2.2 Translations of Sinusoidal Functions

Study Guide and Exercise Book Pages
27 to 29

Tools
- grid paper
- graphing calculator
- coloured pens, pencils, or markers

Related Resources
- G–5 Trigonometric Graph Paper
- T–4 The TI-Nspire™ CAS Calculator
- T2–4 How to Do Section 2.2 #11 and 12 Using TI-83 Plus/ TI-84 Plus and TI-Nspire™ CAS

Key Terms
- parameters
- phase shift or horizontal translation
- vertical shift or vertical translation

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Have students add new terminology to their chapter reference sheets.
- Discuss with students how the range has changed.
- Students may be confused by the term *phase shift*. Remind students that the phase shift is the horizontal shift. Have students research the word *phase* on the Internet in relation to waves, and make a literacy connection with the term *phase shift* for sinusoidal functions.

Example

- You may wish to review translations of quadratic functions. Have students graph $y = x^2$ by plotting five points, and ask them what happens to the parabola if the equation changes to $y = (x - 2)^2 + 3$. Students should remember that the parabola shifts 2 units right and 3 units up. When the translated sine and cosine functions are introduced, point out the similarities: a value in the brackets shifts the graph left or right, while a value at the end of the equation shifts the graph up or down.
- Explain to students that in section 2.1 sine and cosine functions were graphed using many points. Students need to plot only five key points. The five key points are the start, middle, and end of one cycle, and the maximum and minimum points of the cycle.
- When shifting a graph vertically, encourage students to draw a dotted horizontal line through the centre of the graph.
- Encourage students to use the terms *phase shift* and *horizontal translation* interchangeably.
- Encourage students to use the words *vertical shift* and *vertical translation* interchangeably.
- As an extension, have students look at the graph for the **Example** and give one or two other valid equations for it.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- **Questions 1** and **2** will help students become comfortable using a graphing calculator and communicating clearly. Have students list the key features they are expected to find before attempting part **c**).
- Encourage students to use coloured pens, pencils, or markers to draw and label graphs of the functions to which vertical translations or horizontal translations have been applied.
- When using graphing calculators, remind students to set the calculator to degree mode and use a suitable window to display the graph.
- For **question 3**, students may find it helps to rewrite a sinusoidal function such as $y = \sin(x + 45°)$ as $y = \sin(x - (-45°))$, to emphasize that although the sign between the x and the 45° is positive, the function actually shifts to the left.
- For **question 7**, have students describe the key features of sinusoidal functions. Encourage them to explain how the intercepts and the maximum and minimum values of the sinusoidal function change when the function is translated.

Common Errors

- Some students use the wrong direction for the horizontal translation.

R_x Have students make up a mnemonic to help them remember that $(x + d)$ means shift to the left, and $(x - d)$ means shift to the right.

Differentiated Instruction

- Add the **Key Concepts** to the **word wall**.
- Have students make up their own sine function that includes a vertical and a horizontal shift. Have them sketch it neatly on grid paper. Using the **inside/outside circle**, each student shares his or her diagram and the other student describes the transformation in words and with a possible equation. Remind students that there are various correct values for d for a given diagram. Rotate the outside circle until each pair has met and exchanged diagrams and equations.

- Note that for **questions 8** and **9**, students are expected to draw two complete cycles. Encourage students to plan in advance how to choose convenient scales when graphing translated functions. Ensure students label the x-values at the beginning, middle, and end of each cycle, and the x-values where the maxima and minima occur.
- To help students graph the sine functions and cosine functions that have been translated, it may be helpful to have students create tables of values with five key points for the base function $y = \sin x$ or $y = \cos x$, and then create a second table of values with five key points for the translated function.
- All students should complete **questions 14** and **15**.

Technology Suggestions

- You may wish to have copies of **T–4 The TI-Nspire™ CAS Calculator** available.
- You may wish to have students use a TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to complete **questions 1** to **4**. Students should compare the graphs of the translated functions to the graph of $y = \sin x$ or $y = \cos x$.
- For **questions 5** to **9**, set Xmax = 720 to allow for two cycles of the transformed functions to be seen.
- For **question 11**, encourage students to graph the equations they develop, and compare the graphs to the descriptions of the functions given in the question. You may wish to have students use **T2–4 How to Do Section 2.2 #11 and 12 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS**.
- When graphing the function for **question 14a**) using TI-83 Plus/TI-84 Plus, there will appear to be gaps because of the pixel resolution of the screen. The curve intersects the x-axis at multiples of 0° and multiples of 180°, but the x-intercepts are not visible because the pixel width for drawing the curve is too wide to plot accurately. Suggest that students construct the graph for 0° to 180°.
- When graphing the function for **question 15a**) using TI-83 Plus/TI-84 Plus, there will appear to be gaps because of the pixel resolution of the screen. Reducing Xmax to 180° will permit students to see that the curve intersects the x-axis.
- For videos that demonstrate translations of sine and cosine functions, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Ongoing Assessment

Ask students questions, such as

- What questions did you find easy? difficult? Why?
- How often did you check the **Example** in the Study Guide and Exercise Book to help you with questions? For which questions?

Process Expectation	Selected Questions
Problem Solving	n/a
Reasoning and Proving	14, 15
Reflecting	7, 10, 12
Selecting Tools and Computational Strategies	14, 15
Connecting	10
Representing	13
Communicating	7, 10, 12–15