1.1 Sine, Cosine, and Tangent of Special Angles

Study Guide and Exercise Book Pages
1 to 3

Tools
- geometry set
- four quadrant grid paper
- computer with dynamic geometry software
- graphing calculator

Related Resources
- G–3 Four Quadrant Grids
- T–2 *The Geometer's Sketchpad*® 4
- T–4 The TI-Nspire™ CAS Calculator
- BLM 1–1 Chapter 1 Prerequisite Skills
- BLM 1–2 Chapter 1 Self-Assessment Checklist
- BLM 1–3 Trigonometric Ratios of Special Angles
- T1–1 How to Do Section 1.1 #15 Using TI-Nspire™ CAS
- T1–2 How to Do Section 1.1 #15 Using *The Geometer's Sketchpad*®
- A–4 Selecting Tools and Computational Strategies

Key Terms
- unit circle
- origin
- initial arm
- terminal arm
- reference angle
- rotation angle
- standard position
- quadrant
- special angles

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

- Before students begin the chapter, you may wish to have them complete **BLM 1–1 Chapter 1 Prerequisite Skills** to activate their prior skills.
- Give students **BLM 1–2 Chapter 1 Self-Assessment Checklist** to keep track of their skills and knowledge. Have students refer to it throughout the chapter.
- For further teacher support for this chapter, go to the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Key Concepts

- Have students create a reference sheet that they can use throughout this chapter.
- Review the Pythagorean theorem with students to help them better understand how to find the lengths of the sides for the two special triangles.
- Explain how to construct a 30°-60°-90° triangle by dividing an equilateral triangle with side length 2 in half, and use this triangle to determine exact values for sin 30°, cos 30°, sin 60°, and cos 60°.
- Explain how to construct a 45°-45°-90° triangle with equal sides one unit long to determine exact values for sin 45° and cos 45°.
- Students can draw and label a copy of the unit circle on their reference sheet.
- Review with students how to find the equation of a circle, centre (0, 0), with radius r ($x^2 + y^2 = r^2$), and how to find the equation of a unit circle with radius 1, ($x^2 + y^2 = 1$). Students may not automatically make the connection between "unit" and "1."
- Review with students the primary trigonometric ratios. For a given point (x, y) on a unit circle, discuss why $\cos\theta = x$, $\sin\theta = y$, and $\tan\theta = \frac{y}{x}$.

Example

- Remind students that the coordinates at the origin are (0, 0).
- Students may have difficulty determining the reference angle in quadrant I. The reference angle in quadrant I is equal to the acute angle between the x-axis and the terminal arm.
- Emphasize the difference between an exact value, such as $\frac{\sqrt{3}}{2}$, and an approximate value, such as 0.8660.
- Point out that $\tan\theta$ equals the slope of the terminal arm. This provides another context for undefined tangents, such as tan 90° and tan 270°.

Questions

- You may wish to provide students with **G–3 Four Quadrant Grids**.
- As students work through the assigned questions, have them begin to complete the summary sheet of the exact trigonometric ratios for special angles given in **BLM 1–3 Trigonometric Ratios of Special Angles**. Encourage students to look for patterns in the table.
- For most of the questions, students are expected to work with pencil and paper, but a scientific calculator can be used to check whether answers are correct. You may wish to demonstrate alternative technology, such as the TI-Nspire™ CAS.

Differentiated Instruction

- Use **graffiti** to see what students remember from their past studies of trigonometry. Put students into groups of three. Give each group a whiteboard marker or a piece of chalk. Allow them to draw trigonometry graffiti on one or two boards in the classroom.
- Construct a **word wall** of terms relevant to this chapter. Start with words listed under the Key Terms section. Include diagrams.

Common Errors

- Some students may still have trouble labelling the opposite and adjacent sides and the hypotenuse of a right triangle.

R_x Have students identify the right angle first. The hypotenuse is the side opposite the right angle, which is the longest side. Then, have students identify the angle of interest. The side opposite the angle of interest can be found by drawing a line inside the triangle from the angle to the opposite side. The side adjacent to the angle forms one of the arms of the angle.

