1.6 Solving Problems Using the Cosine Law

Study Guide and Exercise Book Pages
19 to 22

Tools
- geometry set
- string
- tape
- computer with dynamic geometry software
- graphing calculator

Related Resources
- T–2 *The Geometer's Sketchpad®* 4
- T–4 The TI-Nspire™ CAS Calculator
- BLM 1–4 Chapter 1 Review
- BLM 1–5 Chapter 1 Practice Test
- BLM 1–6 Chapter 1 Case Study
- A–5 Connecting

Key Terms
- contained angle
- cosine law

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Common Errors
- Some students have trouble entering a long formula into their calculator.

R_x Since calculator functions vary, remind students that it is important to bring their own calculator to class every day and to be familiar with how their calculator works.

Teaching Suggestions

Key Concepts

- Have students add the formula for the cosine law to their chapter reference sheets.

Example

- Students may be unaware that a calculator follows the order of operations. It is important to discuss how to enter the formula in line two.
- When students are determining angles using the cosine law, some students may prefer to substitute first, and then rearrange. Other students may prefer to do the opposite. Either method is acceptable.
- Have students complete another problem that requires them to use the cosine law to determine a side length.
- As an extension, you may want to complete a problem that involves the cosine law, the primary trigonometric ratios, and the sine law:
  Fatima is a forest ranger in a lookout tower 50 m above the ground. Directly north of her is the ranger station, at an angle of depression of 2°. The station has just informed Fatima of a lost hiker. Fatima spots the hiker's campfire 55° west of north at an angle of depression of 1.4° from the tower. How far is the tower from the ranger station? How far is the tower from the lost hiker? How far is the hiker from the ranger station? In which direction should the rescue team leave the ranger station to reach the lost hiker?
- Students may benefit from modelling a three-dimensional problem, such as the forest ranger problem, by standing in the three locations and holding pieces of string: one student represents the tower, and the others represent the station and the lost hiker.
- Remind students that the primary trigonometric ratios can be used only for right triangles. The sine law requires AAS (any two angles and a side) or SSA (two sides and an angle opposite one of the sides). The cosine law requires SAS (two sides and a contained angle) or SSS (all three sides). Students may want to add this memory aid to their chapter reference sheets.

Questions

- Students should be encouraged to sketch and label a diagram for each of **questions 3** to **15**.
- For **questions 5** and **8**, students should find the contained angle first.
- For **question 7**, the 6-in. length represents the slanted side of the cone. As an extension, you may wish to ask students how the answer would change if the 6-in. length represented the vertical distance from the mouth of the funnel to the tip.
- Students may find **question 14** very difficult to visualize and sketch. Using tape and string to model this problem would be a valuable exercise.
- Ask students to identify the extraneous information in **question 15**. They may wish to build the three-dimensional model with a partner.
- You may wish to take students on a field trip to learn more about some occupations in which trigonometry is used, such as surveying. Alternatively, you may wish to invite guest speakers to talk about their careers.

Differentiated Instruction

- Construct a **decision tree** for determining when to solve trigonometric problems using primary trigonometric ratios, the sine law, and the cosine law.

Common Errors

- Some students may think of 1 min as 0.1 h.

R_x Guide students to understand that 1 min is $\frac{1}{60}$ h. You may wish to use a diagram of an analog clock to show them the fractions.

- You may wish to use **BLM 1–4 Chapter 1 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 1–5 Chapter 1 Practice Test** to prepare them for the chapter test.

Case Study

- You may wish to have students complete **BLM 1–6 Chapter 1 Case Study**, which incorporates the learning from Chapter 1.
- Students may need help distinguishing between air traffic controllers and airport ground crews. IFR (instrument flight rules) controllers direct air traffic for en route aircraft, while VFR (visual flight rules) controllers give takeoff and landing instructions from airport towers. Airport ground crews direct aircraft into and out of the terminal using light sticks.
- For **question 1**, students should consider senses such as sight and hearing.
- For **question 2**, you might emphasize how the safety of people on flights relies on the performance of the ATC.
- For **question 3a)**, students should choose the cosine law to determine the distance between CJA and HBI.
- For **question 3b)**, some students might not realize that they should multiply the airplane speed by time to determine the distance travelled. This question illustrates how quickly the situation can change due to the high speeds of airplanes.

Technology Suggestions

- Provide copies of **T–2 *The Geometer's Sketchpad*® 4** and **T–4 The TI-Nspire™ CAS Calculator**.
- For **question 5**, students may wish to construct a diagram using a TI-Nspire™ CAS **Graphs & Geometry** page or *The Geometer's Sketchpad*®. A useful scale is 1 unit to 2 km.
  - Construct a vertical segment near the bottom right corner of the screen.
  - Construct two points to the left of the segment.
  - Measure an angle by clicking on the top end of the vertical segment, then on the bottom end, and then on one of the points. Repeat this process to measure another angle formed using the other point. Drag the points until the angles are 60° and 70° northwest.
  - Construct a line from the bottom end of the vertical segment through each of the points.
  - Construct a point on each of the new lines. Measure the distance to the bottom end of the vertical segment from each new point. Make the distances 11.9 units along the 60° line of sight and 12.6 units along the 70° line of sight.
  - Measure the distance between the two new points.
- For **questions 8** and **9**, students may wish to construct a diagram using a TI-Nspire™ CAS **Graphs & Geometry** page or *The Geometer's Sketchpad*®.
- For **question 10**, students may wish to construct a diagram using a TI-Nspire™ CAS **Graphs & Geometry** page or *The Geometer's Sketchpad*®.
  - Construct a vertical line after clicking on the screen near the top centre of the screen. Construct another line that intersects the vertical line. This line represents the pendulum.
  - Construct a horizontal line through any point on the vertical line.

– Construct a point at the intersection of the horizontal line with the line that represents the pendulum. Make the horizontal distance between the end of the pendulum and the vertical 2.5 units.
– Make the length of the line that represents the pendulum 10 units.
– Construct a reflection of the point at the bottom of the pendulum in the vertical line. Measure the angle from the image point to the top of the pendulum to the bottom of the pendulum on the other side of the vertical line.

Mathematical Process Expectations

Ongoing Assessment

- Use **A–5 Connecting** to assess students' responses to **question 7**.

Summative Assessment

- You may wish to use the chapter test that you can find in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	5–15
Reasoning and Proving	11
Reflecting	11
Selecting Tools and Computational Strategies	7–10
Connecting	5, 7, 10, 11, 13–15
Representing	11, 14
Communicating	4, 10, 11