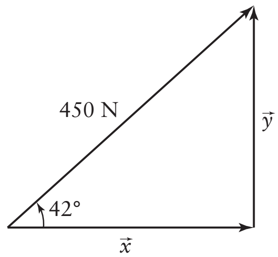
c) Example: The values for the horizontal component and the vertical component are the same.

d) $300\cos 40° \doteq 229.8$; $300\sin 50° \doteq 229.8$

e) $300\cos 50° \doteq 192.8$; $300\sin 40° \doteq 192.8$

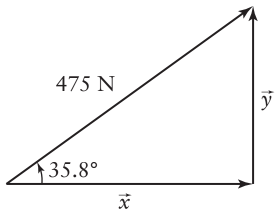
f) Example: The values for the horizontal component and the vertical component of a vector may be calculated using either the angle to the horizontal with the cosine ratio for the horizontal component and the sine ratio for the vertical component, or by using the angle to the vertical that is the complement of the angle to the horizontal with the sine ratio for the horizontal component and the cosine ratio for the vertical component.

5. a)



b) 334.4 N **c)** 301.1 N

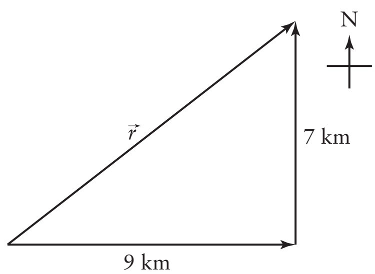
6.



horizontal: 385.3 N; vertical: 277.9 N

7. a) horizontal: 264.9 N; vertical: 140.8 N
b) More force would be required to push the mower forward.

8. a)



b) 11.4 km; 38°

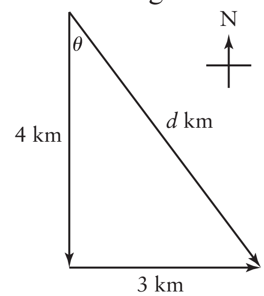
9. a) horizontal: 50.9 N; vertical: 31.8 N
b) the angle would decrease; the vertical force required would decrease

10. a) Determine the horizontal component:
$|\vec{x}| = 160 \cos 15°$
$\doteq 154.5$
The speed of the shadow is approximately 154.5 mph.

b) Determine the vertical component:
$|\vec{y}| = 160 \sin 15°$
$\doteq 41.1$
The rate of climb is approximately 41.1 mph.

Determine the height after 2 min, or $\frac{1}{30}$ h: Height $= 41.1 \times \frac{1}{30}$
$= \frac{41.1}{30}$
$\doteq 1.4$
After 2 min, the height of the aircraft is approximately 1.4 mi.

11. Draw a diagram to represent this situation.



Determine d, the magnitude of the displacement vector: $d^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
$d = 5$

Determine θ, the direction of the displacement vector: $\tan \theta = \frac{3}{4}$
$\theta = \tan^{-1}\left(\frac{3}{4}\right)$
$\doteq 36.9°$

The displacement of the ship can be represented by a vector with magnitude 5 km and approximate direction S36.9°E.

12. Determine the eastern component:
$\left|\overrightarrow{F_E}\right| = 50 \sin 25.7°$
$\doteq 21.7$
The eastern component is approximately 21.7 km.
Determine the northern component:
$\left|\overrightarrow{F_N}\right| = 50 \cos 25.7°$
$\doteq 45.1$
The northern component is approximately 45.1 km.

13. horizontal: 110.2 N; vertical: 68.9 N

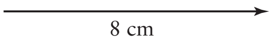
14. a) horizontal: 29.1 m/s; vertical: 136.9 m/s
b) 2738.8 m

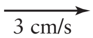
15. a) 32 909.0 kg·m/s south, 19 000 kg·m/s east
b) No, because we do not know the mass of each car

16. force perpendicular to the surface of the ramp: 123.6 N; force of friction, parallel to the surface of the ramp: 40.2 N

17. a) 94.9 N, 284.7 N
b) 18.4°
c) Yes, because the vertical and horizontal magnitudes of the force would also change

3.3 Adding Vectors

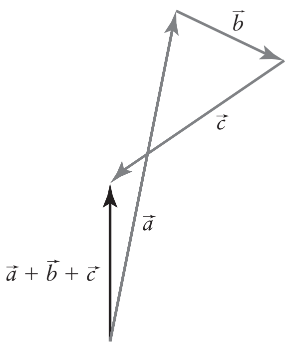
1. a) 

   b) 

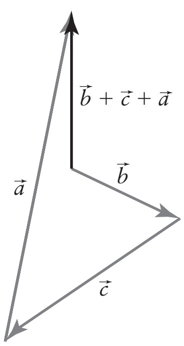
   c) 

2. **a)** distance travelled: 140 m; displacement: 80 m
   b) Different; the distance travelled is a scalar quantity and has magnitude 140 m; the displacement is a vector quantity, 80 m to the east.
   c) The vectors would need to be parallel and in the same direction.

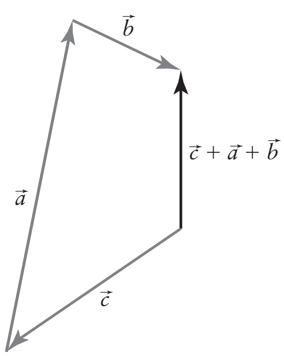
3. **a) i)**



ii)



iii)



   b) The resultant vectors are the same.
   $\vec{a}+\vec{b}+\vec{c}=\vec{b}+\vec{c}+\vec{a}=\vec{c}+\vec{a}+\vec{b}$

4. **a)** $\overrightarrow{AC}$
   b) $\overrightarrow{AB}$
   c) $\overrightarrow{CA}$
   d) $\overrightarrow{BC}$

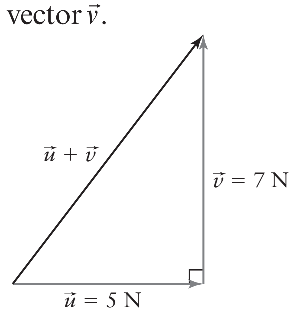
5. **a)** They are the same.
   b) The endpoints of the resultant are the tail of the first vector and the tip of the second vector. For $\overrightarrow{AO}+\overrightarrow{OC}$, the resultant is $\overrightarrow{AC}$.

6. **a)** $\vec{b}=\vec{a}+\vec{c}$
   b) $\vec{e}=\vec{f}+\vec{d}$

7. **a)** $\overrightarrow{BD}$
   b) $\overrightarrow{BD}$
   c) $\overrightarrow{AD}$

8. **a)** $\overrightarrow{HG}$
   b) $\overrightarrow{EF}$
   c) $\overrightarrow{EG}$
   d) $\vec{0}$

9. **a)** Join the tail of vector $\vec{u}$ to the tip of vector $\vec{v}$.



   Determine the magnitude of the resultant vector, $\vec{u}+\vec{v}$:

$$
\begin{aligned}
(\vec{u}+\vec{v})^2 &= (\vec{u})^2+(\vec{v})^2\\
&= 5^2+7^2\\
&= 25+49\\
&= 74\\
\vec{u}+\vec{v} &\doteq 8.6
\end{aligned}
$$

   The magnitude of the resultant vector $\vec{u}+\vec{v}$ is approximately 8.6 N.

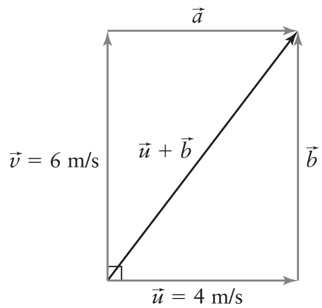
   b) Determine the direction of the resultant vector, $\vec{u}+\vec{v}$:
   Let θ be the angle between vector $\vec{u}$ and the resultant vector, $\vec{u}+\vec{v}$.

$$
\begin{aligned}
\tan\theta &= \frac{7}{5}\\
\theta &= \tan^{-1}\left(\frac{7}{5}\right)\\
&\doteq 54.5°
\end{aligned}
$$

   The direction of the resultant vector is approximately 54.5° counterclockwise from the horizontal.

10. **a)** Complete the parallelogram that has $\vec{u}$ and $\vec{v}$ as two of its sides and two other sides $\vec{a}$ and $\vec{b}$, such that $\vec{a}$ and $\vec{u}$ are equivalent vectors and $\vec{b}$ and $\vec{v}$ are equivalent vectors.

Join the tail of $\vec{u}$ to the tip of $\vec{b}$ to find the resultant vector, $\vec{u} + \vec{b}$.



Determine the magnitude of the resultant vector, $\vec{u} + \vec{b}$:

$$\begin{aligned}(\vec{u} + \vec{b})^2 &= (\vec{u})^2 + (\vec{b})^2 \\ &= 4^2 + 6^2 \\ &= 16 + 36 \\ &= 52 \\ \vec{u} + \vec{b} &\doteq 7.2\end{aligned}$$

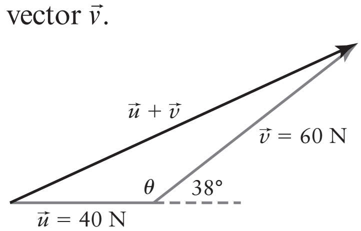
The magnitude of the resultant vector $\vec{u} + \vec{b}$ is approximately 7.2 m/s.

b) Determine the direction of the resultant vector, $\vec{u} + \vec{v}$:
Let θ be the angle between vector $\vec{u}$ and the resultant vector $\vec{u} + \vec{b}$.

$$\begin{aligned}\tan\theta &= \frac{6}{4} \\ \theta &= \tan^{-1}\left(\frac{6}{4}\right) \\ &\doteq 56.3°\end{aligned}$$

The direction of the resultant vector is approximately 56.3° counterclockwise from the horizontal.

11. a) Join the tail of vector $\vec{u}$ to the tip of vector $\vec{v}$.



Determine the angle between $\vec{u}$ and $\vec{v}$:

$$\begin{aligned}\theta &= 180° - 38° \\ &= 142°\end{aligned}$$

Use the cosine law to determine the magnitude of the resultant vector, $\vec{u} + \vec{v}$:

$$\begin{aligned}(\vec{u} + \vec{v})^2 &= (\vec{u})^2 + (\vec{v})^2 - 2(\vec{u})(\vec{v})\cos\theta \\ &= 40^2 + 60^2 - 2(40)(60)\cos 142° \\ &\doteq 8982.45 \\ \vec{u} + \vec{v} &\doteq 94.8\end{aligned}$$

The magnitude of the resultant vector $\vec{u} + \vec{v}$ is approximately 94.8 N.

b) Use the sine law to determine the direction of the resultant vector, $\vec{u} + \vec{v}$:
Let α be the angle between vector $\vec{u}$ and the resultant vector, $\vec{u} + \vec{v}$.

$$\begin{aligned}\frac{\sin\alpha}{60} &= \frac{\sin 142°}{94.8} \\ \sin\alpha &= \frac{60\sin 142°}{94.8} \\ \alpha &\doteq 22.9°\end{aligned}$$

The direction of the resultant vector is approximately 22.9° counterclockwise from the horizontal.

12. a) 63.6 N
b) 28.8° counterclockwise from the horizontal

13. a) 50° **b)** 15.3 km **c)** N37°E

14. 46.1 N on a bearing of 160°

15. Example:
a) i) $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$
ii) $\overrightarrow{BC} = -\overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA} + \overrightarrow{OA}$
iii) $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OB}$
iv) $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{BO} + \overrightarrow{OC}$
b) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ and $\overrightarrow{AC} + \overrightarrow{CA} = \vec{0}$, so $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

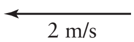
16. a) N3°W **b)** 2 h 15 min

17. Example: if $\vec{u}$ and $\vec{v}$ are parallel, $|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$. For all other cases, $|\vec{u} + \vec{v}| < |\vec{u}| + |\vec{v}|$ because the shortest distance between two points is a straight line.

18. Example: $\overrightarrow{SA} + \overrightarrow{AP}$ will have the same magnitude and direction as $\overrightarrow{RC} + \overrightarrow{CQ}$, so $\overrightarrow{SP}$ and $\overrightarrow{RQ}$ are equivalent vectors. Similarly, $\overrightarrow{PB} + \overrightarrow{BQ}$ will have the same magnitude and direction as $\overrightarrow{SD} + \overrightarrow{DR}$, so PQ and $\overrightarrow{SR}$ are equivalent vectors.

3.4 Subtracting Vectors

1. **a)** 

b) 

c) 

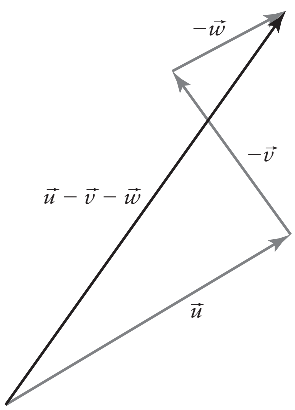
2. **a)** $\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$

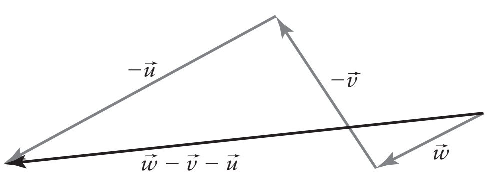
b) $\vec{w} = \vec{v} - \vec{u}$

c) $\vec{p} = \vec{q} - \vec{r}$

d) $\overrightarrow{YZ} = \overrightarrow{XZ} - \overrightarrow{XY}$

3. Example: Add the resultant vector to the opposite of the other known vector, to get the third vector.

4. **a)** 

b) 

5. **a)** $\overrightarrow{BC}$ **b)** $\overrightarrow{HG}$

c) $\overrightarrow{GE}$ **d)** $\overrightarrow{EF}$

6. **a)** $\overrightarrow{EB}$

b) $\overrightarrow{CB}$

c) $\overrightarrow{AB}$

d) $\overrightarrow{AD}$

7. Example:

$\overrightarrow{UV} = \overrightarrow{UO} + \overrightarrow{OV}$

$\overrightarrow{UV} = \overrightarrow{OV} + \overrightarrow{UO}$

$\overrightarrow{UV} = \overrightarrow{OV} - \overrightarrow{OU}$

8. **a)** $\overrightarrow{UV} = \overrightarrow{UO} + \overrightarrow{OV}$

$= -\vec{u} + \vec{v}$

$= \vec{v} - \vec{u}$

b) $\overrightarrow{OW} = \overrightarrow{UV}$

$= \vec{v} - \vec{u}$

c) $\overrightarrow{WO} = -\overrightarrow{OW}$

$= -(\vec{v} - \vec{u})$

$= -\vec{v} + \vec{u}$

$= \vec{u} - \vec{v}$

d) $\overrightarrow{XY} = -\overrightarrow{UV}$

$= -(\vec{v} - \vec{u})$

$= -\vec{v} + \vec{u}$

$= \vec{u} - \vec{v}$

9. **a)** $\overrightarrow{EG} = \overrightarrow{AG} - \overrightarrow{AE}$

b) $\overrightarrow{CG} = \overrightarrow{FG} - \overrightarrow{FC}$

c) $\overrightarrow{HD} = \overrightarrow{BH} - \overrightarrow{DB}$

d) $\overrightarrow{CA} = -\overrightarrow{EC} - \overrightarrow{AE}$

10. **a)** $\overrightarrow{CB}$

b) $\overrightarrow{DB}$

c) $\overrightarrow{CE}$

d) $\overrightarrow{DE}$

11. Example: $\overrightarrow{BC} = \overrightarrow{BE} - \overrightarrow{CE}$; $\overrightarrow{BC} = \overrightarrow{BD} - \overrightarrow{CD}$; $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$; $\overrightarrow{BC} = \overrightarrow{AD} - \overrightarrow{AB} - \overrightarrow{CD}$

12. **a)** $\overrightarrow{F_1} = 101.2$ N on a bearing of 169°

b) $\overrightarrow{F_1} = 125.3$ N on a quadrant bearing of N76°W

c) $\overrightarrow{F_1} = 94.6$ N on a quadrant bearing of S62°W

d) $\overrightarrow{F_1} = 23.2$ N on a bearing of 110°

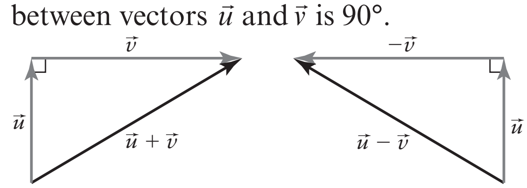
e) $\overrightarrow{F_1} = 232.7$ N on a bearing of 081°

13. $\overrightarrow{ST} - \overrightarrow{SQ} - \overrightarrow{QT} = \overrightarrow{ST} - (\overrightarrow{SQ} + \overrightarrow{QT})$

$= \overrightarrow{ST} - \overrightarrow{ST}$

$= \vec{0}$

14. The expression is true when the angle between vectors $\vec{u}$ and $\vec{v}$ is 90°.



15. **a)** Example: If $\vec{b}$ goes south and $\vec{c}$ goes S45°W, then $\vec{b} - \vec{c}$ will go east.

b) Example: If $\vec{b}$ goes N45ºE and $\vec{c}$ goes N45°W, then $\vec{b} - \vec{c}$ will go east.

d) $|\vec{c}| = 7.4$ m/s on a bearing of S9°E

3.5 Solving Problems Involving Vectors

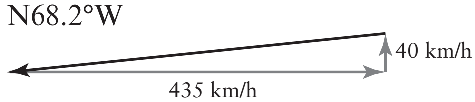
1. **a)** 19.6 m; N39.0°E
b) 390.5 N; N39.8°E
c) 65.3 km; S50.0°W
d) 32.3 km/h; N68.2°W

2. **a)** 26.3 N
b) 325.3 N

3. **a)** 8.6 km
b) 144.5°

4. **a)** 215.4 km
b) N68.2°W

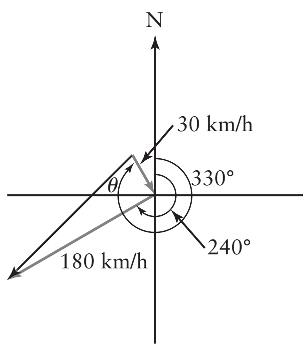
5. **a)**



b) 436.8 km/h, N84.7°E

6. 0

7. **a)**



b) Since 330° − 240° = 90°, the vector triangle is a right triangle.
Determine the magnitude of the resultant velocity, $|\vec{R}|$:

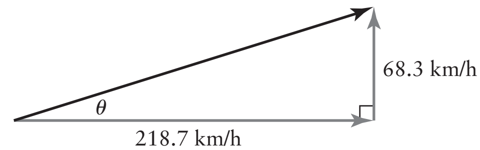
$$|\vec{R}|^2 = 180^2 + 30^2$$
$$= 33\,300$$
$$|\vec{R}| \doteq 182.5$$

Determine the direction of the resultant velocity:
Let θ represent the angle from the resultant velocity, $\vec{R}$, to the wind vector.

$$\tan\theta = \frac{180}{30}$$
$$\theta = \tan^{-1}\left(\frac{180}{30}\right)$$
$$\doteq 80.5°$$

The direction of the wind is actually 150°. The bearing of $\vec{R}$ is 150° + 80.5° = 230.5°.
The resultant velocity, which is the airplane's ground velocity, is 182.5 km/h on a bearing of 230.5°.

