1.3 Trigonometry of Angles

Study Guide and Exercise Book Pages

8 to 10

Tools

- four quadrant grid paper
- graphing calculator
- computer with dynamic geometry software

Related Resources

- G–3 Four Quadrant Grids
- T–2 *The Geometer's Sketchpad*® 4
- T–4 The TI-Nspire™ CAS Calculator
- A–6 Representing

Common Errors

- Some students may not have their calculator set to degree mode.

R$_x$ Make students aware of their calculator settings and how to adjust them to degrees.

- Some students add or subtract reference angles to/from 270° to translate them into quadrant III or IV.

R$_x$ Have students review how triangles were drawn in section 1.2. No triangle is attached to the *y*-axis, and therefore no angle in quadrant III or IV should be calculated using 270°.

Teaching Suggestions

Key Concepts

- Have students add new terminology to their chapter reference sheet.
- Ask students to explain why exactly two angles between 0° and 360° have the same sine, cosine, or tangent ratio.

Example

- The **Example** explores the application of the CAST rule.
- Before beginning, have students describe clearly what the $\sin^{-1}$ function on their calculator does. Remind students that not all calculators are the same. Some students may evaluate $\sin^{-1}(0.6)$ by entering $\sin^{-1}$, and then 0.6. Other students may evaluate $\sin^{-1}(0.6)$ by entering 0.6, and then $\sin^{-1}$.
- Review the concept that the *x*-values, or cosine values, are negative to the left of the *y*-axis (in quadrants II and III) and positive to the right of the *y*-axis (in quadrants I and IV). Similarly, the *y*-values, or sine values, are negative below the *x*-axis (in quadrants III and IV) and positive above the *x*-axis (in quadrants I and II).
- Remind students that when a triangle is drawn in any quadrant, it is always on the *x*-axis. The reference angle is either subtracted from or added to 180°, or it is subtracted from 360°.
- As an extension, complete a problem similar to the **Example**, but with a negative sign on the right side. Ask students to calculate $\sin^{-1}\left(-\frac{3}{5}\right)$ and $\sin^{-1}\left(\frac{3}{5}\right)$ and compare the results. Students will recognize the importance of calculating the reference angle in the first quadrant before finding their final answers.

Questions

- You may wish to provide students with **G–3 Four Quadrant Grids**.
- In **question 2**, students have an opportunity to explain the CAST rule.
- For help with **question 3**, refer students to the **Key Concepts**.
- Remind students that they are not solving for θ in **question 4**. They are only to write the sine, cosine, and tangent ratios for θ.
- To answer **questions 7** to **10**, students need a solid understanding of the signs of the trigonometric ratios and the special triangles.
- For **question 11**, it may be valuable to discuss the values that $\sin\theta$, $\cos\theta$, and $\tan\theta$ can take. Note that $\sin\theta$ and $\cos\theta$ are always between -1 and 1, and $\tan\theta$ can take on any value. Refer students to the lengths of the opposite side, the adjacent side, and the hypotenuse of a right triangle.
- For **question 13**, most students will find the required angle first, and then find the other two trigonometric ratios. If students take this approach, discuss how rounding affects the answers. As an extension to part **a**), you may wish to find the solution by having students draw the triangle in quadrant I, labelling the opposite side 0.7312 and the hypotenuse 1. The adjacent side can be found using the Pythagorean theorem, and then cos A and tan A can be calculated.

Differentiated Instruction

- Use **think-aloud** to have one or two students lead the class through an example: Determine the measures of all angles θ from 0° to 360° for which $\tan\theta = -1.5$. Have another group lead the class through a similar example.

- For **question 18,** have students sketch the appropriate triangle in quadrant III. Note that ∠A is at the origin and ∠S is the other acute angle in the triangle. In part **c)**, students may not understand that ∠B can be in quadrant III or IV.
- For **questions 19** and **20**, some students may have trouble remembering how to factor. For **question 20b)**, remind students to consider both the positive and negative roots, $\tan\theta = \pm\sqrt{3}$.
- For **question 21**, remind students that $\tan x = \frac{\sin x}{\cos x}$ and that restrictions are values for which $\tan\theta$ is undefined, when $\cos x = 0$.

Technology Suggestions

- For **question 17**, it may be useful to construct a graph of $y = \cos\left(\frac{\pi x}{180°}\right)$ with $0° \leq x \leq 360°$ and $-1 \leq y \leq 1$. The fraction $\frac{\pi}{180°}$ converts the function $y = \cos x$ from radians to degrees. Graphing the horizontal line $y = \frac{1}{3}$ reveals that there are two points of intersection with the cosine curve.
- For **question 18**, it may be useful to construct a *The Geometer's Sketchpad*® or TI-Nspire™ CAS document that shows a grid. Have copies of **T–2 *The Geometer's Sketchpad*® 4** and **T–4 The TI-Nspire™ CAS Calculator** available. Plot S(–5, –6). Construct a right triangle with a vertical leg joining S to the x-axis. The angle nearest the origin is a reference angle for ∠A. Determine the distance from the origin to point S. After measuring the acute angle, 180° is added to find the principal angle, ∠A. Determine ∠B by using $\sin^{-1}$ (sin A).
- For **questions 19** and **20**, TI-Nspire™ CAS can be used to find the zeros. Press (home) and choose: **Calculator**. Press (menu), choose **Algebra**, then **Factor**. Type the expression and press (enter). Once the factors are determined, insert a **Graphs & Geometry** page and graph the function and a vertical line at each zero.
- To access a TI-Nspire™ CAS file or *The Geometer's Sketchpad*® file to investigate angles between 0° and 360° with the same sine, cosine, or tangent ratios, access the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7–10, 16, 17, 19, 20
Reasoning and Proving	7–11, 16–18, 21
Reflecting	4, 6, 21
Selecting Tools and Computational Strategies	4, 12, 20
Connecting	6–10, 14, 18
Representing	11, 12, 15
Communicating	2, 11, 12, 19

Ongoing Assessment

- Use **A–6 Representing** to assess students' responses to **question 12**.