2.1 Graphs of Sinusoidal Functions

Study Guide and Exercise Book Pages
23 to 26

Tools
- grid paper
- graphing calculator

Related Resources
- G–5 Trigonometric Graph Paper
- G–6 Graphs of the Sine and Cosine Functions
- T–4 The TI-Nspire™ CAS Calculator
- BLM 2–1 Chapter 2 Prerequisite Skills
- BLM 2–2 Chapter 2 Self-Assessment Checklist
- T2–1 How to Draw the Sine Curve From the Unit Circle Using TI-83 Plus/TI-84 Plus
- T2–2 How to Draw the Cosine Curve From the Unit Circle Using TI-Nspire™ CAS
- T2–3 How to Do Section 2.1 #8 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS
- A–6 Representing

Key Terms
- amplitude
- period
- range
- displacement
- cycle
- sinusoidal function
- intervals of increase and decrease
- domain
- range

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

- Before students begin the chapter, you may wish to have them complete **BLM 2–1 Chapter 2 Prerequisite Skills** to activate their prior skills.
- Give students **BLM 2–2 Chapter 2 Self-Assessment Checklist** to keep track of their skills and knowledge. Have students refer to it throughout the chapter.
- For further teacher support for this chapter, go to the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Key Concepts

- Have students create a reference sheet to refer to throughout this chapter. They can begin by drawing graphs of $y = \sin x$ and $y = \cos x$.
- Have students add the definitions of the Key Terms to their reference sheets.
- Discuss the terms *cycle*, *period*, and *periodic*.

Example

- Ensure students understand that the maximum and minimum values are y-values.
- Remind students that the sine and cosine functions cycle infinitely. We generally draw only one or two cycles for convenience.
- Some students may have difficulty understanding amplitude. It may be helpful for students to draw horizontal lines through the maximum values, the minimum values, and the centre of the sine and cosine functions.
- You may wish to review subtracting integers. For example, $1 - (-1) = 1 + 1$, or 2.
- Review the terms *domain* and *range* when discussing the solution to the **Example**.
- Review notation for domain and range.
- Review with students the direction in which the inequality signs face. Remind students that inequalities such as $-1 \leq y \leq 1$ are typically written with the lesser value on the left and greater value on the right.
- Explain that for a sine or cosine function with a domain of $\{x \in \mathbb{R}\}$, there are actually an infinite number of x-intercepts.
- You may want to have students explain in their own words the meaning of *intervals of increase* and *intervals of decrease*. They may offer suggestions such as, "the x-values on which the graph is going up/going down (as we read from left to right)."
- Ask students to suggest why cosine functions are also called sinusoidal functions.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- Ensure students connect the graphs of $y = \sin x$ and $y = \cos x$ with the unit circle and with the CAST rule by relating **question 1** to the work they did in Chapter 1. Remind students of the special triangles (30°-60°-90° and 45°-45°-90°). Go to the Online Learning Centre at **www.mcgrawhill.ca/books/mct12** for an enlarged copy of the unit circle.

Common Errors

- Some students may have their inequality signs facing backward when writing intervals.

R_x Remind students that the "point" of the inequality sign points toward the smaller value, and the "open mouth" faces the larger value.

- Some students, when drawing one cycle of the cosine function, make the graph look like a V-shape.

R_x Have students practise drawing waves, with rounded bottoms and tops. Remind students that the cosine and sine functions have the same shape but different positions.

Differentiated Instruction

- Build a model of a circle out of cardboard. Pin the centre of it to a bulletin board. Use the circle to model a Ferris wheel. Mark the lowest point on the circle as the lowest car of the Ferris wheel. Make a table of values with columns labelled Degrees and Height. Measure the height from the floor to the car every 10° as the car rotates counterclockwise around the wheel. Have students plot the points to get a cosine function. Discuss how the activity can be modified to get a sine function.
- Construct a **word wall** of terms relevant to this chapter. Start with words listed in the Key Terms section. Include pictures.

- **Questions 1**, **2**, and **4** provide a good opportunity for students to reflect on Chapter 1. Plotting many ordered pairs for the sine and cosine functions helps to reinforce the shapes of the graphs.
- Students having difficulty may benefit from using **G–6 Graphs of the Sine and Cosine Functions**. Have students plot points from **questions 2** and **4** on the sine and cosine curves.
- For **questions 2** to **5**, remind students to have their calculators set to degrees when calculating the sine and cosine ratios.
- For **questions 3** and **5**, remind students to think about appropriate window settings for graphing the sine and cosine functions.
- For **question 6**, you may need to remind students of the definition of a function.
- **Question 7** is a useful thinking/inquiry question. If students have difficulty, refer them to the Exact columns of the tables in **questions 2** and **4**. Alternatively, students could complete this question using a graphing calculator.
- **Questions 8** to **10** will challenge most students. Ensure students know what *horizontal displacement* means. Students may have trouble conceptualizing the problem. A cardboard model will be useful so that students can visualize the horizontal or vertical displacement versus the angle through which you turn for one rotation of the wheel.
- Use **questions 11** and **12** for students who require enrichment. Encourage students to use technology to graph the functions. Remind students how to use a graphing calculator to determine the x- and y-intercepts.
- For **question 12**, have students discuss whether the graphs are sinusoidal.
- Encourage students to explore and graph functions that are more complicated than the functions given in **questions 11** and **12**.