8. **a)** Draw a diagram of the situation.



Determine the magnitude of the resultant velocity, the takeoff velocity:

$$|\vec{R}|^2 = 218.7^2 + 68.3^2$$
$$= 52\,494.58$$
$$|\vec{R}| \doteq 229.1$$

The magnitude of the takeoff velocity is approximately 229.1 km/h.
Let θ be the angle between the resultant velocity and the ground.
Calculate the value of θ.

$$\tan\theta = \frac{68.3}{218.7}$$
$$\theta = \tan^{-1}\left(\frac{68.3}{218.7}\right)$$
$$\doteq 17.3°$$

The takeoff velocity is 229.1 km/h at an angle of 17.3° counterclockwise from the runway.

b) In 6 min the aircraft will travel $229.1 \times \frac{6}{60}$, or 22.9 km and the aircraft's displacement will be 22.9 km at an angle of approximately 17.3° counterclockwise from the runway.

9. 4308.1 N; 68.2° above the horizontal

10. 417.7 N; N42.1°E

11. 665.7 m; N78.9°E

12. 47.4 m; 15.1° from the sideline

13. **a)** 110°
b) 511 km/h on a quadrant bearing of N33.2°E

14. 5.4 km on quadrant bearing of N2.7°E

15. **a)** 54 253 N **b)** 4422.8 N

16. 5.4 units; 292°

Chapter 4 Solve Exponential Equations

4.1 The Exponent Laws

1. **a)** 3^{18} **b)** 6^3 **c)** $(0.25)^2$
d) x^7 **e)** $(-3)^3a^3b^6$ **f)** $\frac{2^2}{5^2}u^4v^2$

2. **a)** $\frac{1}{a^3}$ **b)** $-\frac{14}{m^7}$ **c)** v^5 **d)** $\frac{5}{q^4}$
e) $\frac{1}{h^6}$ **f)** $\frac{d^{12}}{16c^4}$ **g)** $\frac{m^2n^2}{9}$ **h)** $\frac{27b^{12}}{125a^6}$

3. a) $\frac{\frac{1}{2}}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2}$ b) $\frac{125}{64}$

c) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$

4. a) 0.0625   b) –0.0977
c) 136.3063   d) 1.9882
e) –9.8415   f) 10.8536

5. a) $x^{\frac{2}{3}}$ b) $m^{\frac{11}{12}}$
c) $s^{\frac{27}{20}}$ d) $u^{\frac{32}{7}} v^{\frac{3}{2}}$
e) $\frac{1}{y^{\frac{4}{25}}}$ f) $\frac{1}{16c^{\frac{14}{9}}}$

6. Examples:
a) Applying the exponent laws or using a calculator gives $-\left(\frac{1}{16}\right)^{-\frac{1}{4}} = -16^{\frac{1}{4}}$
$= -\sqrt[4]{16}$
$= -2$
b) Applying the exponent laws or using a calculator gives $\left(\frac{25}{49}\right)^{-\frac{1}{2}} = \left(\frac{49}{25}\right)^{\frac{1}{2}}$
$= \frac{7}{5}$ or 1.4

7. Example: They are both correct. It does not matter which operation is done first.

8. Examples:
a) $\frac{x^3}{x^7} = x^{(3-7)}$
$= x^{-4}$
$\frac{x^3}{x^7} = \frac{1}{x^4}$
$= x^{-4}$

b) $\frac{a^5}{a^5} = a^{(5-5)}$
$= a^0$
$\frac{a^5}{a^5} = 1$

9. a) Parts i), iii), and iv) are true for all possible numbers.
b) part ii); positive and negative square root; $\sqrt{x^2} = |x|$

10. a) 2^6 b) 4^3
c) 8^2 d) $16^{\frac{3}{2}}$

11. a) $\sqrt[5]{100}$ or approximately 2.511
b) $\sqrt[6]{30}$ or approximately 1.763
c) $\sqrt[3]{20^2}$ or approximately 7.368

12. Example:
$9^3 = (3^2)^3$
$= 3^6$
$27^2 = (3^3)^2$
$= 3^6$

13. a) divide by 2 or multiply by $\frac{1}{2}$

b)

n	2^n	Result
3	2^3	8
2	2^2	4
1	2^1	2
0	2^0	1
–1	2^{-1}	$\frac{1}{2}$
–2	2^{-2}	$\frac{1}{4}$
–3	2^{-3}	$\frac{1}{8}$

c) Example: They continue the pattern.

14. a)

n	$\sqrt{n}$	$n^{\frac{1}{2}}$
1	1	1
4	2	2
9	3	3
16	4	4

b) Example: The results are the same.
Conclusion: $\sqrt{n} = n^{\frac{1}{2}}$.

c) Example: This relationship is true for all positive values of *n*. It is not possible to determine the positive square root of a negative integer in the real number system.

d)

n	$\sqrt[3]{n}$	$n^{\frac{1}{3}}$
1	1	1
8	2	2
27	3	3
64	4	4

e) Example: The results are the same.
Conclusion: $\sqrt[3]{n} = n^{\frac{1}{3}}$.

f) Example: This relationship is true for all values of *n*. It is possible to determine the cube root of a negative integer or a positive number in the real number system.

g) Example: This relationship is true for all exponent values that are in the real number system.

15. a) $V = \frac{1}{3}\pi r^2 h$

b) $r = \sqrt{\frac{3V}{\pi h}}$

c) 2.6 cm

16. a) $P = \sqrt[3]{\frac{750}{V^2}}$

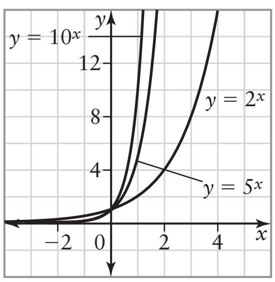
b) 2.3 kPa

c) $V = \sqrt{\frac{750}{P^3}}$

d) 2.4 m^3

4.2 Solving Exponential Equations Graphically

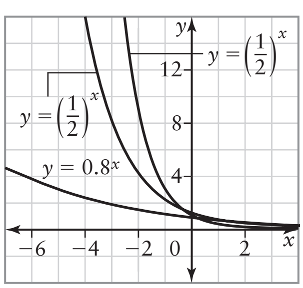
1. a)



b)

	$y = 2^x$	$y = 5^x$	$y = 10^x$
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}, y > 0\}$
Asymptote	$y = 0$	$y = 0$	$y = 0$
***y*-intercept**	1	1	1
***y*-value when $x = 1$**	2	5	10
Rate of increase	grows fast	grows faster	grows fastest

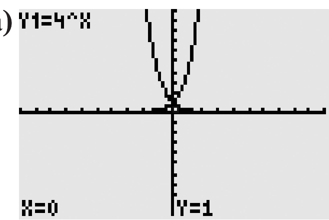
2. a)



b)

	$y = \left(\frac{1}{3}\right)^x$	$y = \left(\frac{1}{2}\right)^x$	$y = 0.8^x$
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}, y > 0\}$
Asymptote	$y = 0$	$y = 0$	$y = 0$
***y*-intercept**	1	1	1
***y*-value when $x = 1$**	$\frac{1}{3}$	$\frac{1}{2}$	0.8
Rate of decrease	decreases fastest	decreases faster	decreases fast

3. a)



Example: The graphs are reflections of each other in the *y*-axis.

b) Example: $4^{-x} = \frac{1}{4^x}$ and $1^x = 1$ for any real number so $\frac{1}{4^x} = \left(\frac{1}{4}\right)^x$

4. Example: As *a* gets larger, between 0 and 1, the graph becomes more flat. At $a = 1$, the graph is a horizontal line. As *a* gets larger than 1, the graph becomes steeper on the right side of the axis.

5. a) Example: graph of $y = \left(\frac{1}{3}\right)^{(x-1)}$

b) Example: Yes; there are many exponential functions with these properties.

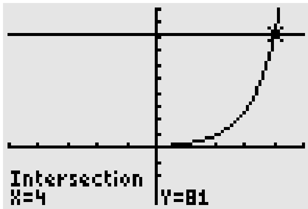
6. a) Example: graph of $y = 2^{(x+2)}$

b) Example: Yes; there are many exponential functions with these properties.

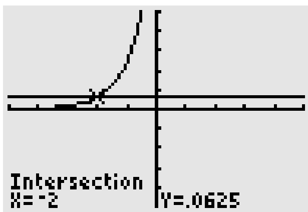
7. $y = 3^x$

8. $y = 0.25^x$

9. a) $x = 4$

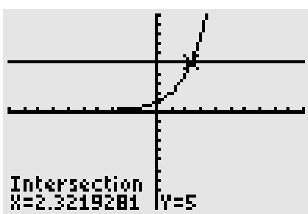


b) $x = -2$



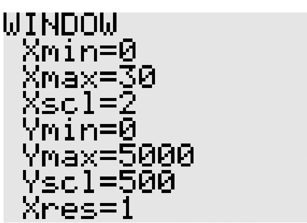
10. a) Example: $x = 2.25$ because $2^2 = 4$ and $2^3 = 8$

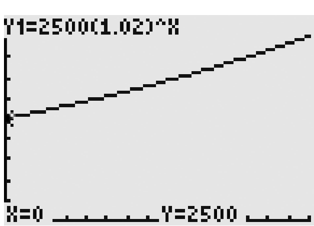
b) $x \doteq 2.3$



11. a) $x = 2$ **b)** $x = 4$
c) $x = 3$ **d)** $x = 0$
e) $x = -5$ **f)** $x = 2$

12. a)

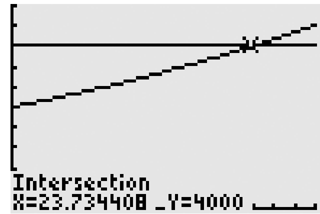




b) The coefficient 2500 represents the y-intercept on the exponential graph.

c) In the context of this situation, 2500 represents the population of Smalltown in the year 2000.

d)

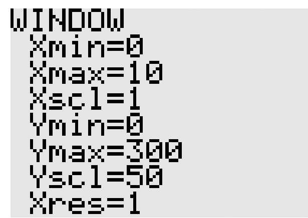


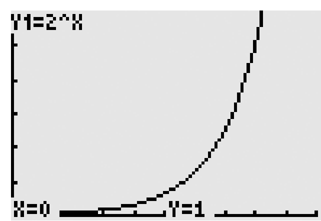
The population will exceed 4000 in the year 2024.

e) If the population grew at 8% per year, the function would be $P = 2500 \times 1.08^n$. If the population were to decrease by 5% per year, the function would be $P = 2500 \times 0.95^n$.

13. a) $N = 2^d$

b)



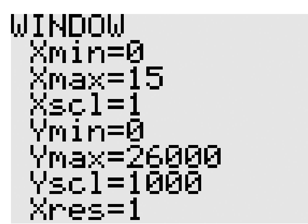


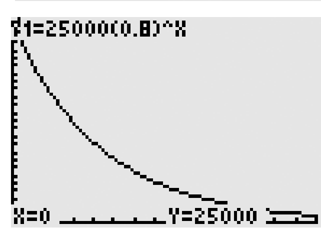
c) January 9

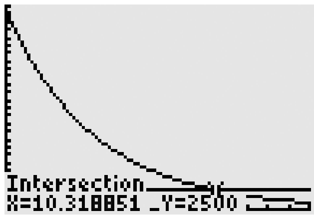
d) 11 days

14. a) Let v represent the value of the car after n years. $v = 25\,000(0.8)^n$

b)

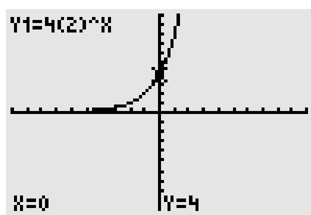




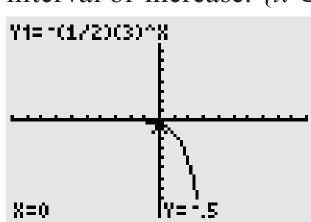
c) 

After approximately 10.3 years, the car will be worth 10% of its initial value.

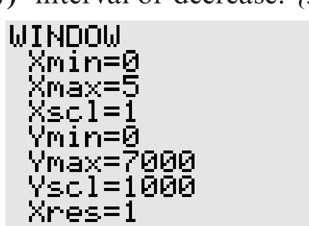
d) During the first year, the car depreciates by \$5000. During the fifth year it depreciates by \$2048. The value of the car after each year is 80% of its value in the previous year.

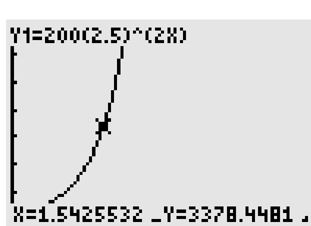
15. a) i) 

ii) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$
iii) no x-intercept; y-intercept = 4
iv) $y = 0$
v) interval of increase: $\{x \in \mathbb{R}\}$

b) i) 

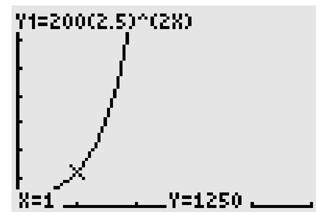
ii) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y < 0\}$
iii) no x-intercept; y-intercept = $-\frac{1}{2}$
iv) $y = 0$
v) interval of decrease: $\{x \in \mathbb{R}\}$

16. a) 

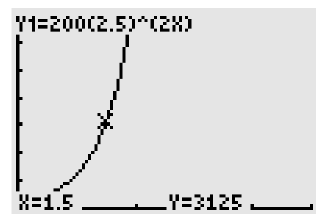


b) The base of the exponential function is greater than 1, indicating that the wheel is speeding up.

c) i) 1250 revolutions per minute



ii) 3125 revolutions per minute



4.3 Solving Exponential Equations Numerically

1. **a)** 3 **b)** 7
c) 4 **d)** 0
e) −3 **f)** 3

2. **a)** $x = 3$ **b)** $x = -2$
c) $w = 3$ **d)** $m = -2$
e) $x = -3$ **f)** $x = 6$

3. **a)** 1.3
b) 1.2
c) 0.8

4. **a)** $x = -\frac{3}{2}$
b) $x = -\frac{2}{3}$
c) $x = \frac{10}{7}$
d) $x = 48$

5. **a)** $x = 1$

b) L.S. $= 3^{7x-3}$ $= 3^{7(1)-3}$ $= 3^4$ $= 81$

R.S. $= 81^x$ $= 81^{(1)}$ $= 81$

L.S. = R.S.

Therefore, the solution is $x = 1$.

c) You can use a graphing calculator and the **Intersect** operation or a table of values to verify your answer to part a).

6.

x	1.03^x	Comment
200	369.355…	Too low. Try a greater value.
235	1039.316…	Still too low, but getting closer.
236	1070.495…	Too low, but close.
236.5	1086.434…	Close, but high.
236.4	1083.228…	This gives a reasonable approximation.

For $1.03^x = 1083$, the solution is approximately 236.4.

7.

x	2.05^x	Comment
15	47 457.8…	Too low. Try a greater value.
16	97 288.5…	Too high.
15.5	67 949.2…	Too low, but closer.
15.6	73 006.2…	Too low, but closer.
15.7	78 439.6…	This gives a reasonable approximation.

For $2.05^x = 78\,440$, the solution is approximately 15.7.

8. **a)** $A = 50\left(\frac{1}{2}\right)^{\frac{t}{4}}$
b) 8.8 mg
c) 12 days

9. $\frac{7}{3}$

10. **a)** $x = 6$
b) L.S. $= 25^{4x + 3}$ R.S. $= 125^{3x}$

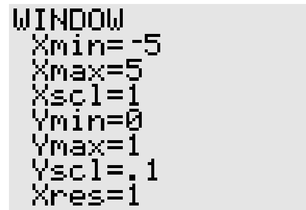
$$\begin{aligned} \text{L.S.} &= 25^{4x+3} \\ &= 25^{4(6)+3} \\ &= 25^{24+3} \\ &= 25^{27} \\ &= (5^2)^{27} \\ &= 5^{54} \end{aligned} \qquad \begin{aligned} \text{R.S.} &= 125^{3x} \\ &= 125^{3(6)} \\ &= 125^{18} \\ &= (5^3)^{18} \\ &= 5^{54} \end{aligned}$$

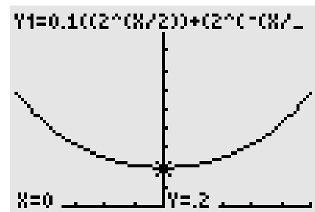
L.S. = R.S.

Therefore, the solution is $x = 6$.

11. **a)** Example: The equations are identical if $a = 2^x$.
b) $x = 0$, $x = 1$

12. **a)**

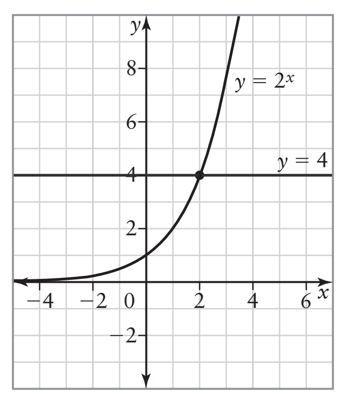




b) Yes, this relation is a function. The x-values do not repeat.
c) parabola
d) (0, 0.2)

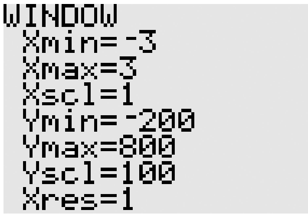
4.4 Points of Intersection

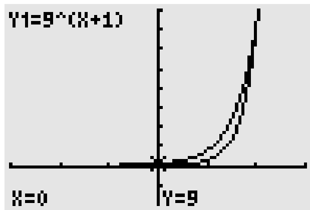
1. **a)** (–5, 0.0016) **b)** (2, 5 764 801)
c) (1, 46 656) **d)** (7, 262 144)
2. **a)** $x = 2$, $y = 4096$
b) $(3, 5^{24})$
3. Example: No, the exponential equation cannot be solved. The graphs of $y = 4^{2x + 3}$ and $y = 2^{4x - 7}$ do not intersect. When equating the exponents of a common base, the variable term cancels out.
4. **a)**



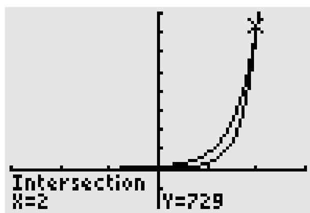
b) (2, 4)
c) $x = 2$
d) Example: Solve using a common base.
e) Example: No; there is no common base.

5. a)





b) (2, 729)

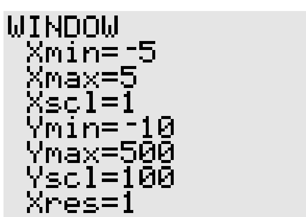


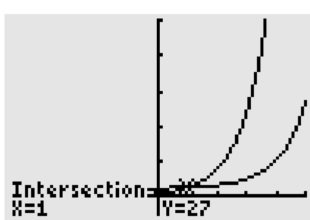
c) $x = 2$

d)
$$9^{x+1} = 27^{x}$$
$$3^{2x+2} = 3^{3x}$$
$$2x + 2 = 3x$$
$$x = 2$$

e) The results are the same.

