- grid paper
- graphing calculator

Related Resources

- G–5 Trigonometric Graph Paper
- T–4 The TI-Nspire™ CAS Calculator
- T2–5 How to Do Section 2.3 #11 and 12 Using TI-Nspire™ CAS

Key Terms

- reflection
- stretch
- compression
- absolute value

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Common Errors

- Some students confuse the meaning of *p* and *k*.

R$_x$ Review the definition of period: the horizontal length of one cycle of the graph. The *k*-value is not the period; it is a variable that helps to find the period. The *k*-value indicates how many complete cycles of the sine and cosine function are found on the interval $0° \leq x \leq 360°$.

Differentiated Instruction

- Add **Key Concepts** to the **word wall**. Include sketches as well.
- Use **think-pair-share** for **question 4**, to improve students' communication skills and to double-check their answers.

Teaching Suggestions

Key Concepts

- You may need to explain the meaning of the absolute value signs.
- Ask students to explain the meanings of the variables *a* and *k* in their own words. For *a*, they may wish to say that the height of one wave will be higher or lower than usual. They may say that the *k*-value indicates how many cycles of the curve are found within 360°.
- Ask students to explain vertical stretch/compression and horizontal stretch/compression in their own words.
- Have students add the **Key Concepts** to their chapter reference sheets.

Example

- Ask students how many cycles are shown in each case. Note that the *k*-value indicates how many cycles there will be on the interval $0° \leq x \leq 360°$.
- If *k* is a fraction between 0 and 1, the graph is horizontally stretched by $\frac{1}{k}$, which is a number greater than 1. Review with students that dividing 360° by a fraction will result in a period of greater than 360°.
- Extend the **Example** by examining $h(x) = -3 \cos 2x$. Have students predict how the graph of the function changes. Have them connect this reflection with their previous knowledge of quadratics: a negative sign in front of the function $y = x^2$ will cause the parabola to be reflected in the *x*-axis.
- As an extension, do an example that includes a reflection in the *y*-axis, since this is a concept students have not learned in previous years.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- Before sketching graphs for this section, have students think about how to determine convenient scales, particularly with horizontal stretches and compressions.
- Encourage students to identify the values *a*, *k*, *d*, and *c* in their solutions.
- For **question 1**, remind students that amplitude is always positive.
- For **questions 1** and **2**, it may be helpful for students to use a mirror to demonstrate the reflection of the cosine functions in the *x*- and *y*-axes.
- For **questions 2c**) and **d**), students should recognize that a reflection in the *y*-axis does not affect the *k*-value or the period.

- When determining the horizontal stretch or horizontal compression, encourage students to write the steps as a part of their solution. For example, if $k = \frac{1}{2}$, then the horizontal stretch is

$$\begin{aligned}\frac{360°}{|k|} &= \frac{360°}{\left|\frac{1}{2}\right|} \\ &= \frac{360°}{\frac{1}{2}} \\ &= 360° \div \frac{1}{2} \\ &= 360° \times \frac{2}{1} \\ &= 720\end{aligned}$$

- To help students graph the sine and cosine functions that have been stretched or compressed horizontally and/or vertically, it may be helpful to have students create a table of values with five key points for the base function, and then create a second table of values with five key points for the stretched or compressed function.
- When sketching two cycles of a graph for **questions 6** to **8**, students may begin at any x-value they wish.
- As an extension, you may wish to sketch two different sine functions with amplitude 3 on the interval $-780° \leq x \leq -560°$, and ask students to write an equation for each.
- For technology **questions 4**, **5**, and **9**, remind students to check that the graphing calculator is in degree mode and the window values are set appropriately.
- For questions such as **question 11**, some students may have difficulty determining the equations of a sine or cosine function when given information about the function. Remind students to list the key features clearly and show steps as a part of their solution.
- As an extension to **questions 14** and **15**, you may wish to have students answer questions such as this: A cosine function has triple the period and half the amplitude of $y = 3 \sin (0.4x)$. Write an equation of the cosine function. Graph both functions on the interval $0° \leq x \leq 360°$, and find the number of points of intersection of the two graphs.
- For **question 16**, the units for the x-axis are not degrees, but days. As an extension, students may wish to calculate how many days old they are, and when their next good day will occur. You may wish to review how many days there are in each month of the year. Have students use a graphing calculator to assist them.
- Challenge students to use the Internet to learn more about the applications of sinusoidal functions in real-world situations.

Technology Suggestions

- You may wish to have copies of **T–4 The TI-Nspire™ CAS Calculator** available.
- For **question 1**, students may use TI-83 Plus/TI-84 Plus to help them understand how changing the value of a affects the graph of the function. Press **WINDOW** and enter Xmin = 0, Xmax = 360, Xscl = 90, Ymin = −4, Ymax = 4, and Yscl = 1. You may wish to have students compare the graphs for parts **a**) and **e**) to see the effect of the negative a-value.

- Using TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus for **question 2** will help students understand that the k-value helps determine the number of cycles on the interval $0° \leq x \leq 360°$.
- For **questions 2** and **3**, students can use TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to graph $y = \sin x$ or $y = \cos x$ with the given function. Comparing the graphs will help students determine how changing the parameters affects the graph.
- For **questions 11** and **12**, encourage students to graph the equations they develop and compare the graphs to the descriptions of the functions given in the question.
- Students may benefit from using **T2–5 How to Do Section 2.3 #11 and 12 Using TI-Nspire™ CAS** as they work on **questions 11** and **12**.
- For **question 14**, students may wish to use TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to graph the functions and determine the number of points of intersection. Students may need help determining how doubling the period affects the equation.
- For **question 16**, students may wish to use TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to graph the three functions for 150 days. For TI-Nspire™ CAS, it is not necessary to enter the degree symbol since the independent variable is not measured in degrees. Appropriate window settings are XMin = 0, XMax = 150, XScl = 30, YMin = −1.1, YMax = 1.1, and YScl = 0.5.
- For movies that demonstrate stretches, compressions, and reflections of sine and cosine functions, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	16
Reasoning and Proving	14, 15
Reflecting	3, 11–13
Selecting Tools and Computational Strategies	14–16
Connecting	11, 12, 16
Representing	14, 15
Communicating	2, 3–5, 10, 14–16

Ongoing Assessment

- Use Assessment Masters **A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.