Technology Suggestions

- To change the TI-83 Plus/TI-84 Plus to degree mode, press **MODE** and select **Degree**.
- You may wish to have copies of **T–4 The TI-Nspire™ CAS Calculator** available.
- To change the TI-Nspire™ CAS to degree mode, press (home), choose **System Info**, and then **Document Settings**. Press (tab) to **Angle:**, press (doc) and choose **Degree**.
- To set the window on a TI-83 Plus/TI-84 Plus, press **WINDOW** and enter appropriate values.
- To set the window on a TI-Nspire™ CAS calculator, press (home) and choose **Graphs & Geometry**. Press (menu). Choose **Window**, then choose **Window Settings**, and then enter appropriate values.
- When using TI-Nspire™ CAS in **AUTO** mode and **RAD** mode, you can find the trigonometric ratios for angles measured in degrees by typing a degree symbol after the angle measure. Press (ctrl) '. If the calculator is in **AUTO** mode, the trigonometric ratios will be exact. If you wish decimal approximations, press (ctrl) (enter).
- When using TI-Nspire™ CAS in **APPROX** mode and **DEG** mode, you can find the trigonometric ratios for angles measured in degrees without having to put a degree symbol in the expression. The trigonometric ratios will be decimal approximations.
- You may wish to use **T2–1 How to Draw the Sine Curve From the Unit Circle Using TI-83 Plus/TI-84 Plus** and/or **T2–2 How to Draw the Cosine Curve From the Unit Circle Using TI-Nspire™ CAS** for this section.
- For **questions 2** and **4**, a fraction template can be used with TI-Nspire™ CAS, which allows students to enter a fraction from the unit circle. Press (ctrl) ÷.

- Some students may wish to use a TI-83 Plus/TI-84 Plus to complete **question 5**. Press **WINDOW**. Use the settings Xmin = 0, Xmax = 360, Xscl = 90, Ymin = −1.1, Ymax = 1.1, Yscl = 0.5. Press **Y=**, enter the function $y = \cos x$, and then press **GRAPH**. Press **2nd**, **TRACE**, **1:value**. Enter the angles from the table, and the corresponding y-value will be displayed.
- Some students may wish to use TI-Nspire™ CAS to complete **question 5**. Press (home) and choose **Graphs & Geometry**. Press (menu). Choose **Window**, and then **Window Settings**. Use the settings XMin = 0, XMax = 360, XScale = 90, YMin = −1.1, YMax = 1.1, YScale = 0.5. Press (cos) × (ctrl) ' (enter). Press (menu). Choose **Trace**, and then **Graph Trace**. Type a degree measure and press (enter). A decimal value will be reported. You will also see the point on the graph that is represented. You could open a **Lists & Spreadsheet** page and enter the angles in the first column with the title Angle. Name the second column Value. In the row under Value, type =. Press (cos) and type angle. Press (enter). The list will populate with the decimal approximations of the cosine ratio.
- For **questions 6** and **7**, once the graph has been constructed, students can insert a vertical line on the graph and move it left and right. On a TI-83 Plus/TI-84 Plus: Press **2nd**, **PRGM**. Choose **4:Vertical**. Press the left and right arrows to move the line left and right. On a TI-Nspire™ CAS: Press (menu). Choose **Construction** and **Perpendicular**. Click on the curve and then on the x-axis to construct the vertical line. Press (esc) to release the tool. Move the arrow over the point on the curve and grab it by pressing and holding (ctrl). Use the arrow keys to move the line.
- Students may benefit from using **T2–3 How to Do Section 2.1 #8 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS** to complete **question 8**.
- For applets that demonstrate how to construct $y = \sin x$ and $y = \cos x$ from the unit circle, go to **www.mcgrawhill.ca/books/mct12** and follow the links.
- For an excellent Ferris wheel demonstration, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7–9
Reasoning and Proving	2, 4, 7
Reflecting	1, 3, 7, 11, 12
Selecting Tools and Computational Strategies	7–9
Connecting	1, 2, 4–7, 12
Representing	3, 5, 9–10
Communicating	2, 3, 5–10, 12

Ongoing Assessment

- Use **A–6 Representing** to assess students' responses to **question 7.**