6. a)

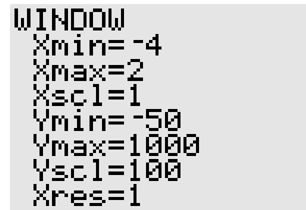


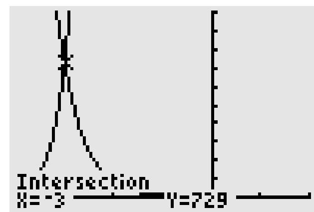


The point of intersection is (1, 27).

b)
$$3^{x+2} = 3^{x} + 24$$
$$3^{x}(3)^{2} - 3^{x} = 24$$
$$3^{x}(3^{2} - 1) = 24$$
$$3^{x} = 3$$
$$x = 1$$

7. a)



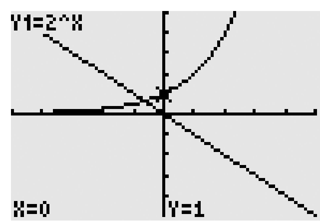


The point of intersection is (–3, 729).

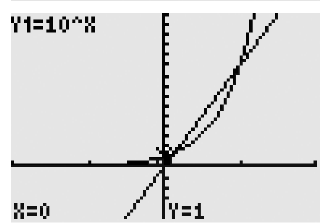
b)
$$9^{-x} = 27^{x+5}$$
$$3^{-2x} = 3^{3x+15}$$
$$-2x = 3x + 15$$
$$x = -3$$

8. Examples:

a)

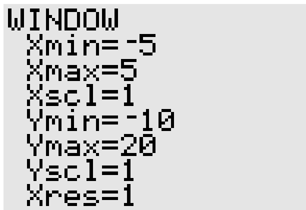


b)

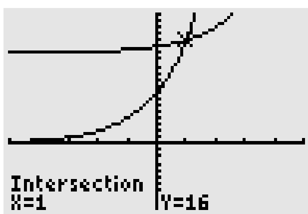


9. $x \doteq -1.3$

10. a) To solve $2^{x+3} = 2^x + 14$, graph the functions $y = 2^{x+3}$ and $y = 2^x + 14$ on the same set of axes on a graphing calculator. Use the window settings shown.



Use the **Intersect** operation to determine the point of intersection of the two functions.



The two graphs intersect at the point (1, 16).

b)

$$\begin{aligned} 2^{x+3} &= 2^x + 14 \\ 2^x(2^3) - 2^x &= 14 \\ 2^x(2^3 - 1) &= 14 \\ 2^x(7) &= 14 \\ 2^x &= 2 \\ x &= 1 \end{aligned}$$

c) The x-coordinate of the point of intersection of the graphs of the two functions is the same as the solution of the exponential equation.

11. a) Let n represent the number of years after 2004. Let E represent the weekly earnings, in dollars.

n	E	Ratio of the E-values
0	550.00	
1	566.50	1.03
2	583.50	1.03
3	601.00	1.03
4	619.03	1.03

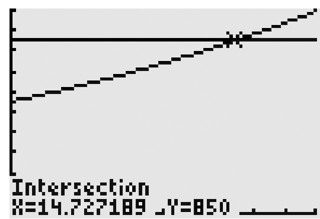
The equation will be of the form $E = ab^n$. Since successive E-values increase by a factor of 1.03, $b = 1.03$. Since the vertical intercept is 550, $a = 550$. Therefore, the function that models the data is $E = 550(1.03)^n$.

b) The year 2012 is represented by $n = 8$.

$$\begin{aligned} E &= 550(1.03)^8 \\ &\doteq 696.72 \end{aligned}$$

Taylor's weekly earnings in 2012 will be \$696.72.

c) Use graphing technology to graph the functions $E = 550(1.03)^n$ and $E = 850$, and determine the point of intersection.



Taylor's weekly earnings will surpass \$850 in 15 years.

12. a) Let n represent the number of years after the car was purchased. Let V represent the value of the car, in dollars.

n	V	Ratio of the V-values
1	12 000	
2	9 600	0.8
3	7 680	0.8
4	6 140	0.8
5	4 920	0.8

The equation will be of the form $V = ab^n$. Since successive V-values decrease by a factor of 0.8, $b = 0.8$. To find the value of a, substitute the coordinates of one point, such as (1, 12 000), and the value of b into $V = a(0.8)^n$.

$$\begin{aligned} V &= a(0.8)^n \\ 12\,000 &= a(0.8)^1 \\ a &= \frac{12\,000}{0.8} \\ a &= 15\,000 \end{aligned}$$

Therefore, the function that models the data is $V = 15\,000(0.8)^n$.

b) For the purchase price, substitute $n = 0$ into the function.
$V = 15\,000(0.8)^n$
$= 15\,000(0.8)^0$
$= 15\,000$
The purchase price of the car was likely \$15 000.

c) The year 2010 is represented by $n = 8$.
$V = 15\,000(0.8)^8$
$\doteq 2516.58$
The value of the car in 2012 will be approximately \$2516.58.

13. a) 157.9 kPa
b) 14 min

4.5 Logarithms

1. a) $7^3 = 343$ **b)** $2^4 = 16$
c) $3^{-4} = \frac{1}{81}$ **d)** $2^{-6} = \frac{1}{64}$
e) $5^0 = 1$ **f)** $b^2 = b^2$

2. Example: Since $a^{\frac{1}{2}} = \sqrt{a}$, $\log_a \sqrt{a} = \frac{1}{2}$.

3. a) $\log_5 25 = 2$ **b)** $\log_2 64 = 6$
c) $\log_4\left(\frac{1}{16}\right) = -2$ **d)** $\log_8 1 = 0$
e) $\log_6 216 = 3$ **f)** $\log_3\left(\frac{1}{27}\right) = -3$

4. a) 7 **b)** 5
c) 4 **d)** −3
e) 3 **f)** −2

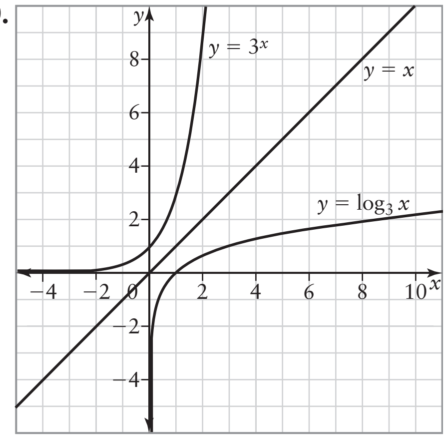
5. a) 0
b) Example: No. The algebraic expression $a^0 = 1$ for $a \neq 0$, so $\log_a 1 = 0$ for $a > 0$, $a \neq 1$.

6. Example: A negative base with a non-integer exponent is undefined, $\log_0 0$ has an infinite number of solutions, and $\log_1 x$ only has meaning for $x = 1$, in which case it has an infinite number of values.

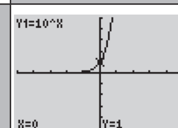
7. a) i) 1 **ii)** 1 **iii)** 1
b) Example: Any base raised to the exponent 1, is itself; $\log_a a = 1$.

8. a) 2 **b)** −1
c) −3 **d)** 4
e) −2 **f)** 6

9. Examples:
a) $\frac{1}{100} = 0.01$
b) The format 0.01 is easier to use because it is easier to enter on a calculator.

10.


11.

	$y = 10^x$	$y = \log x$
Sketch of the graph		
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}, x > 0\}$
Range	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}\}$
***x*-intercept(s)**	none	1
***y*-intercept(s)**	1	none
Vertical asymptote(s)	none	$x = 0$
Horizontal asymptote(s)	$y = 0$	none
Intervals of increase	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}, x > 0\}$
Intervals of decrease	none	none

12. Example: (8, 2); the graphs are reflections of each other in the line $y = x$.

13. a) Example: Find a value of x that approximates $5^x = 13$.
b) 1.6

14. **a)** 1

b) Example: At $y = 0$, $x = 1$ and at $y = 1$, $x = 7$, so at $x = \frac{1}{2}$, y is less than 0.

c) −0.4

15. **a)** 3

b) **i)** 3 **ii)** 3

c) They are the same.

d) approximately 2.73

e) **i)** approximately 2.73

ii) approximately 2.73

f) They are the same.

g) You can evaluate $\log_2 50$ with a calculator using common logarithms. Calculate $\frac{\log 50}{\log 2}$.

16. **a)** $A = P(1.02)^{2t}$

b) 17.5 years

4.6 Solving Problems Using Logarithms

1. **a)** $\log_6 14$ **b)** $\log_9 7$

c) $\log_{\frac{3}{5}}\left(\frac{1}{4}\right)$ **d)** $\log_{\frac{1}{3}}(15)$

2. **a)** 1.362 **b)** 1.369

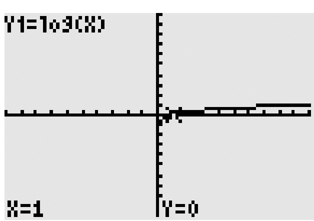
c) 0.565 **d)** −0.627

e) −1.661 **f)** −6.129

3. **a)** 1.89 **b)** 0.58

c) 1.90 **d)** 53.02

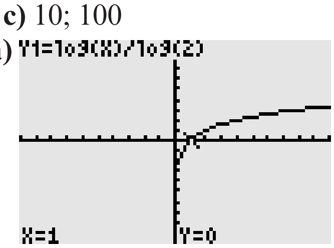
4. **a)**



b) 1

c) 10; 100

5. **a)**



b) 0.5

c) Example: $x = 4$ and $x = 2$; both result in integral y-values, since $x = 2^y$.

6. 13.3 days

7. **a)** $A(t) = 1000(1.05)^t$

b) $1157.63

c) 14.2 years

8. **a)** $A(t) = 2000(1.0225)^{2t}$

b) $2186.17

c) approximately 20.6 years

9. **a)** To determine the pH of lemon juice, substitute the hydronium ion concentration for lemon juice into the equation.

$$\begin{aligned} pH &= -\log [H^+] \\ &= -\log [0.01] \\ &= -(-2) \\ &= 2 \end{aligned}$$

Therefore, the pH of lemon juice is 2.

b) To determine the concentration of hydronium ions in milk, substitute the pH for milk into the equation.

$$\begin{aligned} pH &= -\log [H^+] \\ 6 &= -\log [H^+] \\ -6 &= \log [H^+] \\ 10^{-6} &= [H^+] \\ 0.000\,001 &= [H^+] \end{aligned}$$

Therefore, the hydronium ion concentration in milk is 0.000 001 mol/L.

10. Let the sound level of the rock concert speaker be β_2 and the sound level of the symphony, at its peak, be β_1.
Substitute 150 for β_2 and 120 for β_1 in the equation.

$$\begin{aligned} \beta_2 - \beta_1 &= 10\log\left(\frac{I_2}{I_1}\right) \\ 150 - 120 &= 10\log\left(\frac{I_2}{I_1}\right) \\ 30 &= 10\log\left(\frac{I_2}{I_1}\right) \\ 3 &= \log\left(\frac{I_2}{I_1}\right) \\ 10^3 &= \frac{I_2}{I_1} \\ 1000 &= \frac{I_2}{I_1} \end{aligned}$$

Therefore, the rock concert speaker sound is 1000 times as intense as the symphony sound at its peak.

11. Let the sound level of a shout be β_2.
Substitute 60 for β_1 and 100 for $\frac{I_2}{I_1}$ in the equation.

$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

$\beta_2 - 60 = 10\log 100$

$\beta_2 = 10\log 100 + 60$

$\beta_2 = 10(2) + 60$

$\beta_2 = 20 + 60$

$\beta_2 = 80$

Therefore, the sound level of a shout is 80 decibels.

12. a) 3162 times as intense
b) 4

13. a) 48%
b) 17.7 years
c) Example: $[(1.04)^{10} - 1]$ represents the increase in part a), and $(1.04)^t = 2$ represents when the investment will double.

14. Example: If the half-life of an element is 3 years, after 3 years, $\frac{1}{8}$ of the original amount will be left.

15. $x = -2$, $x = 5$

16. Examples:
a) John Napier in 1614
b) pH, decibels, Richter scale, astronomical apparent magnitude, musical intervals
c) 1970

Chapter 5 Polynomial Functions

5.1 Identifying Polynomial Functions

1. a) 4 **b)** 2 **c)** 5 **d)** 6 **e)** 0 **f)** 3

2. a) 3 **b)** 1 **c)** 2 **d)** 4

3. a) 2 **b)** 4 **c)** 0 **d)** 3 **e)** 1 **f)** 5

4. a) Yes; the x-values do not repeat.
b) Yes; the x-values do not repeat.
c) Yes; the x-values do not repeat.
d) No; the x-values have more than one y-value.
e) Yes; the x-values do not repeat.
f) Yes; the x-values do not repeat.
g) No; the x-values have more than one y-value.
h) No; the x-values have more than one y-value.

5. Example: The equation contains no y-variable or the y-variable is squared.

6. a) Yes; each value of x is mapped onto exactly one value of y.
b) Yes; each value of x is mapped onto exactly one value of y.
c) No; there are two y-values for $x = 2$.
d) No; there are two y-values for $x = 1$.

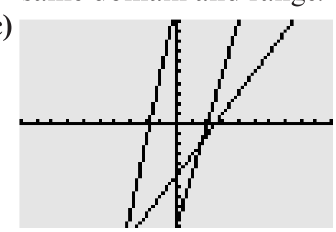
7. a) polynomial function
b) polynomial function
c) exponential function
d) rational function
e) not a function
f) sinusoidal function

8. a) exponential function
b) polynomial function
c) sinusoidal function
d) square root function
e) polynomial function
f) sinusoidal function
g) polynomial function

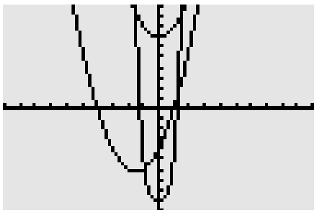
9. Example: No; the exponent is negative.

10. Example: Yes, once it is expanded. The factored form makes determining the x-intercepts easy.

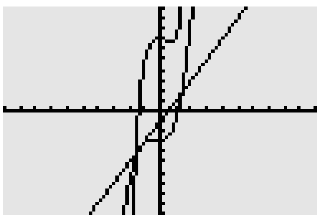
11. Examples:
a) These are polynomial functions of degree 1.
b) The three graphs are of linear functions with a positive slope, one x-intercept, and one y-intercept. They all have the same domain and range.
c)



As $x \to \infty$, $y \to \infty$; as $x \to -\infty$, $y \to -\infty$.

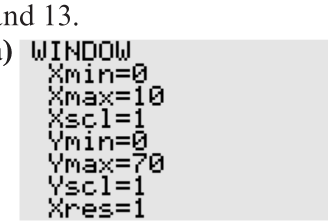
12. Examples:
 a) These are polynomial functions, two of degree 2 and one of degree 4.
 b) The graphs all appear to be parabolas that open up. They all have the same domain. The range is restricted by the minimum value.
 c) 

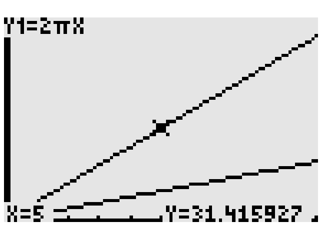
As $x \to \infty$, $y \to \infty$; as $x \to -\infty$, $y \to \infty$.

13. a) 

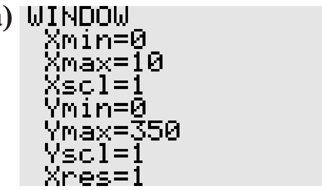
Examples:
 b) Each has one x-intercept and one y-intercept. They all have the same domain and range. For all graphs, as $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to -\infty$.
 c) These graphs have no minimum or maximum value. The end behaviours differ.

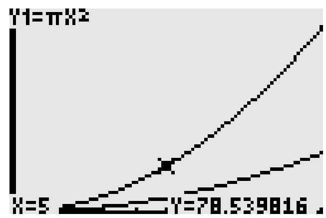
14. Example: For $y = 2^x$, as $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to 0$. For $y = \sin x$, y has maximum and minimum values. Both $y = 2^x$ and $y = \sin x$ have different ranges from the functions in questions 11, 12, and 13.

15. a) 



 b) domain $\{r \in \mathbb{R}, 0 \le r \le 10\}$; range $\{C \in \mathbb{R}, 0 \le C \le 62.8\}$
 c) Example: Similarities: The functions are both linear, with positive leading coefficients. Both graphs pass through the origin, (0, 0).
 Differences: The graph of $C(r) = 2\pi r$ has a restricted domain. All points other than the point (0, 0) are different.

16. a) 



 b) domain $\{r \in \mathbb{R}, 0 \le r \le 10\}$; range $\{C \in \mathbb{R}, 0 \le C \le 314.2\}$
 c) Similarities: The functions are both quadratic, with positive leading coefficients. Both graphs pass through the origin, (0, 0).
 Differences: The graph of $A(r) = \pi r^2$ has a restricted domain. All points other than the point (0, 0) are different.

5.2 Graphs of Polynomial Functions

1. a) 7
 b) −5
 c) 0.4
 d) $\frac{1}{4}$

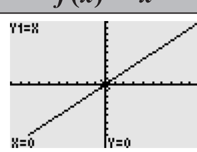
2. a) i) negative
 ii) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}\}$
 iii) as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to -\infty$
 b) i) negative
 ii) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}, y \le 7\}$
 iii) as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to -\infty$
 c) i) positive
 ii) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}, y \ge -3\}$
 iii) as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to \infty$

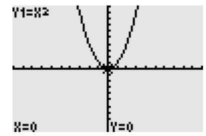
3. a) degree, 4, is even; leading coefficient, 3, is positive; therefore, as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$
b) degree, 3, is odd; leading coefficient, 2, is positive; therefore, as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$
c) degree, 2, is even; leading coefficient, −5, is negative; therefore, as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to -\infty$
d) degree, 5, is odd; leading coefficient, $-\frac{1}{4}$, is negative; therefore, as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to -\infty$
e) degree, 1, is odd; leading coefficient, −4, is negative; therefore, as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to -\infty$
f) degree, 2, is even; leading coefficient, 3, is positive; therefore, as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$

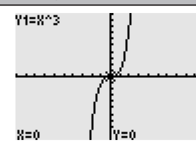
4. a) 3; example: As $x \to -\infty$, $y \to -\infty$, as $x \to \infty$, $y \to \infty$, and the graph has three x-intercepts.
b) 4; example: As $x \to -\infty$, $y \to \infty$, as $x \to \infty$, $y \to \infty$, and has four x-intercepts.
c) 2; example: As $x \to -\infty$, $y \to -\infty$, as $x \to \infty$, $y \to -\infty$, and has two x-intercepts.
d) 1; example: As $x \to -\infty$, $y \to \infty$, as $x \to \infty$, $y \to -\infty$, and has one x-intercept.

