

## Chapter 4 Test

- Simplify the following expressions. Write your answers using only positive exponents.
  - $a^2 \times a^4$
  - $b^3 \div b^{-2}$
  - $(4x^{-3})^{-2}$
  - $\left(\frac{a^5}{b^2}\right)^{-3}$
  - $(2ab^3)^4$
  - $(6x^{-2})(-5x^{-4})$
  - $\frac{48k^{-3}}{8k^{-1}}$
- Evaluate each of the following expressions, without the use of a calculator. Where appropriate, express your answer as a fraction.
  - $(3 \times 4^5)^0$
  - $3^{-2}$
  - $\left(\frac{4}{5}\right)^3$
  - $16^{\frac{3}{4}}$
  - $27^{\frac{1}{3}}$
- Simplify the following expressions.
  - $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$
  - $(9a^6)^{\frac{1}{2}}$
  - $\frac{54w^{\frac{3}{2}}}{9w^{\frac{1}{4}}}$
- Use a calculator to evaluate the following expressions. Where appropriate, express your answer to two decimal places.
  - $200 \times (1.015)^{20}$
  - $144^{\frac{3}{4}}$
  - $(-3)^{-2}$
  - $\sqrt[3]{(-5)^4}$
- Explain the difference between  $(-5)^2$  and  $-5^2$ .
- Express  $100^{\frac{3}{4}}$  as a radical.
  - Express  $\sqrt[3]{(-64)^5}$  as a power with a rational exponent.
- Describe the values of  $x$  for which the following statement is true:  $\sqrt{x^2} = x$ .
- A formula for the volume of a sphere is  $V = \frac{4}{3}\pi r^3$ . Determine the radius of a sphere with a volume of  $200 \text{ cm}^3$ . Express your answer to the nearest tenth of a centimetre.
- For the graph defined by  $y = 3^x$ 
  - state the domain
  - state the range
  - state the  $y$ -intercept
  - explain why the graph has no  $x$ -intercepts
- For the graphs defined by  $y = 2^x$ ,  $y = 5^x$ , and  $y = 10^x$ 
  - state how they are alike
  - state how they are different
- Describe how to use a graphing calculator to solve  $2^x = 25$ .
- Write an equation relating the amount,  $A$ , of a \$100 initial investment that grows at a rate of 1% per month for  $n$  months.
  - What will the investment be worth after 24 months?
  - How long, to the nearest month, will it take for the investment to double in value?
- Explain why  $y = \left(\frac{1}{2}\right)^x$  and  $y = 2^{-x}$  represent the same graph.
- For functions of the form  $f(x) = a^x$ , state which values of  $a$  will result in each of the following types of graph.
  - a graph that represents an increasing function
  - a graph that represents a decreasing function
  - a graph that cannot be defined



15. The value of a house was \$200 000 when it was built at the end of 2009. It is expected to increase in value at a rate of 4% per year.

- a) Write an equation using  $V$  for the value of the home and  $n$  for its age.
- b) Use a graphing calculator to determine the value of the house when it is 30 years old.

16. Due to a day of frost, 10% of a colony of beetles died. There were initially 2000 beetles in the colony.

- a) Write an equation for the number of beetles,  $n$ , after  $d$  days of frost.
- b) Use a graphing calculator to determine how many days of frost it will take for only 20% of the beetles to remain.

17. The following table shows the value of a motorcycle  $n$  years after it was purchased.

| Year | Value (\$) |
|------|------------|
| 1    | 15 000.00  |
| 2    | 10 500.00  |
| 3    | 7 350.00   |
| 4    | 5 145.00   |
| 5    | 3 601.50   |

- a) State the percentage by which the motorcycle is losing value each year.
- b) Determine the value of the bike when it was new.
- c) Write an exponential function relating the value,  $V$ , to the age,  $n$ .
- d) Determine the value of the motorcycle when it is 10 years old.
- e) Use a graphing calculator to determine how old the motorcycle will be when it is worth \$1000. Express your answer to the nearest tenth of a year.

18. Solve the following equations.

- a)  $2^x = 64$
- b)  $3^x = \frac{1}{81}$
- c)  $64^x = 16^{x+3}$

19. a) Solve  $2^{x+3} = 4^{x-1}$ .

- b) Describe how the point of intersection of the graphs defined by  $y = 2^{x+3}$  and  $y = 4^{x-1}$  is related to the equation in part a).

20. a) Write  $\log 1000 = 3$  in exponential form.

- b) Write  $2^x = 128$  in logarithmic form.

21. Evaluate without using a calculator.

- a)  $\log_3 81$
- b)  $\log 0.01$
- c)  $\log_5 5^{1/2}$
- d)  $\log_2 0.25$
- e)  $\log_a \sqrt[3]{a^2}$

22. Explain why  $\log_a 1 = 0$  when  $a > 0$ .

23. a) Describe how points from the graph defined by  $y = 2^x$  can be used to graph  $y = \log_2 x$ .

- b) How are the domain and range of  $y = 2^x$  related to the domain and range of  $y = \log_2 x$ ?

24. a) Without using a calculator, evaluate  $\log_4 64$ .

- b) Use a calculator to evaluate  $\frac{\log 64}{\log 4}$ .
- c) Use your results from parts a) and b) and a calculator to evaluate  $\log_5 40$  to one decimal place.

25. The number of bacteria in a culture doubles every hour. It is estimated there were 500 bacteria at noon on Monday.

- a) Write an exponential equation relating the number of bacteria,  $n$ , and the number of hours,  $h$ , since noon Monday.
- b) Later, there are 20 000 bacteria. Using this information, rewrite the equation in logarithmic form.
- c) Use the method from #24c) to determine when there will be 20 000 bacteria. Express your answer to the nearest minute.