- For **question 1**, students should use the special triangles to determine the exact values of the primary trigonometric ratios.
- For **questions 2** and **3**, you may wish to review rounding with students when they are determining the approximate values of the sine, cosine, and tangent ratios. Students may have trouble conceptualizing a triangle on the unit circle to evaluate the sine, cosine, and tangent ratios of 90°. Exact answers for trigonometric ratios of special angles that fall along the x- or y-axis can be evaluated on a calculator. Alternatively, use a calculator to evaluate sin 85°, sin 87°, sin 89°, sin 89.9°, sin 89.99°, and so on, to show how the value of sine approaches 1 as the angle approaches 90°. Do a similar activity for cos 90°, tan 90°, sin 0°, cos 0°, and tan 0°.
- Refer students to the **Example** when completing **questions 4** to **6**.
- Students may wish to add new terminology to their reference sheets as they encounter it in their work.
- For the **B** questions, encourage students to sketch diagrams for the application problems. The visual representation will enhance their understanding.
- Students will typically complete **question 11** using primary trigonometric ratios. You may wish to make students aware that this question can be solved with ratios by comparing their diagram to one of the special triangles.
- For **question 12**, ensure students do not confuse the slanted length of the ramp with the length of the base of the ramp.
- **Question 14** gives students the opportunity to connect a new concept to previous knowledge. Encourage students to draw a large diagram and perhaps work with a partner to reason through a possible solution to this problem.
- Use **question 15** to help students develop the CAST rule. Encourage students to add this diagram to their chapter reference sheet. Have copies of **T–2** ***The Geometer's Sketchpad*® 4** available for students.
- For **question 16**, students should write the exact value of each trigonometric ratio, and then multiply and add as indicated. You may wish to review the rules for multiplication and addition of fractions.
- **Question 17** gives students an opportunity to think backward. Be sure to discuss the fact that there are two answers for part **a**) and four answers for part **b**), due to a positive and negative root on the right-hand side. Remind students that numbers have a positive square root and a negative square root, but the radical or square root symbol, $\sqrt{\ }$, indicates the positive square root of a number.
- For **question 19**, students must remember rules for division of fractions while developing an identity.

Technology Suggestions

- For students using a TI-83 Plus/TI-84 Plus calculator for **question 1**, the mode must be changed to degrees, since radians is the default. Press **MODE**, scroll to Degree, and press **ENTER**.
- Students using TI-Nspire™ CAS may benefit from using **T–4 The TI-Nspire™ CAS Calculator**.
- If TI-Nspire™ CAS is in RAD mode, the value of a trigonometric ratio can be found for angles measured in degrees. Enter the angle measure and then press (ctrl)(/)(enter). If TI-Nspire™ CAS is in AUTO mode, an exact answer will always be provided. If a decimal approximation is desired, press (ctrl)(/)(ctrl)(enter).
- To change TI-Nspire™ CAS to DEG mode from the default RAD mode, press (home), choose **System Info** then **System Settings**. Press (tab) repeatedly until the **Angle:** box is highlighted. Select **Degree**.

- For applets that may assist students to complete **question 2**, go to **www.mcgrawhill.ca/books/mct12** and follow the links. The applets provide decimal values for angles that are manually entered. There is also a visual that may be helpful to students.
- For applets that may assist students to complete **questions 15,** go to **www.mcgrawhill.ca/books/mct12** and follow the links. The applets provide the decimal values of sine, cosine, and tangent ratios, and their reciprocals for any angle. The sine and cosine ratios are the default. Remember to click on the check box for tangent.
- You may wish to have students use **T1–1 How to Do Section 1.1 #15 Using TI-Nspire™ CAS** or **T1–2 How to Do Section 1.1 #15 Using *The Geometer's Sketchpad*®**. Students produce a unit circle with a point on it, and then drag the point around the circle and observe the how the sine, cosine, and tangent ratios change as the angle changes.
- To access a TI-Nspire™ CAS file or *The Geometer's Sketchpad*® file to help students investigate **question 15**, go to **www.mcgrawhill.ca/books/mct12** and access the Online Learning Centre.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	11–14
Reasoning and Proving	15, 17, 19, 20
Reflecting	2, 3, 7, 17
Selecting Tools and Computational Strategies	4–6, 13, 14
Connecting	4–6, 13, 17
Representing	11–13
Communicating	7, 11, 13–15

Ongoing Assessment

- You may wish to use **A–4 Selecting Tools and Computational Strategies** to assess students' responses to **question 13**.