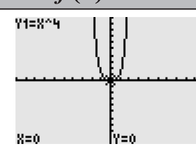
5. a) positive; odd degree; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$
b) positive; even degree; as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$
c) negative; even degree; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to -\infty$
d) negative; odd degree; as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to -\infty$

6. a)

	$f(x) = x$
Sketch	
Degree	1
Type of Function	linear
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}\}$
End Behaviour	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$

	$f(x) = x^2$
Sketch	
Degree	2
Type of Function	quadratic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y \geq 0\}$
End Behaviour	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to \infty$

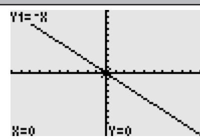
	$f(x) = x^3$
Sketch	
Degree	3
Type of Function	cubic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}\}$
End Behaviour	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$

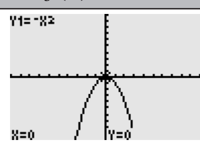
	$f(x) = x^4$
Sketch	
Degree	4
Type of Function	quartic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y \geq 0\}$
End Behaviour	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to \infty$

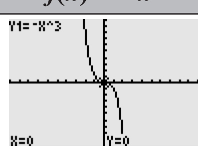
b) Example: The key features of the graphs of $f(x) = x$ and $f(x) = x^3$ are the same.
c) Example: The key features of the graphs of $f(x) = x^2$ and $f(x) = x^4$ are the same.

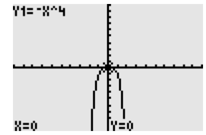
7. a) point symmetry about (0, 0)
b) line symmetry about $x = 0$
c) point symmetry about (0, 0)
d) line symmetry about $x = 0$

8. Example: They have a negative leading coefficient.

	$f(x) = -x$
Sketch	
Degree	1
Type of Function	linear
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}\}$
End Behaviour	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to -\infty$

	$f(x) = -x^2$
Sketch	
Degree	2
Type of Function	quadratic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y \le 0\}$
End Behaviour	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to -\infty$

	$f(x) = -x^3$
Sketch	
Degree	3
Type of Function	cubic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}\}$
End Behaviour	as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to -\infty$

	$f(x) = -x^4$
Sketch	
Degree	4
Type of Function	quartic
Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R}, y \le 0\}$
End Behaviour	as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to -\infty$

9. **a)** $y = -x^4 + 2x^2 - 3$ is represented by graph ii).

b) $y = -x^3 - 3x^2 + 2x - 1$ is represented by graph iv).

c) $y = x^3 + 2x^2 - 3x$ is represented by graph i).

d) $y = x^2 - 3x + 2$ is represented by graph iii).

10. a)

x	y	First Differences
−2	−3	
−1	−1	2
0	1	2
1	3	2
2	5	2

b)

x	y	First Differences	Second Differences
−2	4		
−1	1	−3	
0	0	−1	2
1	1	1	2
2	4	3	2

c)

x	y	First Differences	Second Differences	Third Differences
−2	−8			
−1	−1	7		
0	0	1	−6	
1	1	1	0	6
2	8	7	6	6

d)

x	y	First Differences	Second Differences	Third Differences
−2	16			
−1	2	−14		
0	0	−2	12	
1	−2	−2	0	−12
2	−16	−14	−12	−12

e) The first differences of a linear function are constant. The second differences of a quadratic function are constant. The third differences of a cubic function are constant.

f) same sign

g) For any polynomial function of degree n, the nth differences are equal to $a[n \times (n-1) \cdots \times 2 \times 1]$.

11.

x	y	First Differences	Second Differences	Third Differences
−3	94			
−2	37	−57		
−1	10	−27	30	
0	1	−9	18	−12
1	−2	−3	6	−12
2	−11	−9	−6	−12
3	−38	−27	−18	−12

a) Since the third differences are equal, the degree of the polynomial is 3.

b) Since the sign of the third difference is negative, the sign of the leading coefficient is negative.

c) The value of the leading coefficient is the value of a such that $-12 = a[n \times (n-1) \cdots \times 2 \times 1]$.
Substitute 3 for n in the equation.
$-12 = a[3 \times 2 \times 1]$
$-12 = a[6]$
$-12 = 6a$
$-2 = a$
Therefore, the value of the leading coefficient is −2.

12. a) 4 **b)** positive **c)** 1

13. a) 3
b) positive
c) as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$
d) The third differences are equal.
e) This is an odd-degree polynomial, which has no maximum or minimum value.

14. a) 4
b) negative
c) as $x \to -\infty$, $y \to -\infty$; as $x \to \infty$, $y \to -\infty$
d) The fourth differences are equal.
e) This is an even-degree polynomial with a negative leading coefficient, so there is a maximum value.

15. a) polynomial function of degree 2; quadratic function
b) second differences
c) −2
d) The number of cameras sold must be greater than or equal to zero.
e) The x-intercepts, 1 and 8, represent the profit break-even points.
f) $6000

16. a) $S(r) = 15\pi r^2$; domain $\{r \in \mathbb{R}, r \geq 0\}$; range $\{S \in \mathbb{R}, S \geq 0\}$; as $x \to -\infty$, $y \to \infty$
b) $V(r) = 7\pi r^3$; domain $\{r \in \mathbb{R}, r \geq 0\}$; range $\{V \in \mathbb{R}, V \geq 0\}$; as $x \to -\infty$, $y \to \infty$

17. Example: Since the leading coefficient is positive, and the degree is odd, as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.

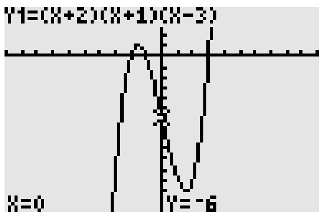
18. Example: Because as $x \to -\infty$, $y \to -\infty$, and as $x \to -\infty$, $y \to -\infty$, or because as $x \to \infty$, $y \to -\infty$, and as $x \to -\infty$, $y \to \infty$, the graph must cross the x-axis.

19. a) Example: Because as $x \to -\infty$ and as $x \to \infty$, $y \to -\infty$ or $y \to \infty$, there must be either a maximum or minimum value for y.

b) Example: If the leading coefficient is positive, the degree is even, and the minimum value is greater than zero, there is no x-intercept. Similarly, if the leading coefficient is negative, the degree is even, and the maximum is less than zero, there is no x-intercept.

5.3 Comparing Polynomial Functions

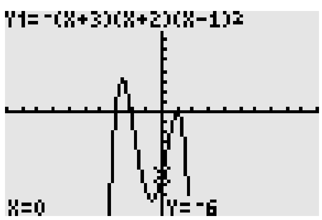
1. a) i)



ii) three x-intercepts; no maximum or minimum point; one local maximum and one local minimum point

iii) $x = -2$, $x = -1$, $x = 3$; the x-intercepts fall where each factor is equal to zero.

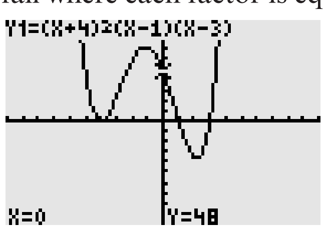
b) i)



ii) three x-intercepts; a maximum point; one local maximum and one local minimum point

iii) $x = -3$, $x = -2$, $x = 1$; the x-intercepts fall where each factor is equal to zero.

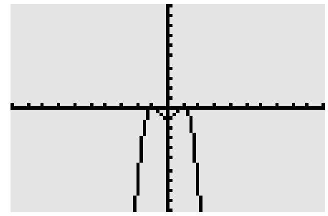
c) i)



ii) three x-intercepts; a minimum point; one local maximum and one local minimum point

iii) $x = -4$, $x = 1$, $x = 3$; the x-intercepts fall where each factor is equal to zero.

d) i)



ii) two x-intercepts; no maximum point; one local maximum point

iii) $x = -1$, $x = 1$; the x-intercepts fall where each factor is equal to zero.

2. a) degree: 3; positive
b) degree: 4; negative
c) degree: 4; positive
d) degree: 4; negative

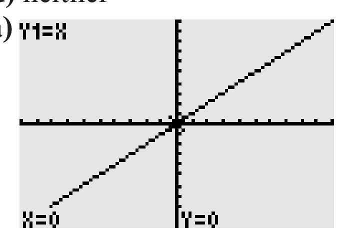
3. a) x-intercepts: $x = -3$ (odd order), $x = -1$ (odd order), $x = 2$ (odd order)
b) x-intercepts: $x = -2$ (even order), $x = 1$ (even order)
c) x-intercepts: $x = -3$ (even order), $x = 2$ (odd order)
d) x-intercepts: $x = -2$ (odd order), $x = 1$ (odd order), $x = 3$ (even order)

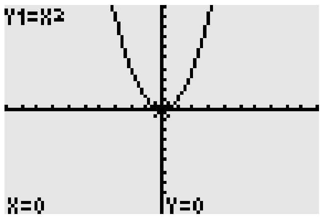
4. a) zeros: $x = -3$ (order 2), $x = 1$ (order 2), $x = 3$ (order 1)
b) zeros: $x = -2$ (order 3), $x = 4$ (order 2)
c) zeros: $x = -1$ (order 2), $x = 2$ (order 1), $x = 3$ (order 2)
d) zeros: $x = -4$ (order 1), $x = 1$ (order 3)

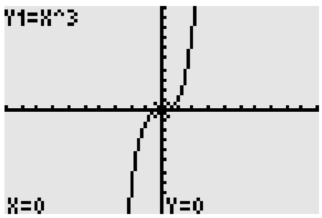
5. Example: If the degree of the factor is odd, the graph crosses the x-axis; if the degree of the factor is even, the graph just touches the x-axis.

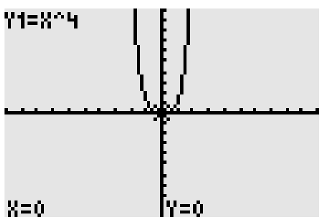
6. a) even
b) odd
c) neither
d) neither

7. a)









b) $f(x) = x$: degree: 1; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$

$f(x) = x^2$: degree: 2; as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$

$f(x) = x^3$: degree: 3; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$

$f(x) = x^4$: degree: 4; as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$

c) Example: As $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.

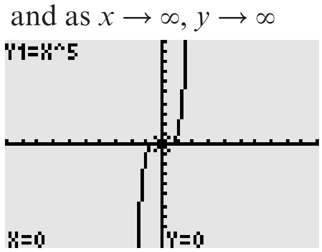
d) Example: As $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$.

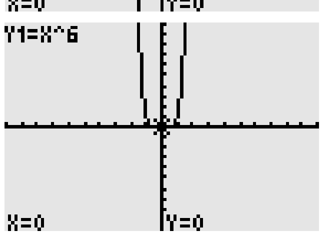
e) Example: Prediction:

$f(x) = x^5$: degree: 5; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$

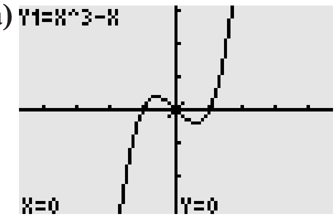
$f(x) = x^6$: degree: 6; as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$

f)



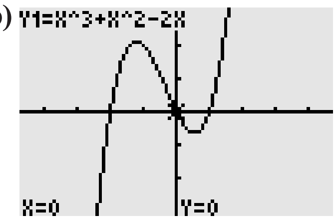


8. a)



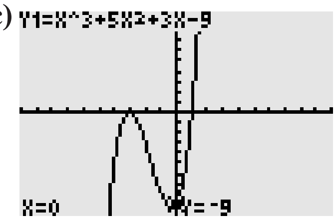
x-intercepts: $x = -1$, $x = 0$, $x = 1$;
no maximum or minimum point;
local maximum $\doteq 0.38$;
local minimum $\doteq -0.38$

b)



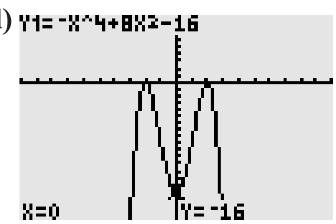
x-intercepts: $x = -2$, $x = 0$, $x = 1$;
no maximum or minimum point;
local maximum $\doteq 2.11$;
local minimum $\doteq -0.63$

c)



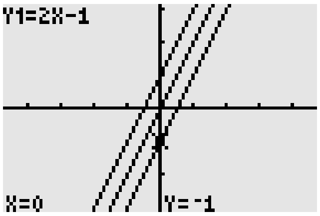
x-intercepts: $x = -3$, $x = 1$; no maximum or minimum point;
local maximum = 0;
local minimum $\doteq -9.48$

d)

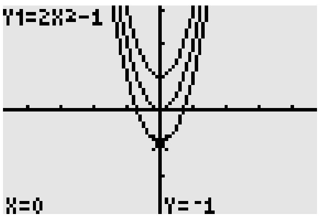


x-intercepts: $x = -2$, $x = 2$;
maximum = 0; local minimum = -16

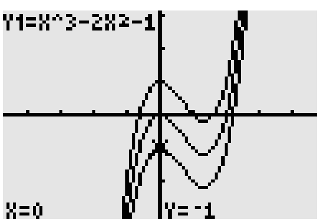
9. b) Example: The graph of a polynomial of degree 4 has the same end behaviour as the graph of a quadratic function, but can have more x-intercepts and a local maximum or minimum. The graph of a polynomial of degree 5 has the same end behaviour as the graph of a cubic function, but can have more x-intercepts, local maximums, and local minimums.

10. a) 

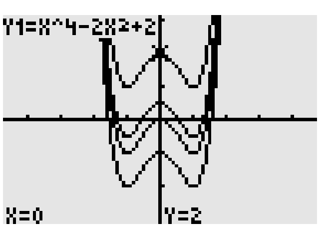
b) Each function has one x-intercept.
c) The maximum number of x-intercepts that a linear function can have is one.

11. a) 

b) $f(x) = 2x^2 - 1$: two x-intercepts; $g(x) = 2x^2$: one x-intercept; $h(x) = 2x^2 + 1$: no x-intercepts
c) The maximum number of x-intercepts that a quadratic function can have is two.

12. a) 

b) $f(x) = x^3 - 2x^2 - 1$: one x-intercept; $g(x) = x^3 - 2x^2$: two x-intercepts; $h(x) = x^3 - 2x^2 + 1$: three x-intercepts
c) The maximum number of x-intercepts that a cubic function can have is three.

13. a) 

b) $f(x) = x^4 - 2x^2 + 2$: no x-intercepts; $g(x) = x^4 - 2x^2 - 1$: two x-intercepts; $h(x) = x^4 - 2x^2$: three x-intercepts; $k(x) = x^4 - 2x^2 + 0.5$: four x-intercepts
c) The maximum number of x-intercepts that a quartic function can have is four.

14. a) zeros: $x = -3$, $x = -2$, $x = 2$, $x = 3$
b) even; satisfies the property that $f(-x) = f(x)$

15. $x = 1.28$

16. Example: For a function to be odd, it must have point symmetry about the origin. If a function has a constant term, it cannot have point symmetry about the origin.

17. Example: Even functions can have a constant term because a constant moves the graph vertically up or down; the function still has line symmetry about the y-axis.

5.4 Evaluating Polynomial Functions

1. **a)** 248
b) −4
c) 8
d) −4
e) 2
f) −2

2. **a)** 291
b) 1587
c) 61
d) 613
e) 21
f) −3

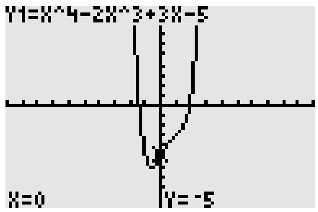
3. **a)** 2.67
b) −0.83
c) 10.5
d) −70.83
e) 0
f) 0.17

4. **a)** −7.33
b) 14.25
c) −33
d) 81
e) −0.33
f) 5.67

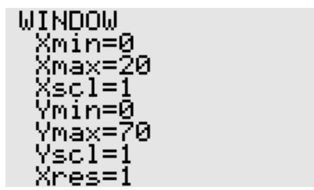
5. **a)** 5
b) −42
c) 4
d) −84

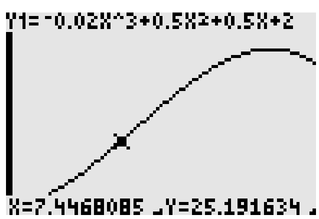
6. **a)** 71
b) 658
c) −264
d) 8
e) −365
f) −5

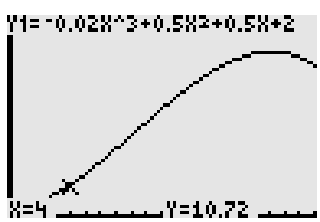
7. **a) i)** 0 **ii)** −5 **iii)** 1 **iv)** −5 **v)** 4
b) Example: The y-intercept is the constant term of the polynomial function.

8. **a) i)**


ii)–iv) $y = -5$
b) $y = -5$

9. **a)**


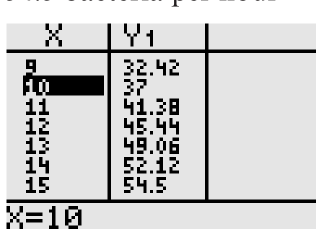


b)


10.72 bacteria per hour

c)

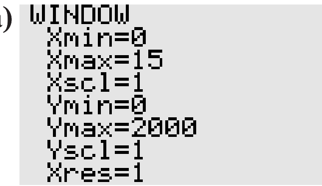

54.5 bacteria per hour

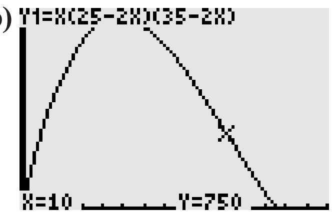
d)


X	Y1
9	32.42
10	37
11	41.38
12	45.44
13	49.06
14	52.12
15	54.5

X=10

10 h

10. **a)**


b)


If $x = 10$, then $V(x) = 750$. The volume of the box is 750 cm^3.

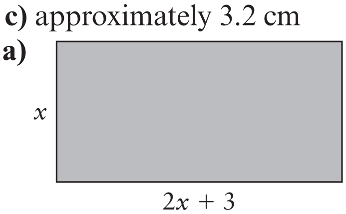
c) Substitute 10 for x in $V(x) = x(25 - 2x)(35 - 2x)$.

$$
\begin{aligned}
V(10) &= 10(25 - 2(10))(35 - 2(10)) \\
&= 10(25 - 20)(35 - 20) \\
&= 10(5)(15) \\
&= 750
\end{aligned}
$$

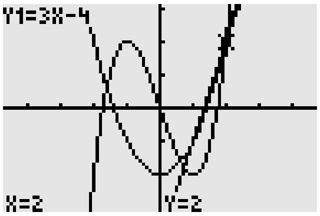
The volume of the box when the side length of each square is 10 cm is 750 cm^3.

d) Use the table of values on the graphing calculator. For the required volume of the rectangular box to be 1875 cm^3, the size of the squares to be cut from each box should be 5 cm.

