2.4 Combining Transformations of Sinusoidal Functions

Study Guide and Exercise Book Pages
33 to 35

Tools
- grid paper
- graphing calculator
- coloured pens, pencils, or markers

Related Resources
- G–5 Trigonometric Graph Paper
- T–4 The TI-Nspire™ CAS Calculator
- A–6 Representing

Teaching Suggestions

Key Concepts

- Have students complete a simple example with all transformations on their chapter reference sheet.
- Have students include an example with a reflection in the x- and y-axes on their reference sheets.
- Have students explain, using proper terminology, what the variables a, k, d, and c stand for.

Example

- Remind students that if the equation in the **Example** were $g(x) = -2\sin[3(x + 30°)] - 1$, the d-value would be negative.
- Have students draw a sketch of the transformed function.
- You may wish to show students a variety of methods for sketching functions with more than one transformation:
  - **Method 1:** Start with a sketch of $y = \sin x$. Determine the new period and amplitude. Draw a sine graph with the new period and the new amplitude. Reflect this graph in the x- or y-axis if necessary. Label the five key points. Translate each key point 30° right and one unit down.
  - **Method 2:** Write the five key ordered pairs for $y = \sin x$: (0, 0), (90°, 1), (180°, 0), (270°, −1), (360°, 0). Multiply each y-value by a. Divide all x-values by k. Add d to all x-values. Add c to all y-values. Graph the five resulting points.
  - **Method 3:** Sketch an upside-down sine curve on an unlabelled grid. Label the highest y-value $(|a| + c)$. Label the lowest y-value $(-|a| + c)$. The cycle begins at $x = d°$ and ends at $x = \left(d + \frac{360°}{k}\right)$. The halfway point is the mean of the first two x-values.
- Review absolute value with students. Remind students that a negative value for a indicates a reflection in the x-axis.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- When using graphing calculators, remind students to set the calculator to degree mode and use a suitable window to display the graph.
- Ensure that students have equal divisions and accurate scales on their graphs.
- Using different colours may help students to visualize the transformations more clearly.
- For **questions 4** and **5**, have students double-check that their phase shift is in the correct direction.
- Some students may state that the amplitude for **question 4b**) is −4; review the concept of absolute value, and explain why amplitude cannot be negative.
- For **questions 6** and **7**, students may have difficulty determining the range of a sinusoidal function. Have students draw horizontal lines through the maximum and minimum points and state the maximum and minimum values.
- As an extension to **questions 6** and **7**, have students model the final graph with a cosine function.

Common Errors

- Some students apply transformations in the wrong order.

R_x Remind students that when graphing, they must follow the order of operations: multiplying and dividing first, adding and subtracting last (stretches, compressions, and reflections first, translations last). Encourage students to check their answers with graphing technology.

Differentiated Instruction

- Use **placemat** to summarize the steps in graphing transformations.
- Have students work in **cooperative task groups** and create their own sine or cosine graph on chart paper. Have them determine the equation. You may wish to use a modified **carousel** to display the diagrams and have groups circulate to check the equations and add a different equation that models the same graph.

- For **question 11**, students must factor the expression in brackets to determine the d-value.
- Remind students that functions are written in the form $f(x) = 4 \sin [2(x + 45°)] + 3$ to help identify the phase shift.
- Review x- and y-intercepts. Remind students that for **questions 10** and **12**, they can find the y-intercept by setting $x = 0$ and solving for y.
- As an extension to **questions 10** and **12**, you may wish to find the x-intercepts algebraically by setting $y = 0$. This method finds one x-intercept. Students must use reasoning to determine the other x-intercepts.
- For **question 17**, you may wish to tell students that most real-world applications of sinusoidal functions have units other than degrees along the x-axis. Ensure students understand that in this context x is the horizontal distance, not a rotation angle, and therefore it is not reasonably measured in degrees.
- Encourage students to challenge themselves by creating a problem of their own that is similar to **question 17**.

Technology Suggestions

- You may wish to have copies of **T–4 The TI-Nspire™ CAS Calculator** available.
- Some students may wish to use TI-Nspire™ CAS to complete **question 12c**). Press ⌂. Choose **Graphs & Geometry**. Type the expression $4 \sin (2x° + 30) - 2$ in the command prompt. Press menu. Choose **Window**, and then **Window Settings**. Set XMin = 0, XMax = 360, XScale = 90, YMin = −7, YMax = 3, and YScale = 1. Press menu. Choose **Points & Lines** and **Point On**. Click on the curve twice near the x-intercepts closest to the y-axis and two points will appear. Drag the points to the x-intercept and note their values. Use another **Graphs & Geometry** page. Compare the x-intercepts for $y = 4 \sin 2x° - 2$ to those for $y = 4 \sin (2x° + 30) - 2$.
- Some students may wish to use TI-83 Plus/TI-84 Plus to complete **question 12c**). Press **Y=**. Enter the expression $4 \sin (2x + 30) - 2$ in Y1. Press **WINDOW**. Set Xmin = 0, Xmax = 360, Xscale = 90, Ymin = −7, Ymax = 3, and Yscale = 1. Press **2nd**, **TRACE**, **2:zero**. Move the cursor to the left of the first x-intercept and press **ENTER**. Move the cursor to the right of the x-intercept and press **ENTER** twice. Repeat this process for the second x-intercept. Record the values. Clear the equation in Y1. Type the expression $4 \sin 2x - 2$. Find the x-intercepts. Compare the x-intercepts for the two functions.
- For applets that students can use to explore transformations of sine and cosine functions, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Ongoing Assessment

- Provide students with some graphs of transformed sine and cosine functions. Have them write two possible equations for each graph. Have students calculate the y-intercepts for each graph using each equation, and then state the domain and range of each function. Use **A–6 Representing** to assess students.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	n/a
Reasoning and Proving	n/a
Reflecting	10, 11
Selecting Tools and Computational Strategies	15, 16
Connecting	10, 11
Representing	17
Communicating	8, 9, 18, 19