11. **a)** $V = 2\pi r^3$
b) approximately 785 cm^3
c) approximately 3.2 cm

12. **a)**


b) $A = x(2x + 3)$
c) 44 m^2
d) approximately 4.3 m

13. **a)** $f(2) = 2$, $g(2) = 2$, $h(2) = 2$
b) Example: The three values are the same. The graphs of the three functions will intersect at the point (2, 2).
c) 

d) when $x = 2$, $y = 2$

14. **a)** $\sqrt{5}$
b) $\frac{1}{16}$
c) $\frac{27}{8}$
d) $\frac{1}{22}$

15. **a)** $f(a) = a^2 + 2a$
b) $f(x + 2) = x^2 + 6x + 8$
c) $f(x - 3) = x^2 - 4x - 3$
d) $f(x^2 + 2x) = x^4 + 4x^3 + 6x^2 + 4x$

5.5 Solving Problems Involving Polynomial Functions

1. **a)** 40.4 ft
b) 27.5 ft
c) 0 ft
d) approximately 6.1 s

2. **a)** 45.9 m
b) 249.9 m
c) 10 s
d) Example: Time cannot be negative.

3. **a)** \$242 580
b) \$264 000
c) approximately 17 400
d) $\{n \in \mathbb{R}, 0 \le n < 27\}$; where the graph is above the x-axis in the first quadrant

4. **a)** 6146.6 L; 1633 L; 280 L
b) 28 min
c) Example: The function allows you to determine that the t-intercept will be 28. So, the domain is $\{t \in \mathbb{R}, 0 \le t \le 28\}$.

5. **a)** quartic function
b) fourth differences
c) as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to \infty$
d) Domain: $\{x \in \mathbb{R}, x \ge 0\}$; the number of cameras sold cannot be negative.

6. **a)** Since the degree of the polynomial is 3, this is a cubic function.
b) Since the degree of the polynomial is 3, the third differences are equal.
c) as $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to -\infty$
d) The domain is restricted to the interval for which the graph of the function is above the x-axis and is positive. The domain is $\{t \in \mathbb{R}, 0 \le t \le 1.67\}$.

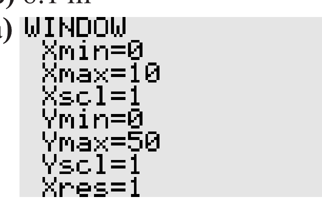
7. **a)** 13 000
b) 254 125

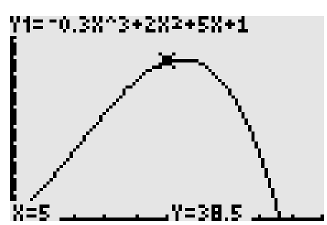
8.

Time (h)	Amount of Water Remaining (L)	First Differences	Second Differences
0	17 750		
1	16 280	−1470	
2	14 870	−1410	60
3	13 520	−1350	60
4	12 230	−1290	60
5	11 000	−1230	60

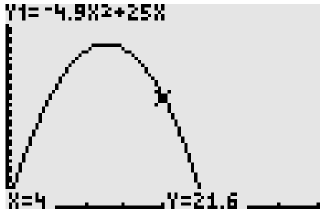
Since the second differences are equal, the type of polynomial function that best models this situation is a quadratic function.

9. **a)** as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to -\infty$
b) 8.1 m

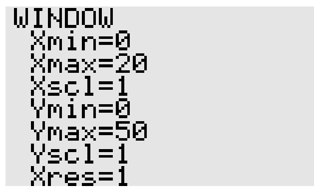
10. **a)** 

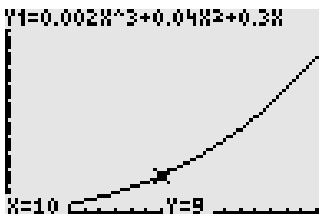


b) \$25.90

11. a) 

b) 30.4 m
c) 21.6 m
d) 5.1 s

12. a) 



b) 9 m
c) 20 s

13. a) 50 m
b) maximum height of 61.25 m occurs at 1.5 s
c) 10 m/s
d) 5 s
e) 30 m

5.6 Factoring Polynomial Expressions

1. a) $7(2x^2 + 3x - 1)$
b) $a^3(5a^2 - 16)$
c) $3b(b^2 - 3)$
d) $5h^2(h - 3)$
e) $4u^3v^2(2u^2 + v^2)$
f) $3m^2(3m^2 - 2m + 4)$

2. a) $(x + y)(a + b)$
b) $(x + y)(c - d)$
c) $(x + y)(x + y) = (x + y)^2$
d) $(a - b)(a - b) = (a - b)^2$

3. a) $(x + 2)(x + 5)$
b) $(j + 3)(j + 9)$
c) $(k + 1)(k + 4)$
d) $(m - 2)(m - 5)$
e) $(y - 1)(y - 4)$
f) $(r - 2)(r - 6)$

4. a) $(x + 3)(x - 1)$
b) $(a - 2)(a - 3)$
c) $(m + 2)(m + 8)$
d) $(d + 8)(d - 3)$
e) $(w - 10)(w + 3)$
f) $(b + 3)(b + 5)$

5. a) i) $(m + 4)(m + 4) = (m + 4)^2$
ii) $(q - 5)(q - 5) = (q - 5)^2$
iii) $(d + 10)(d + 10) = (d + 10)^2$
iv) $(v - 2)(v - 2) = (v - 2)^2$
v) $(s - 6)(s - 6) = (s - 6)^2$
vi) $(r + 3)(r + 3) = (r + 3)^2$
b) Example: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

6. a) $3x^2 - 3x - 18 = 3(x^2 - x - 6)$
$= 3(x + 2)(x - 3)$
b) $4a^2 - 20a - 56 = 4(a^2 - 5a - 14)$
$= 4(a + 2)(a - 7)$
c) $p^3 + 8p^2 + 15p = p(p^2 + 8p + 15)$
$= p(p + 3)(p + 5)$
d) $dm^2 - 9dm + 14d = d(m^2 - 9m + 14)$
$= d(m - 7)(m - 2)$
e) $3ht^2 + 12ht + 9h = 3h(t^2 + 4t + 3)$
$= 3h(t + 3)(t + 1)$
f) $m^3t^2 - 9m^3t + 20m^3 = m^3(t^2 - 9t + 20)$
$= m^3(t - 5)(t - 4)$

7. Example: Her factored form is equal to $3(x + 2)(x - 3)$, but her factored form is not fully factored.

8. a) $3(x + 6)(x - 1)$
b) $4a(a - 1)(a - 3)$
c) $5b(b^2 - 3b + 4)$
d) $y(y - 2)(y + 1)$
e) $2m^2(5m^2 + 2m - 4)$
f) $6z(z^4 - 2z^2 + 3)$

9. **a)** $2m^2 + 11m + 12 = 2m^2 + 8m + 3m + 12$
$= 2m(m + 4) + 3(m + 4)$
$= (m + 4)(2m + 3)$
b) $12a^2 - a - 6 = 12a^2 + 8a - 9a - 6$
$= 4a(3a + 2) - 3(3a + 2)$
$= (3a + 2)(4a - 3)$
c) $6m^2 + 7m - 3 = 6m^2 - 2m + 9m - 3$
$= 2m(3m - 1) + 3(3m - 1)$
$= (3m - 1)(2m + 3)$
d) $4m^2 + 5m - 6 = 4m^2 + 8m - 3m - 6$
$= 4m(m + 2) - 3(m + 2)$
$= (m + 2)(4m - 3)$
e) $20z^2 - 23z + 6 = 20z^2 - 15z - 8z + 6$
$= 5z(4z - 3) - 2(4z - 3)$
$= (4z - 3)(5z - 2)$
f) $8b^2 - 26b + 15 = 8b^2 - 20b - 6b + 15$
$= 4b(2b - 5) - 3(2b - 5)$
$= (2b - 5)(4b - 3)$

10. **a)** length: $(5x + 2)$; width: $(3x + 4)$
b) $x = 1$

11. length: $(x + 2)$; width: $(x + 1)$; height: x

12. **a)** length: $(3x + 1)$, width: $(x + 3)$
b) 16 m by 8 m

13. **a)** length: $(3x + 4)$, width: $(x + 7)$
b) 28 m by 15 m

14. **a)** length: $(7x + 10)$, width: $(3x + 5)$
b) 150 yd by 65 yd

15. **a)** length: $(7x + 1)$, width: $(3x + 4)$
b) 50 m by 25 m
c) 2.5 m

16. **a)** $h = -(5t + 2)(t - 5)$
b) 22 cm

17. Examples:
a) number sold: $20 - x$ or $40 - 2x$
b) price per jacket: $36 + 2x$ or $18 + x$

18. **a)** $(x + 2)(x + 1)(x - 1)$
b) $(x + 3)(x + 2)(x - 2)$

19. **a)** $(x - 2)(x + 2)(x - 1)(x + 1)$
b) $(x - 2)(x + 1)(x - 3)(x + 2)$

5.7 Difference of Squares of Polynomial Expressions

1. **a)** $(x + 5)(x - 5)$
b) $(y + 4)(y - 4)$
c) $(a + 3)(a - 3)$
d) $(m + 7)(m - 7)$
e) $(b + 1)(b - 1)$
f) $(d + 6)(d - 6)$

2. **a)** $(1 + e)(1 - e)$
b) $(5 + h)(5 - h)$
c) $(9 + q)(9 - q)$
d) $(6 + n)(6 - n)$
e) $(4 + s)(4 - s)$
f) $(7 + c)(7 - c)$

3. **a)** $(x + y)(x - y)$
b) $(a + b)(a - b)$
c) $(m + n)(m - n)$
d) $(p + q)(p - q)$
e) $(s + t)(s - t)$
f) $(d + e)(d - e)$

4. **a)** $(a + 2c)(a - 2c)$
b) $(h + 12y)(h - 12y)$
c) $(d + 3m)(d - 3m)$
d) $(f + 10e)(f - 10e)$
e) $(b + 13w)(b - 13w)$
f) $(z + 7u)(z - 7u)$

5. **a)** $(7r + f)(7r - f)$
b) $(8y + x)(8y - x)$
c) $(16g + k)(16g - k)$
d) $(5m + c)(5m - c)$
e) $(14j + l)(14j - l)$
f) $(10m + d)(10m - d)$

6. **a)** $(13x + 3y)(13x - 3y)$
b) $(12m + 7n)(12m - 7n)$
c) $(17a + 9b)(17a - 9b)$
d) $(4w + 19v)(4w - 19v)$
e) $(16k + 15m)(16k - 15m)$
f) $(6h + 5q)(6h - 5q)$

7. **a)** **i)** $(m^2 + 1)(m + 1)(m - 1)$
ii) $(m^2 + 4)(m + 2)(m - 2)$
iii) $(m^2 + 9)(m + 3)(m - 3)$
iv) $(m^2 + 16)(m + 4)(m - 4)$
b) Example:
$m^4 - 625 = (m^2 + 25)(m + 5)(m - 5)$,
$m^4 - 1296 = (m^2 + 36)(m + 6)(m - 6)$

8. **a)** $18x^2 - 32y^2 = 2(9x^2 - 16y^2)$
$= 2(3x + 4y)(3x - 4y)$
b) $q^3 - 25q = q(q^2 - 25)$
$= q(q + 5)(q - 5)$
c) $16m - 64m^3 = 16m(1 - 4m^2)$
$= 16m(1 + 2m)(1 - 2m)$
d) $9m^2n^2 - 81m^4 = 9m^2(n^2 - 9m^2)$
$= 9m^2(n + 3m)(n - 3m)$
e) $125a^4b^4 - 5a^4b^6 = 5a^4b^4(25 - b^2)$
$= 5a^4b^4(5 + b)(5 - b)$
f) $36p^2q^4 - 4p^2 = 4p^2(9q^4 - 1)$
$= 4p^2(3q^2 + 1)(3q^2 - 1)$

9. **a)** $a = 80$ and $b = 2$;
$(80 + 2)(80 - 2) = 6400 - 4 = 6396$
b) i) $(50 + 4)(50 - 4) = 2500 - 16 = 2484$
ii) $(100 + 2)(100 - 2) = 10\,000 - 4 = 9996$
iii) $(70 + 6)(70 - 6) = 4900 - 36 = 4864$

10. **a)** $\frac{1}{4}a^4 - \frac{1}{16}b^2$
$= \left(\frac{1}{2}a^2 + \frac{1}{4}b\right)\left(\frac{1}{2}a^2 - \frac{1}{4}b\right)$
$= \frac{1}{4}\left(a^2 + \frac{1}{2}b\right)\left(a^2 - \frac{1}{2}b\right)$
b) $\frac{1}{8}m^2 - \frac{1}{2}n^2$
$= \frac{1}{2}\left(\frac{1}{4}m^2 - n^2\right)$
$= \frac{1}{2}\left(\frac{1}{2}m + n\right)\left(\frac{1}{2}m - n\right)$

11. **a)** $(5x + 4y)(5x - 4y)$
b) $(3a + 2b)(3a - 2b)$

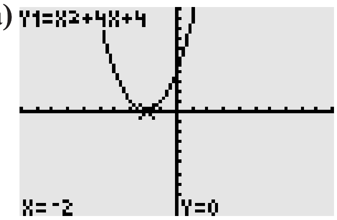
12. **a)** $(a + b + x + y)(a + b - x - y)$
b) $(m + 2n + a - 3b)(m + 2n - a + 3b)$

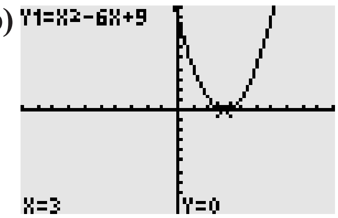
13. **a)** $(x + 2)(x - 2)$; $(x + 3)(x - 3)$
b) i) $(x + \sqrt{5})(x - \sqrt{5})$
ii) $(x + \sqrt{6})(x - \sqrt{6})$
iii) $(x + \sqrt{7})(x - \sqrt{7})$
iv) $(x + \sqrt{8})(x - \sqrt{8})$
$= (x + 2\sqrt{2})(x - 2\sqrt{2})$

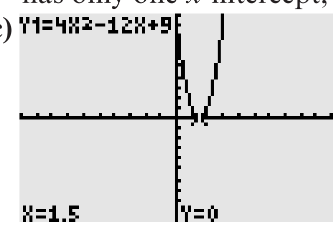
5.8 Intercepts of Polynomial Functions

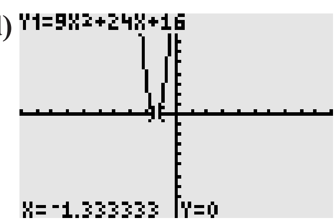
1. **a)** $f(x) = (x + 4)(x - 2)$; x-intercepts: $x = -4$, $x = 2$
b) $f(x) = (x + 3)(x - 3)$; x-intercepts: $x = -3$, $x = 3$
c) $f(x) = (x + 2)(x - 5)$; x-intercepts: $x = -2$, $x = 5$
d) $f(x) = (2x + 5)(2x - 5)$;
x-intercepts: $x = -\frac{5}{2}$, $x = \frac{5}{2}$

2. **a)** $f(x) = (3x + 2)(4x - 1)$;
x-intercepts: $x = -\frac{2}{3}$, $x = \frac{1}{4}$
b) $f(x) = (2x - 3)(4x - 5)$; x-intercepts: $x = \frac{3}{2}$, $x = \frac{5}{4}$
c) $f(x) = x(x + 7)(x - 2)$; x-intercepts: $x = -7$, $x = 0$, $x = 2$
d) $f(x) = x(x + 1)(x - 1)$; x-intercepts: $x = -1$, $x = 0$, $x = 1$

3. **a)** 

Example: The function is a perfect square trinomial $f(x) = (x + 2)^2$, which has only one x-intercept, $x = -2$.
b) 

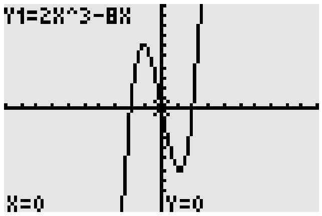
Example: The function is a perfect square trinomial $f(x) = (x - 3)^2$, which has only one x-intercept, $x = 3$.
c) 

Example: The function is a perfect square trinomial $f(x) = (2x - 3)^2$, which has only one x-intercept, $x = \frac{3}{2}$.
d) 

Example: The function is a perfect square trinomial $f(x) = (3x + 4)^2$, which has only one x-intercept, $x = -\frac{4}{3}$.

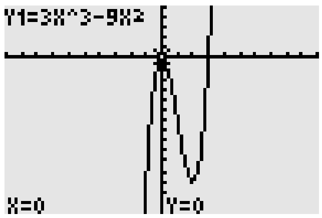
4. **a) i)** $f(x) = 2(x - 3)(x - 4)$; x-intercepts: $x = 3$, $x = 4$
ii) $f(x) = -2(x + 9)(x - 2)$;
x-intercepts: $x = -9$, $x = 2$
b) A numerical common factor does not affect the x-intercepts.

5. a) i)



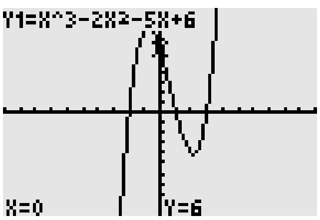
ii) x-intercepts: $x = -2$ (odd order), $x = 0$ (odd order), $x = 2$ (odd order)

b) i)



ii) x-intercepts: $x = 0$ (even order), $x = 3$ (odd order)

6. a)



x-intercepts: −2, 1, 3

b) $f(x) = (x + 2)(x - 1)(x - 3)$

To find the x-intercepts algebraically, let $f(x) = 0$.

$0 = (x + 2)(x - 1)(x - 3)$

$x + 2 = 0$ or $x - 1 = 0$ or $x - 3 = 0$

$x = -2$ $\quad$ $x = 1$ $\quad$ $x = 3$

Therefore, the x-intercepts of the function are $x = -2$, $x = 1$, and $x = 3$.

c) To determine the y-intercept, substitute 0 for x in the function $f(x) = x^3 - 2x^2 - 5x^2 + 6$ and evaluate.

$f(0) = 0^3 - 2(0)^2 - 5(0) + 6$

$= 6$

Therefore, the y-intercept is $y = 6$.

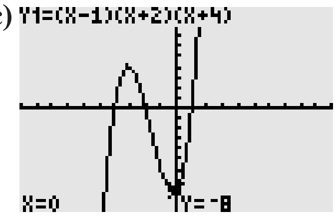
7. Example: Any numerical common factors of a function will not be apparent from the x-intercepts of the graph.

8. a) The zeros of the function are −4, −2, and 1. The zeros divide the x-axis into intervals.

b)

Test Interval	Sign of $f(x)$
$x < -4$	−
$x = -4$	0
$-4 < x < -2$	+
$x = -2$	0
$-2 < x < 1$	−
$x = 1$	0
$x > 1$	+

c)

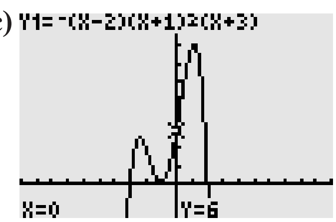


9. a) $x = -3$, $x = -1$, $x = 2$

b)

Test Interval	Sign of $f(x)$
$x < -3$	−
$x = -3$	0
$-3 < x < -1$	+
$x = -1$	0
$-1 < x < 2$	+
$x = 2$	0
$x > 2$	−

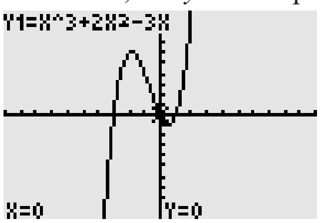
c)

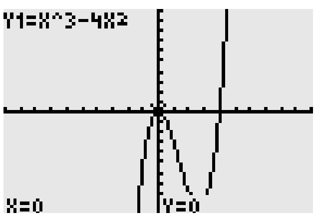


10. a) The degree of the polynomial function is 3. The leading coefficient is 1. The leading coefficient is positive.

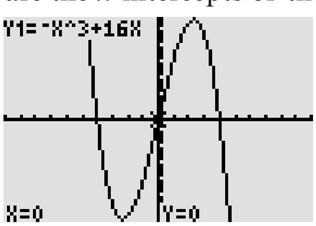
b) Since the polynomial function is of degree 3 and has a leading coefficient that is positive, as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.

c) Let $y = x(x + 3)(x - 1)$.
Substitute 0 for y, and solve for x.
$0 = x(x + 3)(x - 1)$
$x = 0$ or $x + 3 = 0$ or $x - 1 = 0$
$x = -3$ $x = 1$
Therefore, the x-intercepts are –3, 0, and 1.
Substitute 0 for x in $y = x(x + 3)(x - 1)$, and solve for y.
$y = 0(0 + 3)(0 - 1)$
$= 0$
Therefore, the y-intercept is 0.

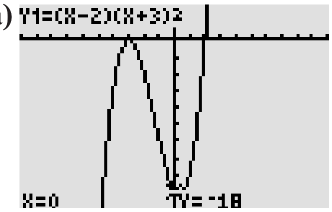
d) 

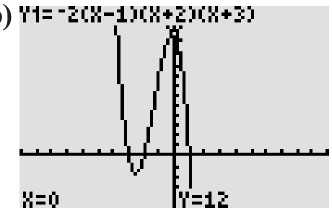
11. a) 

b) x-intercepts: $x = 0$ (even order), $x = 4$ (odd order)
c) real roots: $x = 0$, $x = 4$
d) Example: The real roots of an equation are the x-intercepts of the equation.

12. a) 

b) odd-degree function
c) x-intercepts: $x = -4$, $x = 0$, $x = 4$
d) real roots: $x = -4$, $x = 0$, $x = 4$
e) Example: The real roots of an equation are the x-intercepts of the equation.

13. a) 

b) 

14. a) real roots: $x = -5$, $x = -3$, $x = 3$, $x = 5$
b) Example: The real roots of the polynomial function are the same as the x-intercepts of the graph of the function.

15. Example: $y = -x^3 - 6x^2 - 11x - 6$

Chapter 6 Solve Polynomial Equations

6.1 Simplifying Polynomial Expressions

1. a) $30de$ **b)** a^2b^2 **c)** $12g^3h^4$
d) $-36b^4c^4$ **e)** $0.15c^6e^8$ **f)** $-24a^2b^2$

2. $28x^3y^5$

3. a) $4x^2y^3 + 5x^3y^4$
b) $2m^4n^4 - 6m^3n^5$
c) $-g^2h^4 - gh^5$
d) $-15a^4b^5 - 18a^2b^6$
e) $12c^3e^2 - 20c^2e^4$

4. a) $10x^3 - 6x^2 + 2x$
b) $-6x^4 - 15x^3 + 9x^2$
c) $abx^2 - acxy + axd$

5. Example: Multiply each term in the first binomial by each term in the second binomial.

6. a) $3x^2 + 3x^2 - 90$
b) $-2x^2 + 32x^2 - 126$
c) $-y^2 + 6y^2 + 16$
d) $2k^2 + 20k^2 + 42$

7. $\frac{25}{2}x^2 + \frac{35}{2}x + 6$

8. $4x^2 + 4x + 1$

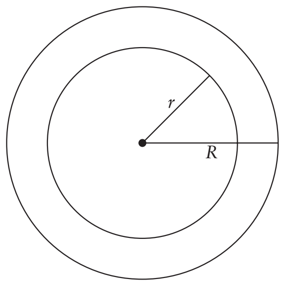
9. $2x^2 + 5x + 3$

10. $\pi x^2 + 4\pi x + 4\pi$

11. a) $\pi(R + r)(R - r) = \pi(R^2 - Rr + Rr - r^2)$
$= \pi(R^2 - r^2)$
$= \pi R^2 - \pi r^2$

Example: The area of a circle is $A = \pi r^2$. The area of the top of the doughnut is:
Area = area of the outer circle − area of the hole in the middle
$= \pi R^2 - \pi r^2$

b)



12. a) $x^2 + 4xy - 21y^2$
b) $x^3 + 6x^2y - 13xy^2 - 42y^3$

13. $2x^2 + 8x + 8$

14. a) $(3 + x)$, $x > 0$
b) $(2 + x)$, $x > 0$
c) Determine the area of the original rectangular platform.
Area = length × width
$= 3 \times 2$
$= 6$
Therefore, the area of the original rectangular platform is 6 ft^2.
The area of the new rectangular platform is double the area of the original rectangular platform, 6 × 2, or 12 ft^2.
Find the dimensions of the new rectangular platform.
Area = length × width
$12 = (3 + x)(2 + x)$
$12 = 6 + 5x + x^2$
$x^2 + 5x - 6 = 0$
$(x + 6)(x - 1) = 0$
$x + 6 = 0$ or $x - 1 = 0$
$x = -6$ $\quad x = 1$
Reject the negative answer.
The amount that the length and width must both be increased to double the area of the original platform is 1 ft.
Therefore, the dimensions of the new rectangular platform are 3 + 1, or 4 ft, by 2 + 1, or 3 ft.
d) Example: Graph $y = 12$ and $y = 6 + 5x + x^2$ and determine the point of intersection. Yes, the method would work with original dimensions 4 ft by 5 ft.

15. a) $6x^2 + 36x + 52$
b) $x^3 + 9x^2 + 26x + 24$

16. a) $42\pi x^2 + 50\pi x + 12\pi$
b) $36\pi x^3 + 69\pi x^2 + 34\pi x + 5\pi$

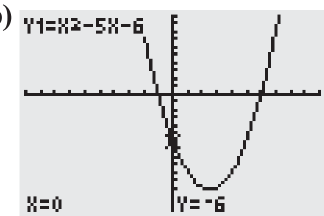
17. $x - 3y$

18. a) $16\pi x^2 + 48\pi x + 36\pi$
b) $\frac{32}{3}\pi x^3 + 48\pi x^2 + 72\pi x + 36\pi$

6.2 Strategies for Solving Polynomial Equations

1. a) −1, 0 **b)** 0, 2 **c)** −5, 0
d) 0, 7 **e)** $-\frac{5}{2}$, 0 **f)** 0, $\frac{4}{3}$

2. a) −1, 6
b)

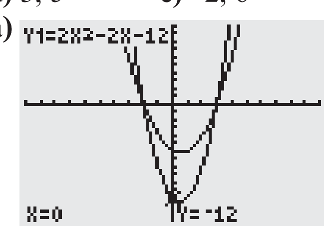


x-intercepts: −1 and 6
c) Example: The roots of the polynomial equation are the same as the x-intercepts of the graph of the corresponding polynomial function.

3. a) −2, 1 **b)** −1, 4 **c)** −2, −1
d) 2, 3 **e)** −7, 2 **f)** −7, 9

4. a) −3, 8 **b)** −5, 2 **c)** −5, −2
d) 3, 5 **e)** −2, 6 **f)** −5, 7

5. a)



x-intercepts for $f(x)$: −2 and 3;
x-intercepts for $g(x)$: −2 and 3
b) Example: The x-intercepts are the same.
c) −2, 3
d) none

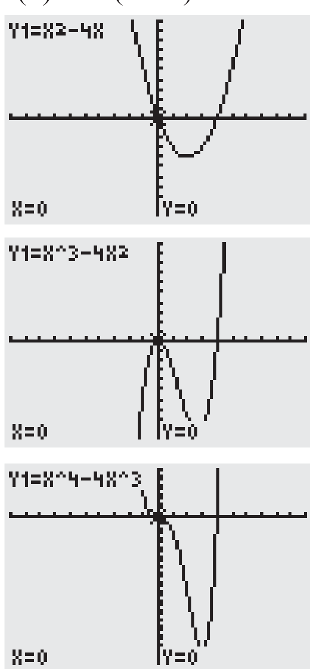
6. Example: Sandy should subtract 4 from both sides and then factor a difference of squares.

7. **a)** $-\frac{3}{2}, \frac{5}{3}$
b) $\frac{4}{3}, \frac{5}{2}$
c) $\frac{3}{2}, \frac{5}{3}$
d) $-3, -\frac{3}{5}$

8. **a)** $\frac{1}{2}, \frac{3}{2}$
b) $-2, \frac{1}{3}$
c) $\frac{1}{5}, 2$
d) $\frac{3}{4}, 2$

9. **a)** Example: It is not in the form $ax^2 + bx + c = 0$. The equation must be rewritten in the form $ax^2 + bx + c = 0$ before it can be factored.
b) $-\frac{3}{4}, \frac{7}{2}$
c) i) $-\frac{2}{3}, -\frac{1}{4}$ **ii)** $-\frac{5}{3}, 1$

10. **a)** $f(x) = x(x - 4)$, $g(x) = x^2(x - 4)$, $h(x) = x^3(x - 4)$
b)



c) Example: When the exponent on the factor is odd, the graph of the function crosses the x-axis. When the exponent on the factor is even, the graph of the function touches the x-axis.
d) Example: Since the degree is 3 and the leading coefficient is positive, as $x \to -\infty, y \to -\infty$, and as $x \to \infty, y \to \infty$. The graph will cross the x-axis at $x = 0$ and touch the x-axis at $x = 1$.

11. Example:
a) $2x^3 + 6x^2 - 20x = 0$
$x^3 + 3x^2 - 10x = 0$ Divide each term by 2.
$x(x^2 + 3x - 10) = 0$ Remove the common factor.
$x(x + 5)(x - 2) = 0$ Factor the trinomial.
Use the zero product property.
$x = 0$ or $x + 5 = 0$ or $x - 2 = 0$
$x = -5$ $x = 2$
Therefore, the roots of the polynomial equation are −5, 0, and 2.
To solve the equation graphically, graph $f(x) = 2x^3 + 6x^2 - 20x$ and determine the x-intercepts.
b) Graphing works in situations where the equation cannot be factored.

12. **a)** −1, 1
b) −2
c) −4, −3, 0
d) −3, 3

13. 8 mm

14. **a)** $0 = -5t^2 + 5t + 10$
b) There are two answers, $t = -1$ and $t = 2$. Since time cannot be negative, only the positive answer is admissible, $t = 2$. The diver hits the water 2 s after diving off the cliff.

6.3 Solving Equations of the Form $x^n = a$

1. **a)** 3 **b)** 3 **c)** 5
d) 4 **e)** 5 **f)** 3

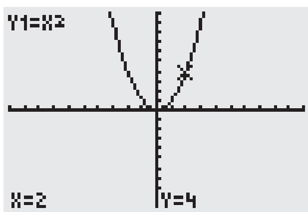
2. **a)** 2 **b)** 3 **c)** 2
d) 5 **e)** 81 **f)** 16

3. **a)** 1.91
b) 2.87
c) 7.62
d) 3.62
e) 9.24
f) 4.97

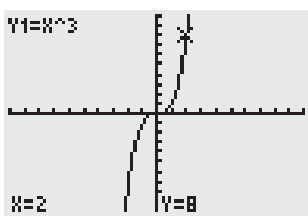
4. **a)** Example:
Start with the polynomial equation $x^5 = 32$.
Raise both sides to the exponent $\frac{1}{5}$.

$$(x^5)^{\frac{1}{5}} = 32^{\frac{1}{5}}$$
$$(x^{5 \times \frac{1}{5}}) = 32^{\frac{1}{5}} \quad \text{Simplify.}$$
$$x = 32^{\frac{1}{5}}$$
$$x = 2$$

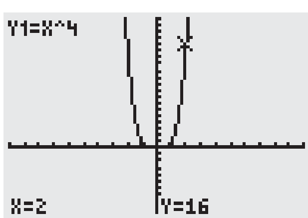
b) Example: No, n must be a whole number.

5. **a)**


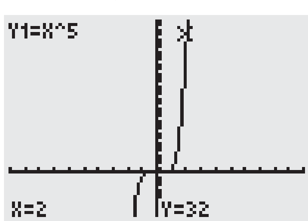
−2 and 2; there are two values of x.

b)


2; there is one value of x.

c)


−2 and 2; there are two values of x.

d)


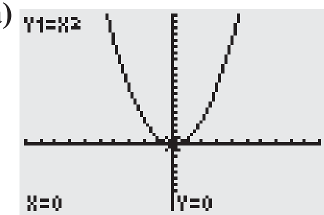
2; there is one value of x.

e) Example: If n is an even integer, there are two solutions to the polynomial equation $x^n = a$.

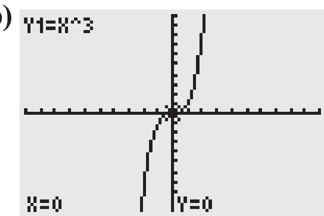
f) Example: If n is an odd integer, there is one solution to the polynomial equation $x^n = a$.

g) Example: The polynomial equation is not defined when $x = 0$ and $n = 0$.

6. Examples:

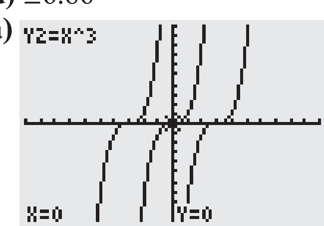
a)


For the polynomial function $y = x^2$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}, y \geq 0\}$. It is possible to solve the polynomial equation $x^2 = 15$ because 15 is in the range of $y = x^2$. It is not possible to solve $x^2 = -15$ because −15 is not in the range of $y = x^2$.

b)


For the polynomial function $y = x^3$, the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}\}$. It is possible to solve the polynomial equation $x^3 = 15$ because 15 is in the range of $y = x^3$. It is also possible to solve $x^3 = -15$ because −15 is in the range of $y = x^3$.

7. **a)** 2.08
b) ±4.24
c) 3.27
d) ±6.86

8. **a)**


b) Example: Similarities: The graphs of the three functions have the same domain $\{x \in \mathbb{R}\}$ and the same range $\{y \in \mathbb{R}\}$. The three graphs all extend from quadrant III to quadrant I. The graphs of the three functions all have one real root. Differences: The three graphs have different x-intercepts—the x-intercept for the function $y = (x + 3)^3$ is −3; the x-intercept for the function $y = x^3$ is 0; the x-intercept for the function $y = (x - 3)^3$ is 3. The function $y = x^3$ is an odd function and has point symmetry about the origin. The other two functions are neither even nor odd.

c) i) −1
ii) 2
iii) 5

d) i) predict: −4; check: −4
ii) predict: 8; check: 8

9. a) 0.89
b) 0.47 and −0.47

10. Answers may vary.

6.4 Functions and Formulas

1. a) Example: The constant is π. The variables are the area, A, and the radius, r.
b) 78.5 cm^2
c) 3.3 m

2. a) Example: The variables are the interest, I, the principal, P, the rate, r, and the time, t.
b) \$150
c) \$112.50

3. a) $m = dv$
b) $v = \frac{m}{d}$
c) 20 m^3

4. a) $a = \sqrt{c^2 - b^2}$
b) 6.9 cm
c) $b = \sqrt{c^2 - a^2}$
d) 7.5 cm

5. a) Example: The constant is k, the rate of change of pressure as the depth increases. The variables are P, the pressure at a depth of h metres underwater; P_0, the pressure at the surface; and h, the depth.
b) 12.5 kPa/m

6. a) 183.3 cm^3
b) 2.1 cm
c) 4.0 cm
d) r
e) h

7. a) Isolate the variable a.

$$d = vt + \frac{1}{2}at^2$$
$$d - vt = \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 = d - vt$$
$$at^2 = 2(d - vt)$$
$$at^2 = 2d - 2vt$$
$$a = \frac{2d - 2vt}{t^2}$$

Substitute 480 for d, 0 for v, and 14 for t.

$$a = \frac{2d - 2vt}{t^2}$$
$$= \frac{2(480) - 2(0)(14)}{14^2}$$
$$= \frac{960 - 0}{196}$$
$$= \frac{960}{196}$$
$$\doteq 4.9$$

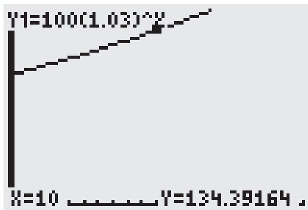
Therefore, the acceleration of the racing car is 4.9 m/s^2.

b) Substitute 480 for d, 0 for v, and 14 for t.

$$d = vt + \frac{1}{2}at^2$$
$$480 = (0)(14) + \frac{1}{2}a(14)^2$$
$$480 = 0 + \frac{1}{2}(196a)$$
$$480 = 98a$$
$$\frac{480}{98} = a$$
$$4.9 \doteq a$$

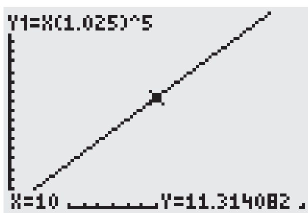
c) Answers may vary.

8. **a)** Example: The graph will be an exponential function in the form $y = a(b)^x$.

b) 

Yes; the prediction is correct.

c) Example: The graph will be a linear function in the form $y = mx$.

d) 

Yes; the prediction is correct.

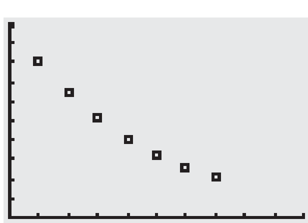
9. No, $2[P(1 + i)^n] \neq P(1 + i)2^n$.
No, $P[(1 + i)^{2n} - 1] \neq 2P[(1 + i)^n - 1]$.

10. **a)** Example: If the radius is kept constant, then a linear function will be generated. For example, if the radius is set at 3 m, then the linear function is $V(h) = 9\pi h$.
b) Example: If the height is kept constant, then a quadratic function will be generated. For example, if the height is set at 3 m, then the quadratic function is $V(r) = 3\pi h^2$.

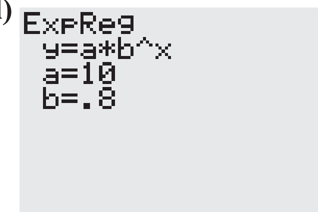
11. Example: No. A symbol is not a variable; π represents the same value wherever it is used.

12. **a)**

2-h Time Intervals	0	1	2	3	4	5	6	7
Amount of Medicine Remaining in Patient's Body (mL)	10	8	6.4	5.12	4.096	3.276 8	2.621 44	2.097 152

b) 

c) Example: The graph appears to model an exponential function.

d) 

$y = 10(0.8)^x$

13. Answers may vary.

14. **a)** $y = R(1.03)^x$, where R represents the monthly rent, x represents the number of years
b) $1066.50
c) Answers may vary.

15. Answers may vary.

16. **a)** 35 086 735
b) and **c)** Answers may vary.

6.5 Solving Multi-Step Problems Using Polynomial Equations

1. **a)** 3 m^2
b) Example: 3.5 m^2

2. **a)** 1.2 m^3
b) 7 m^2
c) Example: nails and hinges

3. Example: For an area of 8.8 m^2, I suggest 9 m^2 for the minimum amount of material.

4. **a)** Example: For an area of 42.625 ft^2, I suggest purchasing 80 tiles.
b) 8 boxes

5. $1260

6. **a)** The width of the deck is 8 ft. Each piece of cedar decking is 6 in., or 0.5 ft in width. Therefore, at least 16 pieces that are 12 ft in length are required. The deck is 15 ft in length. An additional 3 ft is required to be added to each of the 16 boards that are 12 ft in length. Therefore, an additional 3×16 or 48 ft of board length is required. This is equivalent to four additional pieces of decking that are 12 ft in length.
The total number of pieces is 16 + 4 or 20 pieces of wood, plus the purchase of one additional piece of wood is recommended.
Therefore, the minimum number of pieces that Vaughn should purchase is 21 pieces of cedar decking.

b) Cost of the 21 pieces of cedar decking is 21 × \$18.75, or \$393.75.

7. a) \$2700
   b) \$2970
   c) Example: There are costs associated with removing the old shingles, and labour costs.

8. $A_{sides} = 2 \times 7.3 \times 4.5$
   $= 65.7$
   $A_{back} = 3.2 \times 4.5 + 0.5 \times 3.2 \times 1.3$
   $= 16.48$
   $A_{front} = 3.2 \times 4.5 - (2 \times 3) + 0.5 \times 3.2 \times 1.3$
   $= 10.48$
   $A_{total} = A_{sides} + A_{back} + A_{front}$
   $= 65.7 + 16.48 + 10.48$
   $= 92.66$
   To allow for wastage, multiply the total area by 1.10.
   The total area of siding to be purchased is 92.66 × 1.10, or 101.926 m^2.
   The total cost of the white vinyl siding is 101.926 × 10.25, or \$1044.74.

9. a) 425 ft^2
   b)–f) Answers may vary.

10. a) Answers may vary.
    b) Example: the number of coats of paint, labour charge, expected percent of waste

11. a) 2 L b) \$46.18

12. a) \$133 802.35 b) \$634.51

13. a) 6 b) \$75

14. 3200 cm^2

15. approximately 90.5 m^2

16. and 17. Answers may vary.

Chapter 7 Solve Problems Involving Geometry

7.1 Area of Two-Dimensional Objects

1. a) Example: One rounds to two decimal places, another rounds to one to seven decimal places, and a third rounds to any number of decimal places.
   b) Example: When a number is converted from one unit, such as metres, to another unit, such as feet, the more decimal places you have in the converted number, the more accurate the answer is.

2. a) 7.62 m b) 96.52 cm
   c) 274.32 m d) 19.31 km

3. a) 236.22 ft
   b) 100.61 yd
   c) 17.72 in.
   d) 161.56 miles

4. a) 38.28 m^2 b) 96.77 cm^2
   c) 100.33 m^2 d) 72.52 km^2

5. a) 1290.32 cm^2
   b) 5.39 m^2
   c) 102.84 m^2
   d) 7.28 ha
   e) 246.05 km^2

6. a)

Square Inches	Square Centimetres
10	64.45
20	129.03
30	193.55
40	258.06
50	322.58

   b) 1 in.$^2 \doteq 6.45$ cm^2

7. a)

Metres	Feet	Square Metres	Square Feet
1	3.281	1	10.764
2	6.562	2	21.528
3	9.843	3	32.292
4	13.123	4	43.056
5	16.404	5	53.820

   b) Example: The number of square feet per square metre is equal to the number of feet per metre multiplied by 3.280 707 101. Since the number of feet per metre is approximately 3.280 707 101, then the number of square feet per square metre is equal to the number of square metres multiplied by 3.280 707 101 multiplied by 3.280 707 101, or the number of square metres multiplied by 3.280 707 101 squared.
   c) 64.618 ft^2
   d) Example: Yes. The relationship could be used to determine the number of square feet in any number of square metres.

8. Answers may vary.

9. a) Example: Since the units for the area of the hanging will be square centimetres, the radius must be multiplied by itself, or squared. Janet should use $A = \pi r^2$.

b) Example: Since the units for the circumference will be centimetres, the radius is not multiplied by itself, or squared. Therefore, she should use $C = 2\pi r$.

c) area: 706.9 cm^2; circumference: 94.2 cm

10. a) 8.8 m^2 **b)** 150 cm^2

c) 10 km^2 **d)** 24.6 $in.^2$

e) 32.5 ft^2 **f)** 46 m^2

11. c) Example: Sewer access hole covers are usually round. A circle will not pass through a circular hole that has the same diameter.

d) Answers may vary.

12. a) Example: It is easier to convert the dimensions first, and then calculate the area.

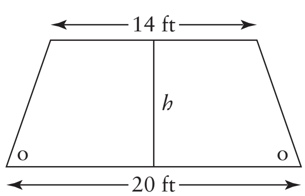
b) $523.85

c) Example: Laura should buy an additional 10% of the vinyl flooring for wastage.

13. Yes

14. area: 94.2 cm^2; width: 5 cm; length: 18.8 cm

15. a)



b) Determine the distance, h, in feet, between the two parallel sides.

$a = 14$, $b = 20$, and $A = 102$

$$A = \frac{1}{2}h(a + b)$$

$$102 = \frac{1}{2}h(14 + 20)$$

$$204 = 34h$$

$$\frac{204}{34} = h$$

$$6 = h$$

The distance between the parallel sides of the deck is 6 ft.

c) Example: A trapezoid and a rectangle are similar in that they both have one pair of parallel sides. A trapezoid and a rectangle are different in that a trapezoid may have at most two right angles, while a rectangle has four right angles.

d) The formula to calculate the area of a trapezoid, $A = \frac{h}{2}(b_1 + b_2)$, is related to the formula used to calculate the area of a rectangle and a parallelogram. Rewrite the formula.

$$A = \frac{h}{2}(b_1 + b_2)$$

$$A = h\frac{(b_1 + b_2)}{2}$$

The formula used to calculate the area of a trapezoid is the height multiplied by the average of the sum of the two parallel sides.

e) The area of the deck is 102 ft^2; 102 $ft^2 \doteq 9.476$ m^2; 1 gallon of stain covers 8 m^2, so Bob needs 2 gallons of stain.

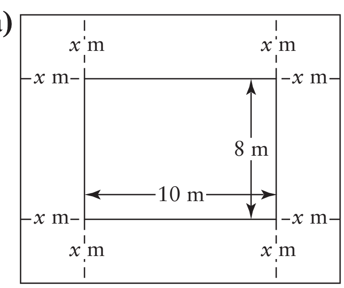
16. a) rhombus **b)** 454 cm^2 **c)** 70 $in.^2$

17. a) 25 tiles **b)** $50.70

18. 2240.7 m^2

19. 24 cm^2

20. a)



b) Calculate the area of the flower garden.

$$A_{garden} = \ell \times w$$
$$= 10 \times 8$$
$$= 80$$

The area of the flower garden is 80 m^2.

Let the width of the grassy area surrounding the flower garden be x m, $x > 0$.

Determine an expression to represent the grassy area that will surround the flower garden.

$$\begin{aligned} A_{\text{grassy area}} &= A_{\text{four corners}} \\ &\quad + A_{\text{two rectangles with side length 10 m}} \\ &\quad + A_{\text{two rectangles with side length 8 m}} \\ &= 4(x^2) + 2(10x) + 2(8x) \\ &= 4x^2 + 20x + 16x \\ &= 4x^2 + 36x \end{aligned}$$

$$\begin{aligned} A_{\text{grassy area}} &= A_{\text{garden}} \\ 4x^2 + 36x &= 80 \\ 4x^2 + 36x - 80 &= 0 \\ x^2 + 9x - 20 &= 0 \end{aligned}$$

Use the quadratic formula.

$$\begin{aligned} x &= \frac{-9 - \sqrt{9^2 - 4(1)(-20)}}{2(1)} \\ &= \frac{-9 - \sqrt{161}}{2} \\ &= -10.844\ldots \\ &\doteq -10.8 \end{aligned}$$

$$\begin{aligned} x &= \frac{-9 + \sqrt{9^2 - 4(1)(-20)}}{2(1)} \\ &= \frac{-9 + \sqrt{161}}{2} \\ &= 1.844\ldots \\ &\doteq 1.8 \end{aligned}$$

Since $x > 0$, the width of the grassy area that will surround the garden is approximately 1.8 m.

21. a) Example: Divide the area into a rectangle with length 8 ft and width 3 ft, and a trapezoid with height 3 ft and side lengths 8 ft and 6 ft.

b) Example: Divide the area into a square with side length 6 ft, and a trapezoid with height 2 ft and side lengths 6 ft and 3 ft.

c) Example: Divide the area into a rectangle with length 8 ft and width 3 ft, a rectangle with length 6 ft and width 3 ft, and a triangle with base 2 ft and height 3 ft.

d) 45 ft^2

22. 19.1 ft^2

23. a) 80.7 ft^2 **b)** 66.4%

24. a) 6814.2 cm^2 **b)** 81 tiles

25. a) 503.6 ft^2 **b)** 46.8 m^2

26. a) length: 7.5 in.; width: 3.75 in.; height: 5 in.; height highest point: 9.8 in.

b) 228.6 $in.^2$

c) Answers will vary.

d) Answers will vary.

7.2 Surface Area of Three-Dimensional Objects

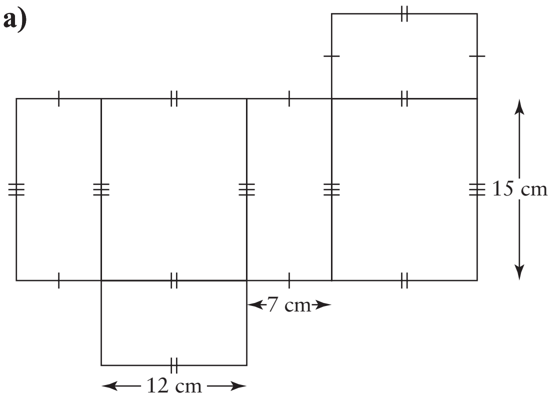
1. a) 45.4 ft^2

b) 120.5 cm^2

c) 93.7 $in.^2$

d) 184.6 m^2

2. a)



b) 738 cm^2

3. a) 347.5 cm^2

b) $237.00

c) Example: The formula would be $SA = \pi r^2 + 2\pi rh$.

4. 1024 $in.^2$; 32 in.

5. a) 6.0 in.

b) No. The area of the vertical side of the candle is halved, but the areas of the top and bottom are unchanged.

6. Substitute $SA = 2827$ in $SA = 4\pi r^2$.

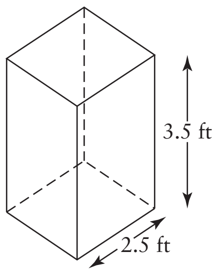
$$\begin{aligned} 4\pi r^2 &= 2827 \\ r^2 &= \frac{2827}{4\pi} \\ &= 224.998\ldots \\ r &\doteq \pm 14.998\ldots \end{aligned}$$

Since $r > 0$, $r \doteq 15.0$.

The radius of the beach ball is approximately 15.0 cm.

The diameter of the beach ball is double the radius of the beach ball, so the diameter is approximately 30.0 cm.

7. a)



b) 47.5 ft^2
c) 190 ft^2; by a factor of 4; answers will vary.
d) 427.5 ft^2, by a factor of 9; answers will vary.

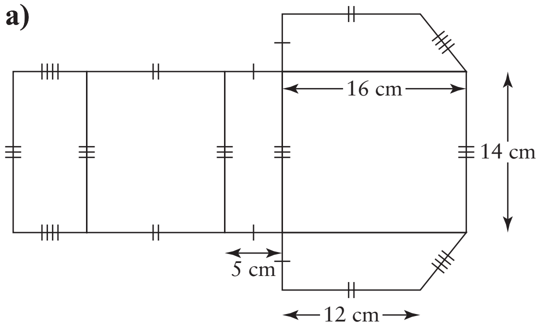
8. a) 153.9 $in.^2$
b) 615.8 $in.^2$
c) $SA_{\text{sphere B}} = 2SA_{\text{sphere A}}$
d) $SA_{\text{sphere B}} = \frac{1}{2}SA_{\text{sphere A}}$
e) 38.5 $in.^2$

9. Yes. Example: There will be enough material to fit over the section of the cylindrical water pipe. The surface area of the insulation is 1020 cm^2, which is greater than the surface area of the section of the water pipe to be covered (1005.3 cm^2).

10. a) $A_{\text{outside of bowl}} = 2\pi r^2$
$= 2\pi(30)^2$
$\doteq 5654.9$
b) $A_{\text{inside of bowl}} = 2\pi r^2$
$= 2\pi(28)^2$
$\doteq 4926.0$
c) $A_{\text{rim}} = \pi(30)^2 - \pi(28)^2$
$= 116\pi$
$\doteq 364.4$
d) Calculate the total area of exposed wood.
$SA \doteq 5654.9 + 4926.0 + 364.4$
$\doteq 10\,945.3$
Convert the surface area to square inches.
10 945.3 cm^2 ≐ 1696.5 $in.^2$
Determine the percent of a bottle of oil needed.
$\frac{169.5}{1800} \times 100 = 94.25$
Reid will use more than 94% of a bottle of oil.

11. 3540.2 cm^2

12. a)



b) 467.6 m^2

13. a) 6415.1 cm^2
b) 6.9 ft^2
c) Answers will vary.

14. 115 $in.^2$

15. 35.9 ft^2

16. a) 7.2 ft
b) 179.2 ft^2

17. 594.4 cm^2

18. a) $s = 18.7$ cm
b) 372.3 cm^2
c) 279.6 cm^2

19. $431.46

7.3 Volume of Three-Dimensional Objects

1. a) 5.7 m^3 b) 1835.4 cm^3
c) 5.4 m^3 d) 56 633.8 cm^3

2. a) 3001.7 ft^3 b) 412.0 yd^3
c) 12.7 $in.^3$ d) 10.6 ft^3

3. a)

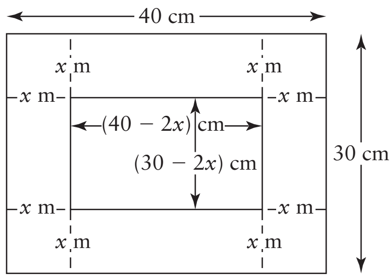
Cubic Centimetres	Cubic Metres
1	0.000 001
2	0.000 002
3	0.000 003
4	0.000 004
5	0.000 005

b) Example: To convert from cubic centimetres to cubic metres, divide the number of cubic centimetres by 100 000. To convert from cubic metres to cubic centimetres, multiply the number of cubic metres by 100 000.

4. Answers will vary. 6.6 L

5. a) 3053.6 cm^3 b) 1.75 $in.^3$ c) 7854.0 ft^3
d) 34.3 m^3 e) 34.5 ft^3

6. 904.8 cm^3
7. 8.2 cm
8. 3.1 yd^3
9. 8 cm
10. 10 983.5 ft^3
11. a) 56 m^3 **b)** 51 m^3
12. a) 432 000 cm^3 **b)** 4.32 m^3
13. a)



b) $V = x(40 - 2x)(30 - 2x)$
c) Example:
Press **Y =**. Type $(40 - 2x)(30 - 2x)$.
Press **WINDOW**. Set Xmin = 0, Xmax = 20, Xscl = 1, Ymin = 0, Ymax = 3500, and Yscl = 500.
Press **GRAPH**. Press **2nd**, **TRACE**, 4.
Use the arrow keys to move to the left of the maximum. Press **ENTER**.
Move the cursor to the right of the maximum. Press **ENTER**, **ENTER**.
The maximum occurs when $x = 6.56$.
d) The maximum volume is 3032.3 cm^3.
e) length: $40 - 2(5.65) = 28.7$;
width: $30 - 2(5.65) = 18.7$; height: 5.65
The dimensions of the box with maximum volume are 28.7 cm by 18.7 cm by 5.7 cm.

14. a) Calculate the surface area of the front of the office building.
Calculate the area of the rectangular area at the front of the office building.

$$\begin{aligned} A_{\text{rectangle}} &= \ell \times w \\ &= 28 \times 12 \\ &= 336 \end{aligned}$$

Calculate the height of the triangular portion using trigonometry.
Consider the right triangle with base 14 ft.

$$\begin{aligned} \frac{h}{14} &= \tan 30° \\ h &= 14 \tan 30° \\ &= 8.082... \\ &\doteq 8.1 \end{aligned}$$

Calculate the area of the triangular portion.

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2}(14)(8.1) \\ &= 113.4 \end{aligned}$$

The total area of the front of the office building is the sum of the rectangular and triangular areas at the front of the building.

$$\begin{aligned} A_{\text{total}} &= A_{\text{rectangle}} + A_{\text{triangle}} \\ &= 336 + 113.4 \\ &= 449.4 \end{aligned}$$

The total surface area is 449.4 ft^2.

Calculate the volume of the building.

$$\begin{aligned} V &= SA_{\text{total}} \times \ell \\ &= 449.4 \times 50 \\ &= 22\,470 \end{aligned}$$

The total volume of air in the building is 22 470 ft^3.
According to the design specifications, the ventilation system is required to exchange the air every 40 min.
The ventilation system exchanges air at the rate of 500 ft^3/min.
In 40 min, the amount of air that will be exchanged is $500 \times 40 = 20\,000$ ft^3/min.
Since the amount of air in the office building is 22 470 ft^3, the ventilation system does not satisfy the design specifications.
b) The minimum air exchange rate that would satisfy the design specifications: 562 ft^3/min

15. a) Example: Container B will hold more water because it is taller than container A.
b) Container A: 141.4 cm^3; container B: 70.7 cm^3; container A

16. Example: It would be better to use package A. The volume of package A is 128 $in.^3$ and its surface area is 104 $in.^2$. The volume of package B is 100 $in.^3$ and its surface area is 160 $in.^2$. Package A will hold more product, and it will be cheaper to make since it has less surface area than package B.

17. b) 10 cm by 10 cm by 10 cm

c) Cube; reasons may vary.

18. a) Sphere: 309.5 cm^2; cube: 384 cm^2; sphere has smaller surface area.

b) Example: Spheres cannot be stacked.

c) Example: Due to surface tension, a drop of water has a near-spherical shape; a sphere has the least possible surface area to volume ratio.

19. a) 2

b) 4

c) $V = \pi r^2 h$, so increasing the radius by factor 2 increases the volume by factor 2^2, or 4.

20. 2

21. a)

	House A
Surface Area	$A_{top} = 8 \times 8$ $= 64$ $A_{sides} = 4 \times 8 \times 6$ $= 192$ $SA = A_{top} + A_{sides}$ $= 64 + 192$ $= 256$
Volume	$V = \ell \times w \times h$ $= 8 \times 8 \times 6$ $= 384$
Surface Area/ Volume Ratio	$256 \div 384$ $= 2 \div 3$ $= 0.\overline{6}$

	House B
Surface Area	$A_{top} = 8 \times 16$ $= 128$ $A_{sides} = 2 \times 3 \times 8$ $= 48$ $A_{front/back} = 2 \times 3 \times 16$ $= 96$ $SA = A_{top} + A_{sides} + A_{front/back}$ $= 128 + 48 + 96$ $= 272$
Volume	$V = \ell \times w \times h$ $= 16 \times 8 \times 3$ $= 384$
Surface Area/ Volume Ratio	$272 \div 384$ $= 17 \div 24$ $= 0.708\overline{3}$

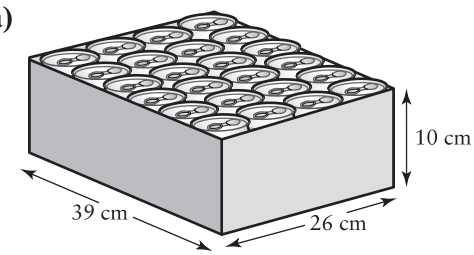
b) House A will have less heat gain and heat loss than house B. The surface area/volume ratio is less for house A than for house B.

c) Example: Other factors include the temperature of the ground, air, or snow with which the building is in contact; the direction and speed of winds blowing at the building; and the solar radiation incident on the building.

22. Answers will vary.

23. a) 418.9 ft^3 **b)** 75%

24. a)



b) length: 39 cm; width: 26 cm; height: 10 cm

c) 2176.1 cm^3

25. a) 7425 cm^3

b) Answers will vary.

c) 2123.9 cm^2

7.4 Properties of Circles

1. a) ii) **b)** iv) **c)** i) **d)** iii)
e) viii) **f)** v) **g)** vi) **h)** vii)

2. 5.2 cm

3. 120°

4. 7 in.

5. 66.4 m^2

6. a) 117.8 ft^2

b) 97.5 ft^2

c) 20.3 ft^2

7. a) Example: The fraction on the left side represents the number of degrees in the central angle, divided by the total number of degrees in a circle. The fraction on the right side represents the arc length divided by the circumference of the circle.

b) Example: The equation $\frac{\theta}{360°} = \frac{a}{2\pi r}$ can be obtained from the equation $a = \frac{\theta}{360°}(2\pi r)$ by multiplying both sides by $2\pi r$.

c) $\frac{\theta}{360°} = \frac{A}{\pi r^2}$

8. Example: $\frac{1}{6}$ of the circle. There are 360° in a circle. In simplest form, the fraction $\frac{60°}{360°} = \frac{1}{6}$.

9. 18 cm

10. 14 cm

11. a) distance $= \frac{3}{4}(2\pi)(32)$

$= 150.796\ldots$

Quinn ran approximately 150.8 m.

b) $\frac{3}{4} \times 360° = 270°$

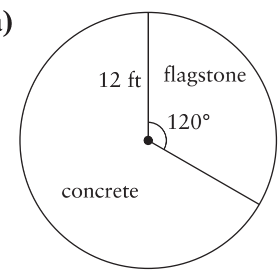
c) distance $= \frac{200°}{360°}(2\pi)(32)$

$= 111.701\ldots$

Isha ran approximately 111.7 m.

12. Dan: 37.7 in.^{2}; Nadine: 31.4 in.^{2}; Vidak: 44.0 in.^{2}

13. a)



b) Calculate the area of the patio that will be concrete.

Since the sector that will be concrete is two thirds of the area, the central angle is 240°.

$$A = \frac{\theta}{360°}\pi r^2$$
$$= \frac{240°}{360°}\pi(12)^2$$
$$\doteq 301.6$$

The area of the patio that will be concrete is approximately 301.6 ft^2.

Calculate the volume of the sector of concrete needed.

Convert the height of the concrete from inches to feet.

Since there are 12 in. in 1 ft, 20 in. $= \frac{5}{3}$ ft.

$$V = SA \times h$$
$$= 301.6 \times \frac{5}{3}$$
$$= 502.666\ldots$$

The volume of concrete needed is approximately 502.7 ft^3.

c) Calculate the area of the patio that will be flagstone.

Since the sector that will be flagstone is one third of the area, the central angle is 120°.

$$A = \frac{\theta}{360°}\pi r^2$$
$$= \frac{240°}{360°}\pi(12)^2$$
$$= 150.796\ldots$$

The area of the patio that will be flagstone is 150.8 ft^2.

Calculate the volume of flagstone needed.

Convert the height of the flagstone from inches to feet.

Since there are 12 in. in 1 ft, 2 in. $= \frac{1}{6}$ ft.

$$V = SA \times h$$
$$= 150.8 \times \frac{1}{6}$$
$$= 25.133\ldots$$

The volume of flagstone needed is approximately 25.1 ft^3.

14. Calculate the area of one segment.

The diameter of the tabletop is 107 cm, so the radius is $\frac{107}{2}$, or 53.5 cm.

$$A = \frac{1}{2}r^2\left(\frac{\pi}{180°}\theta - \sin\theta\right)$$
$$= \frac{1}{2}(53.5)^2\left[\frac{\pi}{180°}(90°) - \sin 90°\right]$$
$$= 816.880\ldots$$

Therefore, the area of one segment is approximately 816.9 cm^2.

The total area that is hinged to fold down is 816.9 × 2, or 1633.8 cm^2.

Calculate the total area of the tabletop.

$$A = \pi r^2$$
$$= \pi(53.5)^2$$
$$= 8992.023\ldots$$

The total area of the circular table top is approximately 8992.0 cm^2.

Find the percent of the surface area that is hinged to fold down.

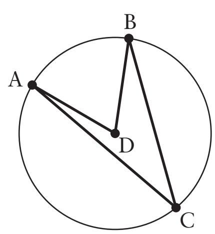
$$\frac{1633.8}{8992.0} \times 100\% = 18.169\ldots$$

Approximately 18.2% of the area of the tabletop is hinged to fold down.

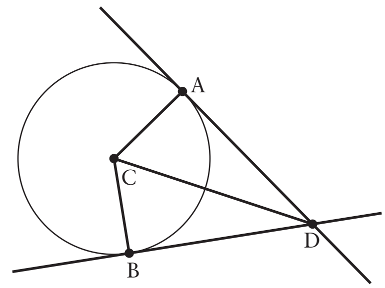
15. Answers will vary.
16. 41 887.9 cm^3
17. $A = \frac{1}{6}\pi r^2$; answers will vary.

7.5 Investigating Properties of Circles

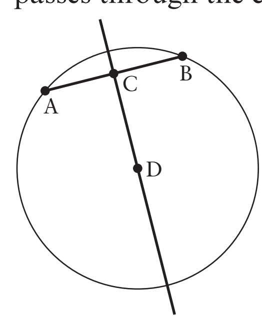
1. **a)** 40°
b) 59°
2. **a)** 3.7 cm
b) 11.2 m
3. **a)** 47°
b) 70°
4. **a)** 15 m
b) 4.0 cm
5. 11.2 in.
6. 6.6 ft
7. Example: The measure of an inscribed angle that is on the same side of a chord is one half the measure of the central angle that is subtended by the same chord.



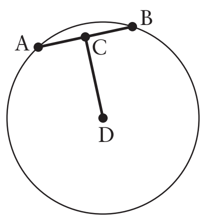
8. Example:
Open *The Geometer's Sketchpad*®.
Click on **New Sketch**.
Draw a circle with radius 4 cm.
Click on two points on the circle.
Label the two points on the circle A and B, and the centre of the circle C.
Draw a line segment for the radius AC and a line segment for the radius BC.
Click on point A and draw a tangent to the circle at point A.
Click on point B and draw a tangent to the circle at point B.
Label the point where the two tangent lines intersect as point D.
Calculate the measure of ∠CAD and ∠CBD to show that they are both equal to 90°.
Calculate the length AD and the length BD to show that they are both equal to 7.66 cm.
Two tangents of equal length can be drawn to a circle from a point that is outside the circle.



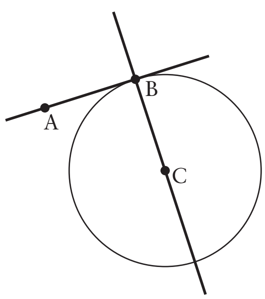
9. **a)** Example:
Open *The Geometer's Sketchpad*®.
Click on **New Sketch**.
Draw a circle with radius 4.98 cm.
Click on two points on the circle.
Label the two points on the circle A and B, and the centre of the circle D.
Draw a line segment for the chord AB.
Determine the midpoint of chord AB.
Label the midpoint C.
Click on point C and draw a perpendicular line through point C.
This line is the right bisector of chord AB.
The right bisector of chord AB passes through point D(0.00, 0.00), which is the centre of the circle.
Therefore, the right bisector of a chord passes through the centre of the circle.



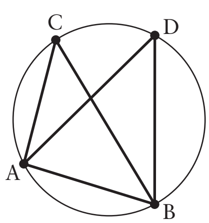
b) Example:
Open *The Geometer's Sketchpad®*.
Click on **New Sketch**.
Draw a circle with radius 4 cm.
Click on two points on the circle.
Label the two points on the circle A and B.
Determine the midpoint, C, of chord AB.
Join the centre of the circle, D, to the midpoint, C, of chord AB.
Determine the measure of ∠ACD = 90°.
Therefore, a line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.



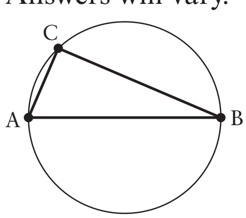
10. Answers will vary.



11. Answers will vary.



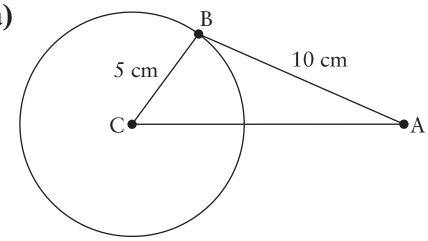
12. Answers will vary.



13. Answers will vary.
14. 16.3 km
15. Answers will vary.
16. 23.6°; answers will vary.

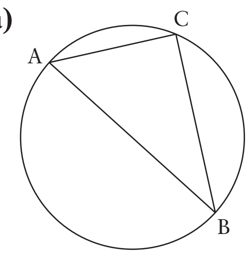
7.6 Solving Problems Involving Properties of Circles

1. a)



b) 11.2 cm

2. a)



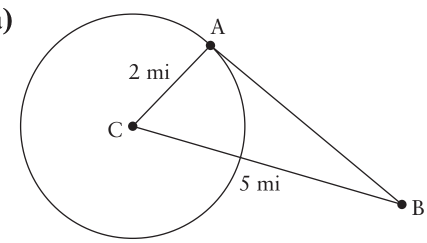
b) Example:
∠ACB = 90°, and line segment AB is the diameter of the circle.

3. KL = PQ = 3 in.

4. a) ∠CED = ∠CFD
b) Yes

5. a) Answers will vary.
b) Answers will vary.

6. a)



b) 4.6 mi

7. 183.3 m

8. 4 in.

9. Example:
Determine the sum of the squares of sides AB and AO.
$AB^2 + AO^2 = 4^2 + 3^2$
$= 16 + 9$
$= 25$
Determine the square of side OB.
$OB^2 = 5^2$
$= 25$
Since $AB^2 + AO^2 = OB^2$, then ∠OAB = 90°.
Therefore, AB is a tangent to the circular area on the field at point A.

10. Calculate the length of CD.

$CD^2 = AC^2 + AD^2$
$CD^2 = 5^2 + 12^2$
$CD^2 = 25 + 144$
$CD^2 = 169$
$CD = \pm\sqrt{169}$
$= \pm 13$

Since CD > 0, CD = 13 ft.

Calculate the area of △ACD.

$A = \frac{1}{2}bh$
$= \frac{1}{2}(5)(12)$
$= 30$

The area of △ACD is 30 ft².

Calculate the area of △BCD.

$A = \frac{1}{2}bh$
$= \frac{1}{2}(5)(12)$
$= 30$

The area of △BCD is 30 ft².

Calculate the total area of the two triangular areas.

Area of △ACD + Area of △BCD
= 30 + 30
= 60

The area of the two triangular areas is 60 ft².

Calculate the area of the circle.

$A = \pi r^2$
$= \pi(5)^2$
$= 78.539\ldots$

The area of the circle, with radius 5 ft, is approximately 78.5 ft².

Calculate the area of sector ABC.

For angle θ_1 in △ACD, the hypotenuse is CD = 13 ft, the opposite side is AD = 12 ft, and the adjacent side is AC = 5 ft.

$\tan\theta_1 = \frac{12}{5}$
$\theta_1 = \tan^{-1}\left(\frac{12}{5}\right)$
$= 67.4°$

For angle θ_2 in △BCD, the hypotenuse is CD = 13 ft, the opposite side is BD = 12 ft, and the adjacent side is BC = 5 ft.

$\tan\theta_2 = \frac{12}{5}$
$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right)$
$= 67.4°$

Determine the central angle, θ, of the sector.

$\theta = \theta_1 + \theta_2$
$= 67.4° + 67.4°$
$= 134.8°$

Calculate the area of a sector.

$A = \frac{\theta}{360°}\pi r^2$
$= \frac{134.8°}{360°}\pi(5)^2$
$= 29.408\ldots$

The area of sector ABC is approximately 29.4 ft².

The sector is a common area for the circle and the two triangles.

Calculate the area of the garden.

Area of the circle
+ Area of the two triangles
− Area of the sector
= 78.5 + 60 − 29.4
= 109.1

The area of the garden is 109.1 ft².

11. blue area: 1.5 in.²; green area: 1.5 in.²

12. Yes

13. 1059.8 m

Practice Exam

1. A
2. C
3. C
4. A
5. B
6. D
7. B
8. A
9. D
10. D
11. B
12. $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$, $\tan 150° = -\frac{1}{\sqrt{3}}$
13. a) 38° **b)** 83° **c)** 66°
14. a) 14.4 m **b)** 25.6 cm **c)** 8.5 km
15. a) 2
b) 90°
c) 50° horizontal to the left
d) 3 units down
e) $\{x \in \mathbb{R}\}$
f) $\{y \in \mathbb{R}, -5 \le y \le -1\}$
16. a) S50°E **b)** 280°

17. a) 75.0 N

b) 26.5°, counterclockwise from the horizontal

18. a) 5 **b)** $\frac{1}{27}$

19. 3

20. a) 4 **b)** −3

21. $\log_2 16 = 4$

22. a) 3

b) positive

c) Since the degree is 3 and the leading coefficient is positive, as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.

d) The third differences are equal.

e) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

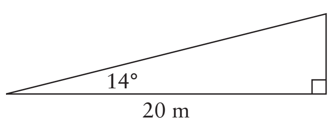
23. 26

24. $16x + 12$

25. 1.1 Canadian gallons

26. $\sin \theta = -\frac{6}{\sqrt{61}}$, $\cos \theta = \frac{5}{\sqrt{61}}$, $\tan \theta = -\frac{6}{5}$; $\theta \doteq 310°$

27. a)



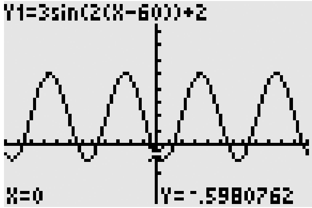
b) Yes. The vertical distance from the ground to the underside of the bridge is approximately 5.0 m. The height of the truck is 4.3 m. The clearance will be approximately 0.7 m.

28. $\angle A \doteq 33.6°$, $\angle B \doteq 50.7°$, $\angle C \doteq 95.7°$

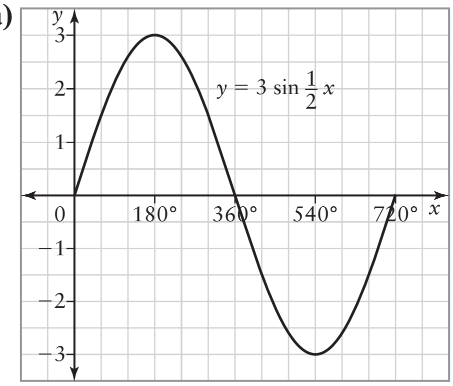
29. a) Apply the amplitude of 3, $y = 3 \sin x$; apply the vertical shift of 2 units up, $y = 3 \sin x + 2$; apply the horizontal compression of factor $\frac{1}{2}$, $y = 3 \sin 2x + 2$; translate the function 60° to the right, $g(x) = 3 \sin [2(x - 60°)] + 2$.

b) period 180°, amplitude 3, phase shift 60° to the right, range $\{y \in \mathbb{R}, -1 \le y \le 5\}$

c)

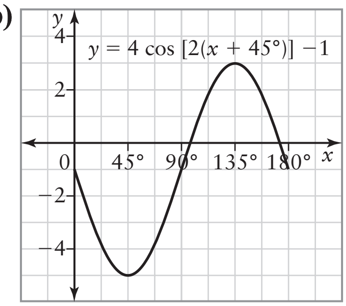


30. a)



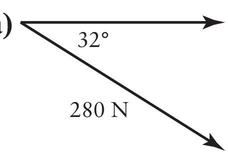
period 720°, amplitude 3, no phase shift

b)



period 180°, amplitude 4, phase shift 45° to the left

31. a)



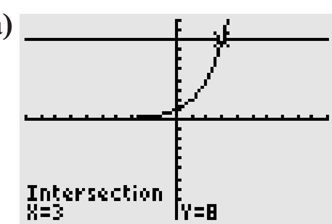
b) 237.5 N **c)** 148.4 N

32. a) 428.3 km

b) N74.7°E

33. 1

34. a)



$x = 3$

b) $x = -2$

35. a) $A(t) = 5000(1.008\ 75)^{4t}$

b) $5551.02

c) yes

d) no

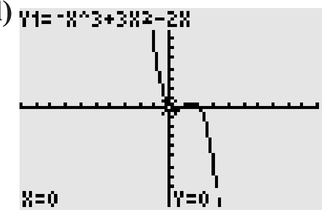
36. even; $f(-x) = f(x)$

37. a) quartic **b)** fourth differences
c) 15 000 **d)** 25 368

38. a) $5x^2y^2(y - 3x^2)$
b) $(x + 9)(x - 3)$
c) $(3m + n)(m + 7n)$
d) $(m + 9)(m + 9) = (m + 9)^2$
e) $2a(a + 3)(a - 2)$
f) $(5x + 7y)(5x - 7y)$

39. a) height: x; width: $(x + 2)$; length: $(x + 3)$
b) height: 2 cm; width: 4 cm; length: 5 cm

40. a) −5, −2 **b)** $-\frac{2}{5}, \frac{3}{2}$ **c)** $-\frac{3}{4}, \frac{1}{3}$

41. a) 3
b) as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to -\infty$
c) x-intercepts: 0, 1, 2; y-intercept: 0
d)


42. a) 150.8 cm^2 **b)** 7.9 m

43. a) 26.2 m **b)** 226.9 cm^2

44. 15 cm

45. $219.96

46. The resultant vector has a magnitude of 0.